Intense gamma-ray generation for a polarised positron beam in a linear collider

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Gamma-ray generation by Compton backscattering in an optical lens series with periodic focal points is considered to produce a polarised positron beam for a linear collider. The lens series is composed of 20 unit cells with a length of 210 mm. Each lens has a hole to pass an electron beam with an energy of 5.8 GeV and the generated $\gamma$-rays. It is shown by diffraction analysis that laser beam loss in the series is very small, and the beam size is periodically reduced to 26 $\mu$m. Electron beam size is reduced to 34 $\mu$m in a superconducting solenoid with a field of 15 T. To get a required $\gamma$-ray yield of $7 \times 10^{15}$ $\gamma$/s, only one circularly polarised CO$_2$ laser source with a power of 24 kW is needed.

Key words: Polarised gamma-rays, Compton backscattering, optical lens series, CO$_2$ laser, linear collider

I. INTRODUCTION

Intense circularly polarised $\gamma$-rays, generated by colliding a circularly polarised laser beam with a high energy electron beam, can be used to produce a polarised positron beam for linear colliders by colliding the $\gamma$-rays with a metal target. Such a $\gamma$-ray generation was originally proposed to pass a very high energy electron beam through a helical undulator [1]. Recently, it was proposed to use 40 sets of 3.2 kW CO$_2$ lasers to provide the power into 40 sets of parabolic mirrors for the collision [2], which is here referred as Proposal I.

Since good optical lenses are commercially available at a wavelength $\lambda_s=10.6$ mm, we consider to pass a pulsed laser beam through an optical lens series with periodic focal points, where the collision is made with a bunched electron beam. In a simple ideal case of no laser beam loss through the lens series and the same sizes of electron and laser beams with those of Proposal I, only one laser source of 4.7 kW is needed for the required $\gamma$-ray yield $Y=7 \times 10^{15}$ $\gamma$/s. In the present paper, we discuss the transmission and profile of the laser beam in a lens series as well as the focusing of the electron beam, and estimate the laser power required for the $\gamma$-ray yield.

Figure 1 shows the schematic structure of the present optical system, which is composed of $N_c=20$ unit cells of focusing lenses with a cell length $L_c=210$ mm. Electron bunches and laser pulses in trains are both separated by 1.4 ns or 420 mm.

II. LASER BEAM OPTICS

A. Aberration

In a geometric optics, a light emitted from a point through a lens is focused at a point if the light ray is close to the beam axis. The light ray in off-axis, however, goes to different
position or spreads in the focal plane, thus an aberration is induced. Among several kinds of aberrations, only the spherical aberration is considered here, since the profile in the focal plane is very small in the present case.

The spherical aberration depends on the curvature of lens surface. Figure 2 shows the aberration $W_{1s}$ and $W_{2M}$ calculated for a conventional lens and a doublet with a form of Meniscus lens, respectively. As shown in the figure, the aberration can be fitted by $W(v \mu m) = \xi v$, with $v \mu m$ the distance from the beam axis. We get $\xi = -9.06 \times 10^{-4}$ and $-2.12 \times 10^{-4}$ for $W_{1s}$ and $W_{2M}$, respectively. As discussed later, the effects of aberration on the profile even with $\xi = \pm 9.06 \times 10^{-4}$ are rather large. In addition, the effects of aberration on the profile are algebraically accumulated in the lens series, so that it is necessary to reduce the aberration significantly.

The aberration $W_{1s}$ and $W_{2M}$ are negative and become larger for a larger $v$. If the radius of surface curvature is gradually increased for a larger $v$, the aberration can be reduced in principle to zero, as shown by $W_{1m}$ in the figure. If the aberration is reduced to $\xi \approx \pm 1.0 \times 10^{-4}$, for instance, the total aberration in the 20 cells might be reduced to about $\xi \approx \pm 5 \times 10^{-4}$.

### B. Diffraction analysis

As shown in Fig. 3, the initial laser beam with a Gaussian distribution with a size $\sigma_{r00} = 2$ mm is expanded with a circular cone to a hollow beam with a hollow radius $r_0 = 3$ mm, deflected by 45 degree with a mirror, and injected into the lens series. Each lens has an aperture $D_a = 2r_2 = 30$ mm with a hole diameter $D_h = 2r_1 = 6$ mm to pass the electron beam and the generated $\gamma$-rays with an energy higher than 1 MeV. The amplitude function of the hollow beam is given by

$$U_0(r) = \frac{1}{\sqrt{2\pi \sigma_{r00}}} \sqrt{\frac{r}{r - r_0}} \exp\left[-\frac{(r - r_0)^2}{4 \sigma_{r00}^2}\right]. \quad (1)$$

Applying the Fresnel-Kirchhoff diffraction analysis, we have the following amplitude function $U_1$ at the first focal point,

$$U_1(\rho, Z) = \frac{2\pi \Gamma(\rho, Z)}{\lambda L} \int J_0(Z_0)U_0(r) \exp\left(-\frac{ikr^2Z}{2L'}\right) \exp[-ikW(r)] r dr,$$  \quad (2)

with

$$\Gamma(\rho, Z) = \exp(-ikZ) \exp[-\frac{ik}{2L'}(X^2 + Y^2 + Z^2)].$$
where \( \rho^2 = X^2 + Y^2 \), \( J_0(Z_0) \) is the zeroth Bessel function with \( Z_0 = k \rho / L' \), \( k = 2 \pi / \lambda \), and \( L' \) is the distance between the lens and the focal point. Intensity distribution of the profile and the integrated intensity are given by

\[
I_1(\rho, Z) = |U_1(\rho, Z)|^2, \quad S_1(Z) = \int I_1(\rho, Z) \rho \, d\rho.
\]

The function \( U_2(\rho, Z) \) at the second focal point is given by

\[
U_2(\rho, Z) = \left( \frac{(2\pi)^2 \Gamma(\rho, Z)}{(\lambda_s L')^2} \right) \int \left[ U_1(r) \exp \left( \frac{ik_s \rho^2}{2L'} (1 + \frac{Z}{L'}) \right) \right] \\
\times J_0(Z_1)J_0(Z_2) \exp \left( \frac{ik_s r^2 Z}{2L'^2} \right) \exp \left[ -ik_s W(r) \right] r \, dr \, d\rho.
\]

where \( Z_1 = k_1 \rho / L' \) and \( Z_2 = (k_1 \rho / L')(1+Z/L') \).

Applying Fourier transformation on the function \( U_n(X, Y, Z=0) \Gamma(X, Y, Z=0) \), we get

\[
F_n(k_x, k_y) = |P(u', v')| \exp \left[ -ink_s W(u', v') \right] U_0(u', v'),
\]

where \( u' = k L'/k_1 \), \( v' = k L'/k_2 \) and \( |P(u, v)| \) is the pupil function of the lenses, which is given by \( |P(u, v)| = 1 \) for \( r_1 < (u+v)^{1/2} < r_2 \) and 0 for others.

The function \( U_n(X, Y, 0) \) at the n-th focal point can be obtained by the inverse transformation. Equation (4) indicates that the aberration effects on the profiles are accumulated algebraically. In addition, we have generally the following relation,

\[
\iint |F_n(k_x, k_y)|^2 \, dk_x \, dk_y = \iint |U_n(X, Y, 0)|^2 \, dX \, dY.
\]

Substituting \( F_n(k_x, k_y) \) of Eq.(4) into the above equation, we find that the integrated intensity of the profile is conserved through the lens series even if there are aberrations.

The focal strength of the first lens is half of that of following lenses downstream. Dividing each lens into two parts except for the first lens, we have a completely periodic structure with symmetric unit cells. Since the focal points are at the centre of each cell, a symmetric amplitude function is expected in all the cells in the case of no aberration, which implies no beam loss in that case. In the presence of aberration, we also expect no beam loss by the consideration of Fourier transformation.

### C. Numerical results

Figure 4 shows the profiles, normalised by the peak value at \( X = Z = 0 \), at the first focal point for various \( Z \) in the case of no aberration of the first lens or \( \xi_1 = 0 \). The profiles are symmetric for \( \pm X \) because of axial symmetry, represent nearly Gaussian distribution for all \( Z \), and decrease nearly to zero around \( X \approx 80 \) mm. As shown by various lines, the profiles can be fitted well by
\[ I_p(X) = a(Z) + b(Z)\exp[-\frac{X^2}{2\sigma_r(Z)^2}]. \quad (6) \]

The parameters \(a(Z), b(Z)\) and \(\sigma_r(Z)\) against \(Z\) are shown in Fig. 5. The parameters are symmetric for \(\pm Z\), and the beam size \(\sigma_r(Z)\) has its minimum \(\sigma_{r0} = 26 \mu m\) at \(Z = 0\), and increases gradually with \(Z\). The peak value \(I_{np}(Z)\) for different \(Z\) is nearly equal to \(b(Z)\) since \(a(Z)\) is very small. The parameters are fitted further by the polynomials of \(Z\) up to the ninth power, as shown with solid lines, which are used for the calculation of collision rate.

Figure 6 shows the parameters at the first focal point in the presence of aberration \(\xi_1 = 9.06 \times 10^{-4}\), which is the magnitude for a conventional lens as given above. The figure indicates that the beam sizes are nearly the same as those for no aberration, but the minimum position of \(\sigma_r(Z)\) is shifted to \(Z \approx 2.8\) mm.

Characteristic features of the profiles at the second focal point can be observed by the peak values \(I_{np}(Z)\), shown in Fig. 7. The peak values for no aberration and those for \(\xi_1 = 9.06 \times 10^{-4}\) and \(\xi_2 = 0\) are almost the same as those at the first focal point, where \(\xi_2\) is the aberration coefficient of the second lens. In the case of \(\xi_1 = \xi_2 = -4.53 \times 10^{-4}\), the peak values are the same as those at the first focal point for \(\xi_1 = 9.06 \times 10^{-4}\), but reversed with respect to \(Z=0\). By comparing the broken and dotted lines in the figure, we observe that the aberration effects are accumulated.

The integrated intensity for \(\rho = 0\) to \(5\) mm is \(S_1 = 0.99979\) and \(S_2 = 0.99351\) at the first and the second focal point, respectively, in the case of no aberration. When \(D_h\) of the second lens is decreased to 5, 4 and 0 mm, we have \(S_1 = 0.99810, 0.99904\) and \(0.99948\), respectively. Since we expect \(S_2 = 1.000\) for \(D_h = 6\) mm by the symmetry consideration, the difference is thought to be ascribed to numerical errors. In the presence of aberration, \(\xi_1 = 9.06 \times 10^{-4}\) and \(-9.06 \times 10^{-4}\), we have \(S_2 = 0.99394\) and \(0.99343\), respectively, which are almost the same as that for no aberration. Therefore, the beam loss by diffraction is negligibly small even in the presence of aberration. To be more decisive, however, it is necessary to investigate the reason of numerical differences.

### III. ELECTRON BEAM FOCUSING

The electron beam with an energy of 5.8 GeV is focused by a superconducting solenoid with a field of 15 T. The beam is first focused by quadrupole magnets in the injection beam line to a minimum size \(\sigma_{x0} = \sigma_{y0} = 34 \mu m\) at the entrance of the solenoid. Then the beam size is kept constant in the solenoid. Rough dimensions of the solenoid are shown in Fig. 8. Since the critical current of the superconducting wire made of NbSn is not very high in such a high field, a large amount of wire is needed for the solenoid.

### IV. GAMMA-RAY YIELD

The electron beam and the laser pulse are assumed to have a Gaussian distribution with a length of \(\sigma_z\) and \(\sigma_{sz}\), respectively. If the transverse sizes are constant with \(\sigma_{x0}(=\sigma_{y0})\) and \(\sigma_{r0}\), the collision rate is given by
\[ N_{t0} = \frac{n_e n_s \sigma_T}{2\pi (\sigma_{x0}^2 + \sigma_{r0}^2)} , \quad (7) \]

where \( n_e \) and \( n_s \) are the number of electrons and photons per bunch and pulse, respectively, and \( \sigma_T \) is the cross section for Thomson scattering.

Taking into account the variation of the laser beam size along the \( Z \) axis, we have the following collision rate \( N_t = N_a + N_b \) between one bunch and one pulse,

\[ N_a = \frac{n_e n_s \sigma_T}{(2\pi)^2 \sigma_z \sigma_{zc}} \int \frac{a(Z)}{\sigma_z(Z)^2} \exp\left(-\frac{z^2}{2\sigma_z^2} - \frac{z'^2}{2\sigma_{zc}^2}\right) dz dz' , \quad (8-a) \]

\[ N_b = \frac{n_e n_s \sigma_T}{(2\pi)^2 \sigma_z \sigma_{zc}} \int \frac{b(Z)}{\sigma_z(Z)^2 + \sigma_r(Z)^2} \exp\left(-\frac{z^2}{2\sigma_z^2} - \frac{z'^2}{2\sigma_{zc}^2}\right) dz dz' , \quad (8-b) \]

where \( Z = (z + z')/2 \), and \( z \) and \( z' \) are taken from the centre of the electron bunch and laser pulse, respectively.

A reduction factor \( F_r \) of collision rate is introduced by \( N_t = F_r N_{t0} \). Numerically, we have \( F_r = 0.60 \) for \( \sigma_z = 0.9 \) mm and \( \sigma_{zc} = 3 \) mm for no aberration, and \( F_r = 0.41 \) for \( \xi_1 = 9.06 \times 10^{-4} \). Thus, the collision rate is considerably affected by such an aberration.

A lens, made of ZnSe, has a refraction index of 2.4028, a bulk absorption coefficient smaller than 0.05 \% / cm and a reflectivity smaller than 0.20\% by special coating.

For a matched collision between the electron bunches and the laser pulses at the focal points, the distance of bunches and pulses are both \( 2L_c \). To make the collision \( N_c \) times with each electron bunch, we need a laser pulse number \( N_b = N_c + N_b - 1 \) per train, where \( N_b \) is the number of electron bunches per train. The \( \gamma \)-ray yield is given by

\[ Y = \frac{n_e n_{s0} \eta N_b N_c f_r \sigma_T F_r}{2\pi (\sigma_{x0}^2 + \sigma_{r0}^2)} , \quad (9) \]

where \( n_{s0} \) is the initial photon number per pulse, \( \eta \) ( = 0.97) is the average transmission factor due to the lens loss by the absorption and reflection, and \( f_r \) is the repetition frequency. In order to get the required \( \gamma \)-ray yield, we need \( n_{s0} = 7.9 \times 10^{19} \) photons per pulse or a pulse energy of \( E_p = 1.5 \) J. Accordingly, a laser power \( P_L = 24 \) kW with \( N_p = 104 \) pulse per train is needed. Heat load in a lens is about 6 W. If the laser pulse length is reduced to \( \sigma_{zc} = 0.9 \) mm, we have \( F_r = 0.87 \), so that the required laser power is decreased to \( P_L = 16 \) kW. The parameters of the present work are summarised in Table I and compared with Proposal I.

V. DISCUSSION AND CONCLUSION

We have considered the case \( \sigma_0 = \sigma_1 = 3 \) mm for a critical discussion. If we set as \( \sigma_0 > \sigma_1 \), we have a safety margin to avoid a beam loss which might be induced by accidental and construction errors. Some of the scattered photons with an energy lower than 1 MeV collide...
the lenses by passing through the holes, the radiation power of which amounts to about 9 W in the end lens. It is required to investigate radiation effects on the lenses. If serious damage is induced, it is necessary to increase the hole size and install radiation shields.

In the present paper, discussions on the basic concept of lens series for an intense $\gamma$-ray generation are presented. The laser beam loss in the lens series is considered to be very small, and we can significantly reduce the laser power required for the $\gamma$-ray yield. It is valuable to make a further investigation on the present system for technical practice. This method can be applied to a similar system for enhancing the collision rate.


### TABLE I. Parameters of Compton backscattering for polarised γ-ray generation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Proposal I</th>
<th>Present work</th>
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<tbody>
<tr>
<td><strong>Electron beam</strong></td>
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<tr>
<td>Beam energy</td>
<td>E(GeV)</td>
<td>5.8</td>
</tr>
<tr>
<td>Normalized emittance</td>
<td>$\varepsilon_x$(mm mrad)</td>
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<tr>
<td>Number of electrons</td>
<td>$n_e$/bunch</td>
<td>$10^{11}$</td>
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<tr>
<td>Number of bunches</td>
<td>$N_b$/train</td>
<td>85</td>
</tr>
<tr>
<td>Bunch length</td>
<td>$\sigma_z$(mm)</td>
<td>0.9</td>
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<tr>
<td>Beam size</td>
<td>$\sigma_x$(µm)</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>$\sigma_y$(µm)</td>
<td>34</td>
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<tr>
<td><strong>CO$_2$ laser beam</strong></td>
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<tr>
<td>Wavelength</td>
<td>$\lambda$(µm)</td>
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<tr>
<td>Pulse length</td>
<td>$\sigma_\omega$(mm)</td>
<td>3 (3) (0.9)</td>
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<tr>
<td>Beam size</td>
<td>$\sigma_r$(µm)</td>
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<tr>
<td>Pulse energy</td>
<td>$E_p$(J)</td>
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<tr>
<td>Number of pulses</td>
<td>$N_p$/train</td>
<td>85</td>
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<tr>
<td>Laser power</td>
<td>$P_L$(kW)</td>
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<td>Number of lasers</td>
<td>$N_L$</td>
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<tr>
<td><strong>Compton backscattering</strong></td>
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<tr>
<td>Repetition frequency</td>
<td>$f$(Hz)</td>
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<tr>
<td>$\gamma$-ray yield</td>
<td>$Y$(γ/s)</td>
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<tr>
<td>Number of cells</td>
<td>$N_c$</td>
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<tr>
<td>Reduction factor</td>
<td>$F_r$</td>
<td>?</td>
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<tr>
<td></td>
<td></td>
<td>0.60 (0.87)</td>
</tr>
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</table>
FIG. 1. Schematic structure of the present optical system. A pulsed laser beam passes through a lens series with periodic focal points, and collides there with a bunched electron beam to generate $\gamma$-rays.

FIG. 2. Aberration $W_{1s}$, $W_{2M}$ and $W_{1m}$ for a conventional lens, a doublet with a form of Meniscus lens, and a modified surface lens, respectively.
FIG. 3. Initial part of the optical system. A Gaussian laser beam is expanded to a hollow beam and focused by lens series.

FIG. 4. Normalized laser beam profiles at the first focal point for no aberration.
FIG. 5. Parameters $a(Z)$, $b(Z)$ and $\sigma_r(Z)$ obtained by the fitting to the laser beam profiles for no aberration.
FIG. 6. Parameters $a(Z)$, $b(Z)$ and $\sigma_r(Z)$ in the presence of aberration with $\xi_1=9.06 \times 10^{-4}$.
FIG. 7. Peak value $I_{np}(Z)$ of normalised profile intensity. Solid line represents $I_{np}(Z)$ of the profiles at the first focal point for the aberration $\xi_1 = 9.06 \times 10^{-4}$ and $\xi_2 = 0$. The broken line and dotted lines represent $I_{np}(Z)$ at the second focal point for $\xi_1 = 9.06 \times 10^{-4}$ with $\xi_2 = 0$ and $\xi_1 = \xi_2 = -4.53 \times 10^{-4}$, respectively.

FIG. 8. Rough dimensions of a superconducting solenoid.