A Study of Evolving Optical Caustics Formed by Evaporating Water Droplets

Samantha Scibelli

Laser Teaching Center, Department of Physics and Astronomy

Stony Brook University

INTRODUCTION

OPTICAL CAUSTICS

- Optical caustics are concentrated envelopes of light rays reflected or refracted by a curved surface of an object, or the projection of that envelope of rays on another surface
- Familiar examples of caustics: rainbows, bright lines in water droplets and on the bottom of a swimming pools, and the sharp light curves formed by a wine glass or coffee mug (Fig. 1)

Fig. 1 – The image to the left is a photograph of caustics reflecting off of a soda bottle. The image to the right was taken from the Optics Picture of the Day website that shows rapidly moving caustics at the bottom of a stream and the distortion of a trout caught by the caustics.

- The same organization patterns of caustics explain the twinkling of stars and underlie the phenomenon of gravitational lensing, which makes visible to us the most distant objects in the universe
- An optical caustic can be referred to more precisely as the locus of the points at which the intensity of the deflected rays becomes infinite [2]
- Experimental example:
  - When rays from a distant point source of light pass through an irregular drop on a dirty glass, they come to focus not at a point but in a system of caustics [5]
  - These seemingly random patterns share an underlying mathematical structure

CATASTROPHE THEORY

- Catastrophe theory, developed by the French mathematician Rene Thom in the 1970's, is the study of how changing control variables leads to qualitative changes in the solutions of an equation
  - It has been applied to different phenomena, such as the stability of ships and bridge collapses; optical caustics, however, are one of the best visual illustrations of how it can be applied
  - Berry [7] first applied the theory to caustics seen by evaporating water droplets
  - Later, Nye [5] performed an experiment of his own looking at water drop caustics in the near field
- For any given number of dimensions in control space, Thom's theorem states that there are a limited number of types of catastrophes [3]
- There are seven 'elementary catastrophes' in control space less than or equal to four (Table 1) [8]
- The degree of polynomial in x and y relates to the max number of light rays which can focus to form a caustic [1]
- Due to their restricted nature, catastrophes can be classified based on how many control parameters are being simultaneously varied
- For example, if there are two controls then the catastrophe known as a cusp is formed
  - For a cusp the standard form looks like:

$$\varphi = \pm(x^4 + ax^2 + bx) + y^2$$

Table 1

The first seven elementary catastrophes

Name	Symbol	Crank	Codimension	Germ	Unfolding terms
Fold	A_2	1	1	$x^3 + y^2$	x
Cusp	A_3^+	1	2	$\pm x^4 + y^2$	x^2, y
Swallowtail	A_4	1	3	$x^5 + y^2$	x^3, x^2, x
Butterfly	A_5^+	1	4	$\pm x^6 + y^2$	x^4, x^3, x^2, x
Hyperbolic umbilic	D_4^+	2	3	$x^3 + xy^2$	$x^2 + y^2$, x, y
Elliptic umbilic	D_4^-	2	3	$x^3 - xy^2$	$x^2 + y^2$, x, y
Parabolic umbilic	D_5	2	4	$x^4 - xy^2$	x^2, y^2, x, y

PURPOSE

- The evolution of caustics from evaporating water droplets provides a good test case where a comparison between the observed features and the mathematical model of the caustics analyzed can be performed [6]
- Studying the formation of these caustics gives an insight into the natural organization and structure of light fields

EXPERIMENTAL METHODS

FAR FIELD VIEW

- When laser light passes through a water droplet the perturbations produce a far field caustic, known as the parabolic umbilic in the catastrophe theory classification [2]
- Laser beam (λ = 633nm) was expanded to about 6mm in diameter with two lenses

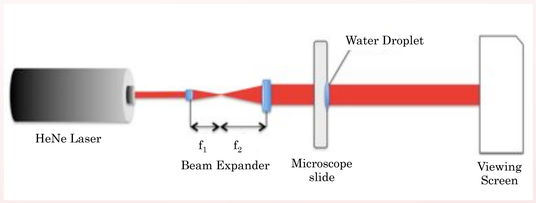

Fig. 2 – Diagram of the far field set-up used for this experiment.

MICROSCOPE SLIDE SETUP

- Microscope slide was placed vertically for the far field viewing method (Fig. 3)
- Pipette was used to place a water droplet close to 6mm in diameter onto dirty slide with 6mm pre-made circular hole outlines about 0.1mm thick (Fig. 4) for each trial
- Slide was illuminated on the side opposite the water droplet with a horizontal broadened laser beam
- At a region about 40cm away caustic features of the parabolic umbilic catastrophe could be seen on a viewing screen

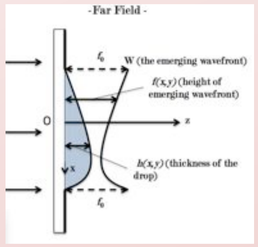

Fig. 3 – A drop of water is in a circular traced hole on a vertical glass slide.

Fig. 4 – The microscope slide used to hold the water droplets. The drops were placed in the hole circled in red.

DATA

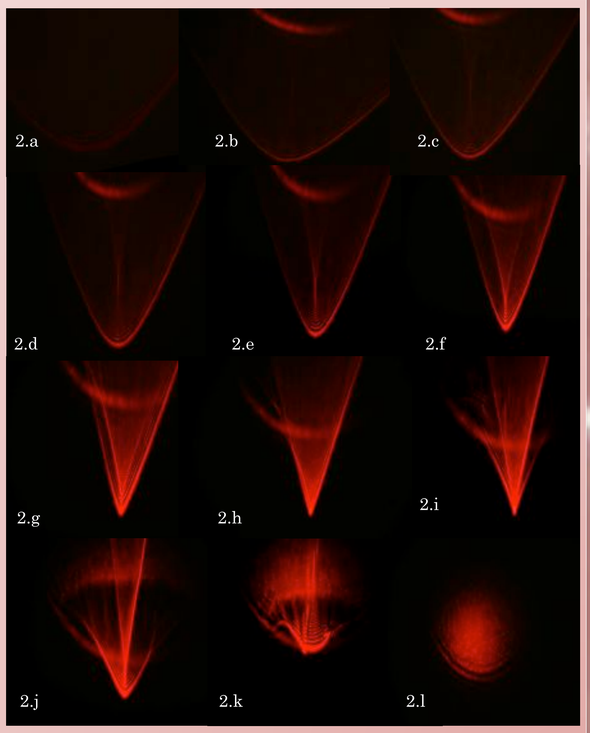

Fig. 5 – Evolution of caustics from evaporating water droplets for trial 2 (2.a – 2.l).

RESULTS

Table 2

Trial #2

Time (sec)	Observations	Picture number
0	-Fold caustic present	2.a
600	- Elliptic umbilic forming	2.b
960	-More pronounced elliptic forming	2.c
1380	-Triangle pattern (elliptic umbilic) pronounced	2.d
1800	-Elliptic moves towards fold	2.e
2280	-Combining of elliptic and fold which turns to cusp	2.f
2580	-Elliptic engulfed into cusp at point not complete	2.g
2760	-Elliptic completely engulfed to cusp point	2.h
2490	-Starts to transform back into elliptic and cusp	2.i
3180	-Elliptic moving away from cusp- fold now	2.j
3360	-Transforms back into fold until caustic disappears	2.k
3420	-Evaporation complete	2.l

- Total of six droplets were observed, only the second trial is shown
- Fig. 5 show pictures of the caustic evolution for the second trial
- Table 2 includes the time, initial observations, and picture number for each transition a caustic had for the second trial that corresponds with the images in Fig. 5
- Each transition sequence for all trials was similar:
  - Fold caustic was first present
  - Diffraction stars (also called elliptic umbilic foci) present
  - Elliptic umbilic would slowly merge into fold
  - Fold and Elliptic umbilic would merge into cusp
  - Process would reverse to a fold until water droplet completely evaporated
- Note: when elliptic umbilic focus lies exactly on the fold this is the parabolic singularity
- This *entire* process is part of the unfolding of the parabolic umbilic catastrophe

ANALYSIS

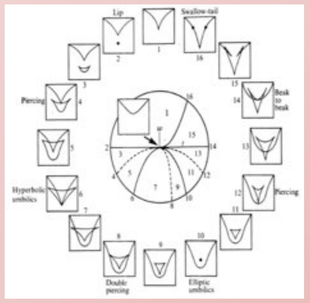

Fig. 6 – Unfoldings of the parabolic umbilic (D_5) from Nye 1978, adapted from Thom 1972.

- Entire evolution observed shows parabolic umbilic catastrophe unfolding, not every step is seen visually because some are blurred out by diffraction (Fig. 6)
- Parabolic umbilic catastrophe unfolding eqn.: $F(x, y, a, b, c, d) = x^4 + xy^2 + ax^2 + by^2 + cx + dy$ of its germ, $f(x,y) = x^4 + xy^2$
- Goal was to derive the caustic curve for each catastrophe by manipulating the generating function (the first three have been studied so far)
- Generating function formed from germ and unfolding terms (see Table 1)
- Each catastrophe may have similar unfolding terms, but each germ is unique to each shape
- Caustic curve derived by setting the 2nd derivative of the generating function, or the formation of the caustic, equal to zero and solving for control variables
- The eqns. were derived and plotted in Mathematica
- Graphs of eqns. for Cusp catastrophe shown here (Fig. 7)

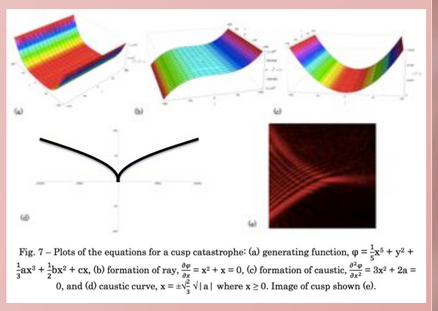

Fig. 7 – Plots of the equations for a cusp catastrophe: (a) generating function, $\varphi = \frac{1}{5}x^5 + y^2 + \frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx$, (b) formation of ray, $\frac{\partial \varphi}{\partial x} = x^2 + x = 0$, (c) formation of caustic, $\frac{\partial^2 \varphi}{\partial x^2} = 3x^2 + 2a = 0$, and (d) caustic curve, $x = \pm\sqrt{\frac{2}{3}}\sqrt{|a|}$ where $x \geq 0$. Image of cusp shown (e).

DISCUSSION

- The evolution of the caustics formed from the evaporating water droplets do follow a similar pattern described by Lock [2], Nye [4], and Berry [8].
- Within these similar patterns, however, the times as well as the appearance of the caustic vary from transformation to transformation
  - Size of the droplet affects how the caustic develops and what it looks like at the beginning of its transition
- The dynamic nature of the caustic formed does not affect the internal structure of the caustic analyzed
- What's striking is that even though each drop is unique, its evolution takes place through the same sequence of steps according to the unfolding process of the parabolic umbilic
- By analyzing the curves a better understanding of how caustics evolved in this experiment as well as the structure of light's overall natural focusing power can be achieved

FUTURE STUDIES

- Study of far field caustic behavior of evaporating water droplets (Fig. 8)
- A clearer understanding of the caustic curves for higher dimension catastrophes
- Investigation of caustics of higher order singularities (ex: symbolic umbilic E_6)

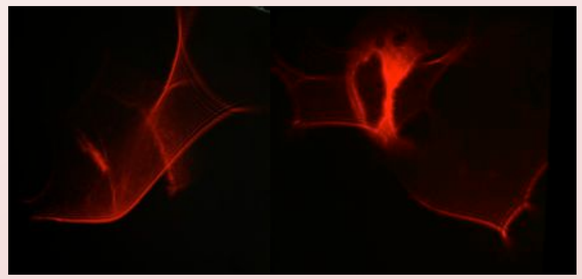

Fig. 8 – Images of separate water drop caustics that formed in the far field as the size of the water droplet changed rapidly as it was absorbed by surrounding tape.

- Investigation of the caustic shapes from the near field view (Fig. 9) where microscope slide is place horizontally and a microscope is used
  - Elliptic umbilic foci can be visually seen evolving in the near field

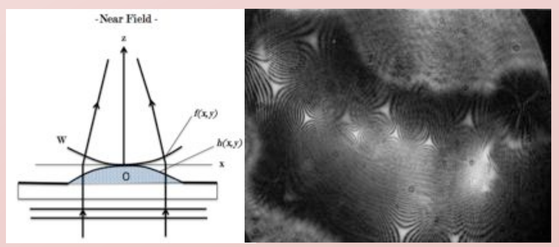

Fig. 9 – The diagram on the left is of the microscope slide setup for the near field. The image on the right is of an irregularly shaped drop of water as it evaporates in the near filed and elliptic umbilic foci appear [5].

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