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# 3D printed hierarchical honeycombs with shape integrity under large compressive deformations

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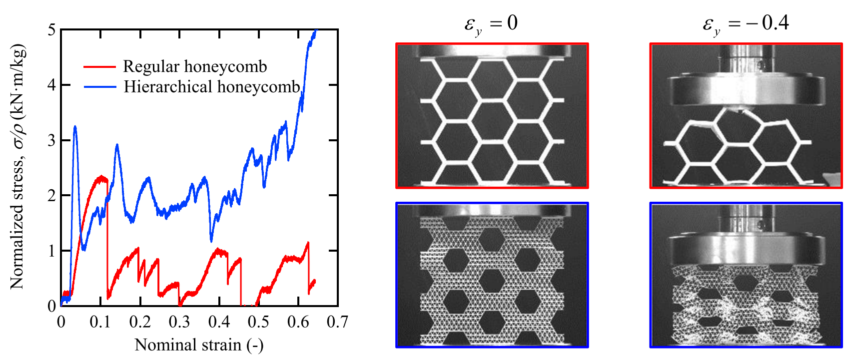
## HIGHLIGHTS

- A new class of hierarchical honeycombs was designed and fabricated using 3D printing technique.
- The hierarchical honeycomb exhibits a progressive failure mode under uniaxial compression.
- Compared with regular honeycombs, improved stiffness and energy absorption have been achieved simultaneously.
- High energy dissipation at large imposed strains (up to 60%) have also been observed under cyclic loading.

## GRAPHICAL ABSTRACT

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## ABSTRACT

We describe the in-plane compressive performance of a new type of hierarchical cellular structure created by replacing cell walls in regular honeycombs with triangular lattice configurations. The fabrication of this relatively complex material architecture with size features spanning from micrometer to centimeter is facilitated by the availability of commercial 3D printers. We apply to these hierarchical honeycombs a thermal treatment that facilitates the shape preservation and structural integrity of the structures under large compressive loading. The proposed hierarchical honeycombs exhibit a progressive failure mode, along with improved stiffness and energy absorption under uniaxial compression. High energy dissipation and shape integrity at large imposed strains (up to 60%) have also been observed in these hierarchical honeycombs under cyclic loading. Experimental and numerical studies suggest that these anomalous mechanical behaviors are attributed to the introduction of a structural hierarchy, intrinsically controlled by the cell wall slenderness of the triangular lattice and by the shape memory effect induced by the thermal and mechanical compressive treatment.



## 1. Introduction

Recent advances in the defense, aerospace, automotive, semiconductor, and energy industries have triggered a tremendous demand for high-performance materials with lightweight and enhanced mechanical properties. A typical example of this quest is the research and

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development activity in the aerospace field, for which structural components with a combination of lightweight, high stiffness, energy absorption, and fracture toughness for extreme loading conditions are continuously sought after [1,2]. On a smaller scale, electrodes for lithium-ion batteries in energy applications require porous structures that exhibit considerable damage tolerance to improve the battery safety and operational life [3,4]. Compared with bulk materials, cellular structures and lattice solids can be designed to possess all these desired properties. In this regard, natural materials such as bone [5–7] and wood [8–11] have offered inspiration to design lightweight materials with remarkable mechanical properties. One of the reasons behind the presence of unusual deformation mechanisms in these biological structural materials is because of their hierarchical cellular configuration at multiscale [12–16]. Several efforts have been devoted to synthesize open or closed foams with similar cellular structures [17–19]. Yet, specific properties such as stiffness, strength, and energy absorption are still limited by the random and stochastic microstructure of these porous materials, which result in structural defects, inhomogeneity, and local stress concentration. In addition, the stiffness of cellular structures is typically characterized by the bending deformation of their ribs, which indicates that the effective mechanical properties decrease dramatically with the decrease of relative density (cube of the relative density) [20].

Structural topologies have a significant impact on the mechanical behavior of cellular structures, and in that sense, efforts have been devoted to designing deterministic materials for new mechanical functionalities. For example, ultralight metallic micro-lattice materials that exhibit excellent energy dissipation and recoverability have been designed and fabricated [21]. Their unusual mechanical properties and recoverability are attributed to their rationally designed hierarchical structure, which ranges from the nano to the millimeter scale. Ultra-low density lattice metamaterials with similar structural features have also been introduced [22–28]. Localized fracture around the nodes of these micro-lattice materials, however, arises during the fabrication and the uniaxial compression. To circumvent this problem a new type of low-density cellular structure (shelluar) has been developed [29–31]. In contrast to the sharp shape changes around the nodes of a classical cellular topology, the shelluar configuration has a three-dimensionally continuous smooth surface that could carry the load and distribute the stress uniformly. Although this new configuration allows reaching an ultralow density range, stiffness, and strength are still low compared to other micro-lattice materials. It is worth pointing out that, besides the rationally designed architectures, these unusual properties are enabled by using state-of-the-art fabrication techniques such as two-photon lithography [22] and projection micro-stereolithography [23]. More recently, a new type of mechanical metamaterial with a combination of cubic and octet foams has shown the possibility to reach the theoretical upper bounds for isotropic elasticity and strain energy storage [32]. This remarkable feature is attributed to the specific topology that allows for a better load transfer between neighboring members. Moreover, this relatively simple design can be fabricated by using widely available 3D printers with a broad range of material selections.

In this work we design and fabricate a new type of hierarchically structured honeycomb with overall part size at centimeter scale, using a commercially available 3D printer. Our purposes here are two-folds. On one hand, we aim to explore the unusual properties of hierarchical honeycombs at large deformations. We also aim to demonstrate the potential of using the commercial 3D printer to facilitate the mass production of materials with complex architectures by applying a post-processing technique to preserve the shape and structural integrity of the printed structures. Honeycomb topologies have been the main topic of an extensive body of research due to their lightweight, novel thermomechanical properties, and energy absorption capability [33–38]. Their mainly bending-dominated mechanical behavior along the in-plane directions poses, however, a great limitation to potential applications. Structural hierarchy has been recently used as a new design strategy to explore improved mechanical and other unusual physical properties in honeycombs. Structural hierarchy is ubiquitous in both architecture designs and nature; typical examples range from the Eiffel Tower to hierarchically architected biological materials with multiple length scales [39–41]. Inspired by these designs researchers have introduced structural hierarchy into conventional honeycombs materials [42–47]. By replacing the cell walls of regular honeycombs with Kagome and triangular lattices it is theoretically possible to increase the stiffness of hierarchical honeycombs by about two orders of magnitude, compared to regular honeycombs [48,49]. In addition, recent studies have also shown that it is possible to control wave propagation by harnessing the multiscale characteristic of hierarchical honeycombs [49–51]. Quite remarkably, the introduction of a structural hierarchy into regular honeycombs results in significant energy absorption under crushing loading conditions [52–54]. Though analytical and numerical approaches have been developed to understand the mechanical response of these hierarchical honeycombs, the mechanical behaviors and associated mechanisms under large deformation are still elusive.

We report in this paper how designed hierarchical honeycombs with particular post-processing techniques may exhibit improved stiffness and energy absorption under uniaxial compression, as well as energy dissipation and shape preservation under cyclic loading. A combination of qualitative and quantitative analysis is performed to reveal the underlying mechanisms responsible for the mechanical behaviors of the hierarchical honeycombs under large deformations. The fabrication of the hierarchically structured cellular solids with features ranging from micrometer to centimeter is made possible by recent advances in additive manufacturing, which allows for fabricating complex topologies with fine features quickly, inexpensively and at a relatively large scale.

## 2. Materials and methods

### 2.1. Characterization of the hierarchical honeycombs

The hierarchical honeycomb topology described in this work is made by replacing the cell walls of regular honeycombs with triangular lattices (Fig. 1(a)–(d)). Two geometric parameters are defined to characterize the hierarchical honeycombs: the hierarchical length ratio $\gamma = l_t/l$ and the number of triangular lattices away from the central axis, $N$. We indicate by $l$ and $l_t$ the lengths of cell walls of the regular honeycomb and the triangular lattice, respectively. The length and thickness of the triangular lattices are determined by a mass equivalence between the regular honeycombs and the hierarchical honeycombs. As a result, the thickness and length of the triangular lattice can be calculated as:

$$\frac{t_t}{l_t} = \frac{1}{\sqrt{3}}\left(1 - \sqrt{1 - \frac{4\sqrt{3}}{3R\gamma^2}\left[\frac{t}{l} - \frac{1}{2\sqrt{3}}\left(\frac{t}{l}\right)^2\right]}\right) \quad (1)$$

where $t$ and $t_t$ are the cell wall thickness of the regular honeycomb and the triangular lattice, respectively. The parameter $R$ stands for the number of triangular lattices, and it is determined by $\gamma$ and $N$.

In view of the resolution of the 3D printer and the size of the compression platen, here we choose $\gamma = 1/5$, $N = 2$, and $l = 3$ cm and each sample consists of $2 \times 3$ representative volume elements. As a result, the dimensions of the prototypes are 180 mm × 157 mm (Fig. 1(e)–(f)). The relative density of the regular and hierarchical honeycombs can be controlled by tailoring the thickness of the cell wall.

### 2.2. Sample fabrication

All samples used in the study have been printed by using an Objet Connex260 multi-material 3D printer (Stratasys). VeroWhite (a glassy polymer) is the constitutive material used for the 3D printing, the mechanical properties of which are shown in Fig. S1. It should be pointed

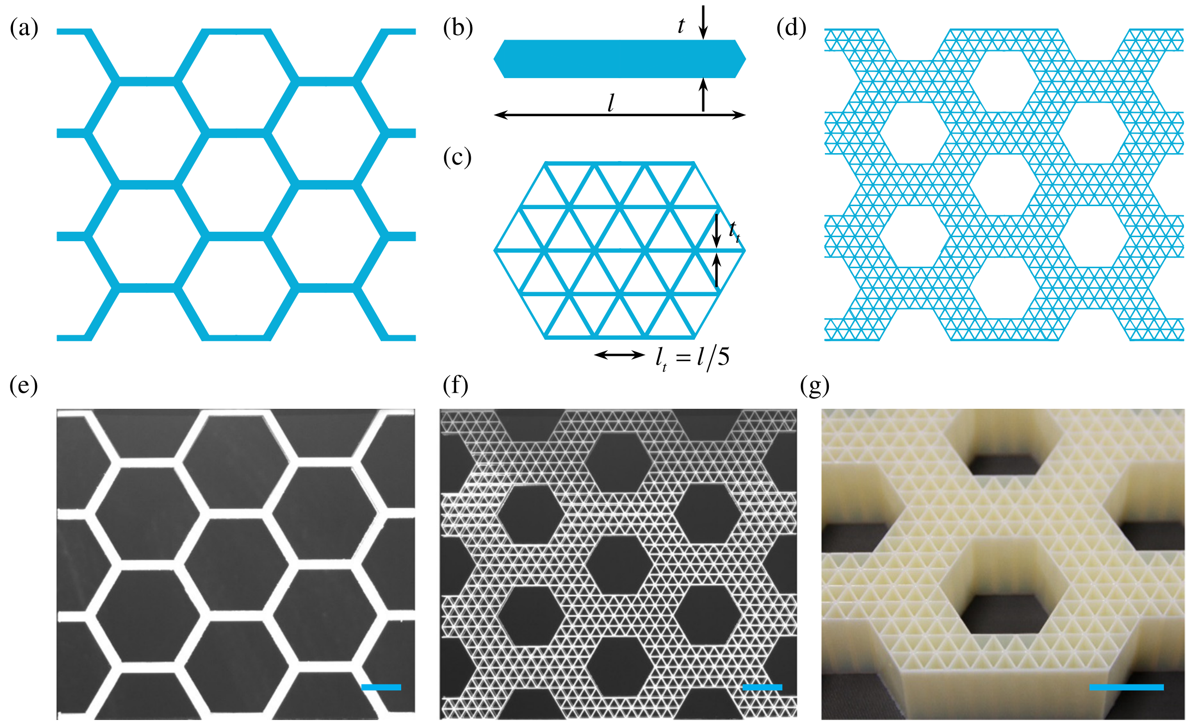


**Fig. 1.** Design and 3D printing of the hierarchical honeycomb. (a) Regular honeycomb. (b) The cell wall of the regular honeycomb. (c) Substructure composed of a triangular lattice. (d) Hierarchical honeycomb. (e) 3D printed regular honeycomb. (f)–(g) 3D printed hierarchical honeycomb. Scale bar: 2 cm.

out that the minimum feature size of cell walls in the 3D printed hierarchical honeycombs is about 400 μm, while the maximum allowable sample size is 26 cm. These requirements allow us to fabricate samples with only 2 × 3 representative volume elements in this work. The hierarchical honeycombs consisting of 2 × 3 representative volume elements were fabricated having an overall dimension of 180 mm × 157 mm × 30 mm (width × height × thickness) for samples with relative densities of 0.16, 0.32 and 0.55 and 252 mm × 219.8 mm × 30 mm (width × height × thickness) for samples with a relative density of 0.11. Within the limitation of 3D printing technology, the layer orientation was found to influence the mechanical properties of the material; therefore, all the samples were printed along the same orientation. For our uniaxial compression and cyclic testing we have fabricated 11 samples, which include one regular honeycomb with a relative density of 0.16, two hierarchical honeycombs with a relative density of 0.11, four hierarchical honeycombs with a relative density of 0.16, two hierarchical honeycombs with a relative density of 0.32, and two hierarchical honeycombs with a relative density of 0.55. The as-fabricated samples were kept at room temperature for seven days to allow for the saturation of the curing process.

### *2.3. Mechanical testing*

To examine the mechanical response of the 3D printed hierarchical honeycombs, uniaxial compressive testing was performed using a universal mechanical testing machine (SHIMADZU AGS-X) with a 300 kN load cell. The experimental setup of the compression testing is shown in Fig. S3. All experiments including the cylinder and hierarchical honeycombs were conducted in a quasi-static regime with a constant strain rate of 0.001 $s^{-1}$. Compression was applied to reach the strain of the catastrophic failure of each sample. The load-displacement curves measured from the uniaxial compressive test were then transferred into nominal stress-strain behaviors based on the measured dimensions of the samples. The Young's modulus and the yield stress have been then obtained from the measured stress-strain curve directly (Fig. S1). Images of the samples at various loading conditions were taken at a rate of 1 FPS (VicSnap, Correlated Solution). It should be pointed out that due to the large-scale feature of the proposed hierarchical honeycombs, we have only conducted repeated tests for the hierarchical honeycombs with a relative density of 0.16 (Fig. S4). We have achieved some very good repeatability within the elastic range, while some differences are observed in the nonlinear domain. These differences may be attributed to the limitation of the 3D printing technique used. In all of our experiments, however, we have recorded the whole deformation process and did not notice any abnormal behaviors, such as out-of-plane deformation.

To study the recoverability and energy dissipation behaviors of 3D printed hierarchical honeycombs, we performed cyclic loading-unloading tests using a universal mechanical testing machine (SHIMADZU AGS-X) with a 300 kN load cell (Fig. S3). The engineering strain rate of 0.001 $s^{-1}$ was applied for both compressive loading and unloading processes. All tests were taken at a room temperature of 23 °C. In the loading-unloading uniaxial compression tests, each sample was compressed to a target strain endpoint for each loading-unloading cycle. The engineering strain targets were 20%, 40%, and 60% for hierarchical honeycombs with a relative density of 0.16. By contrast, hierarchical honeycombs with a relative density of 0.11, 0.32, and 0.55 were compressed to a strain of 20%. Each sample experienced five loading-unloading cyclic tests.

During the unloading procedure, we notice that the samples can recover to 50% of the applied strain immediately and the recovery of the remaining strain took 72 h. To accelerate our experiment procedure, we harness the shape memory effect of the VeroWhite to recover the compressed samples in a more efficient manner. This method was used for the cyclic testing of all of the hierarchical honeycombs. Each cycle includes three stages: (1) loading-unloading compression testing, (2) heating, and (3) cooling.

Heating recovery experiments with 3D printed hierarchical honeycombs after loading-unloading uniaxial compression tests were carried out in an MTS Advantage™ Environmental Chamber (Thermcraft LBO-24-10-10-11-5001) with temperature control (Fig. S5). Samples were immersed at room temperature into the chamber at 75 °C. They transformed into their original permanent configuration within 10 min. After that, samples were taken out of the chamber and placed in room temperature for 10 min to cool to the room temperature.

### 2.4. Finite element analysis

The numerical simulations related to the mechanical response of the hierarchical honeycombs under uniaxial compression are conducted using commercial FE package ABAQUS/Standard (Simulia, Providence, RI). The properties of VeroWhite are characterized by Young's modulus $E_s = 1.56$ GPa, Poisson's ratio $\nu = 0.33$, a yield stress $\sigma_y = 46.3$ MPa, and density $\rho_s = 1174$ kg/m$^3$. The true stress-true strain relation is implemented as the constitutive equation for the mechanical behavior simulations. To save computational time, a smallest repeating representative volume element (RVE) together with periodic boundary conditions is employed in this work (Fig. S6) [55,56], as each representative volume element has 200,000 triangular elements (each beam has 6 elements along thickness direction). All models are generated by plane strain elements CPE4R and meshed after a convergence test. In view of the large deformation of the hierarchical honeycombs, both geometric and material nonlinearities are considered in the finite element simulations. The constituent phases are taken to be either a representative glassy polymer. The stress-strain behavior of the glassy polymer is captured using an elastic-plastic model which is exported from the true stress-strain relation of VeroWhite. Simulated stress-strain relations of regular and hierarchical honeycombs are plotted in Fig. S7.

## 3. Results and discussion

### 3.1. Mechanical behaviors under uniaxial compression

Uniaxial quasi-static compression tests have been performed on both regular and hierarchical honeycombs. For direct comparison, we have considered the case in which the two classes of honeycombs have the same relative density ($\rho/\rho_s = 0.16$) (refer to Videos 1 and 2 in the supporting information for details). Fig. 2(a) shows the measured nominal stress-strain relations. Quantitative comparisons of the effective stiffness and energy absorption between regular and hierarchical honeycombs are plotted in Fig. 2(b) and (c), respectively. The stiffness and energy absorption of the hierarchical honeycombs are 6.6 times and 7.5 times those of the regular honeycombs. The improved mechanical performance can be explained by the unusual behavior of the

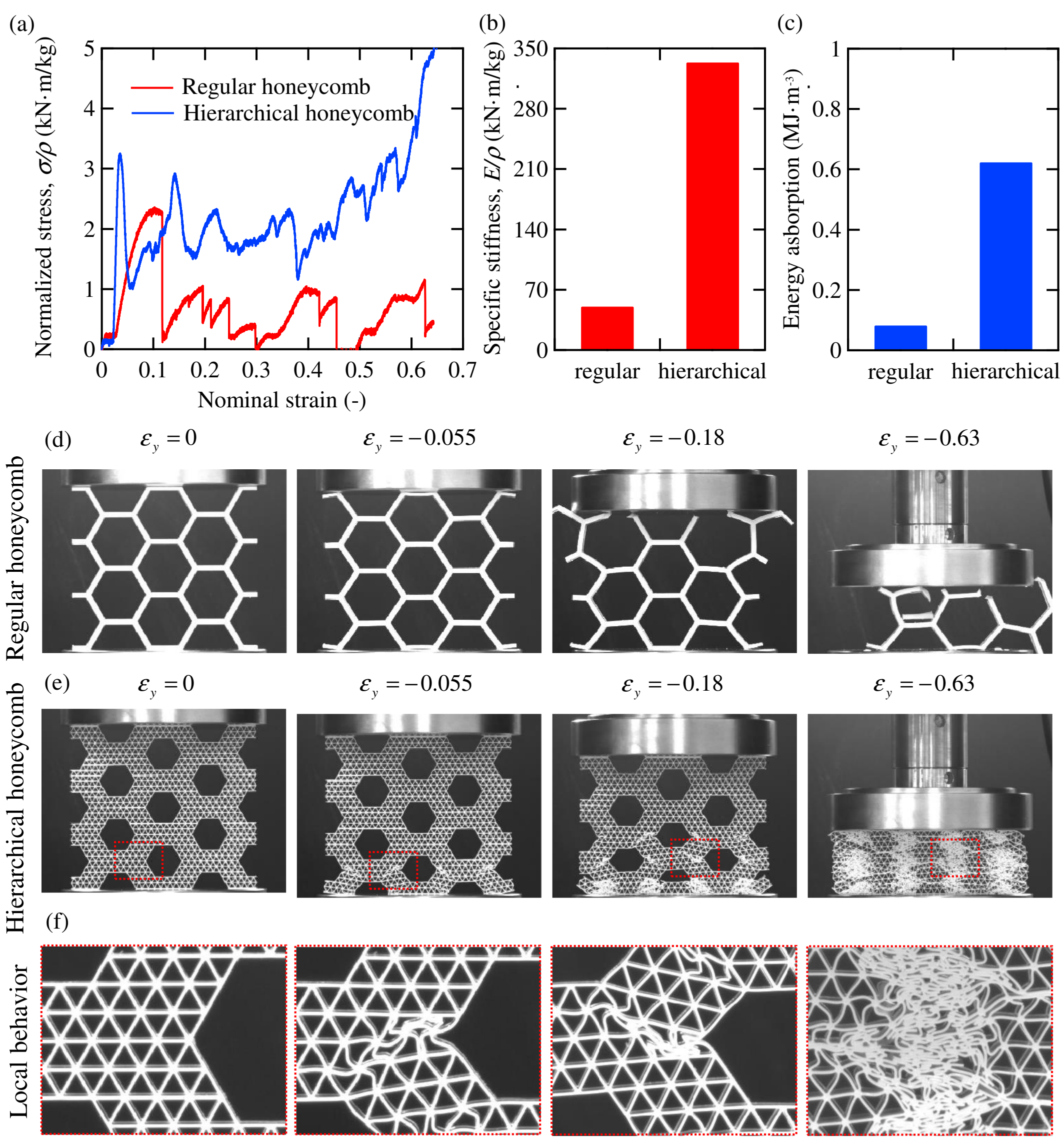


**Fig. 2.** Mechanical response of the regular and hierarchical honeycombs under uniaxial compression. (a) Normalized stress-strain relations for regular and hierarchical honeycombs. (b)–(c) Comparison between specific stiffness and energy absorption of regular and hierarchical honeycombs. (d)–(e) Deformation pattern of regular and hierarchical honeycombs. (f) Local deformation behaviors of hierarchical honeycomb corresponding to those at each strain level in (e).

hierarchical honeycombs. The regular honeycomb shows a sudden stress drop immediately after the first peak stress ($\sim\varepsilon_y = -0.12$). This is due to brittle failure of the laminated plastic 3D printed elements, which are subjected to bending under uniaxial compression (Fig. 2(d)). Subsequently, multiple stress drops arise, mainly attributed to the progressive failure of the cell walls. By contrast, the stress decreases gradually in the hierarchical honeycombs. This gradual and smoother plateau-like behavior is associated with the local buckling of the triangular cell walls (Fig. 2(e)–(f)). The global response tends then to become stiffer because the buckled cell walls increasingly contact with each other and provide additional support to carry the compressive load. One can notice after 15% of compressive strain a recurrent rise and decrease of the stress, which may be attributed to the progressive buckling, fracture, and contact of the triangular cell walls. It is noticeable that densification phenomenon emerges at a larger strain ($\sim\varepsilon_y = -0.6$). Compared with the catastrophic failure of the regular honeycomb, the hierarchical honeycombs tend to produce a global deformation through local buckling and fracture of cell walls, leading to a progressive failure behavior and further facilitating the energy absorption. From a general perspective, the introduction of a cellular structure at a lower hierarchical level provides the equivalent effect of embedding an open cell foam, which produces an elastic-plateau-densification behavior similar to classical foam materials. Moreover, the unusual mechanical responses in these hierarchical honeycombs are attributed to the introduction of the stretched-dominated triangular lattice structure.

It is evident that the enhanced stiffness, energy absorption, and progressive failure mode in the hierarchical honeycombs are all associated with the introduction of the internal triangular lattice structure. The slenderness ratio of the triangular lattice controls these unusual mechanical behaviors, such as buckling. To better understand this, we have fabricated and subjected to flatwise compression hierarchical honeycombs with relative densities of $\rho/\rho_s = 0.16(l_t/t_t = 13.3)$, $\rho/\rho_s = 0.32(l_t/t_t = 6.5)$, and $\rho/\rho_s = 0.55(l_t/t_t = 3.2)$. Hierarchical honeycomb with slightly thicker cell walls ($\rho/\rho_s = 0.32$) exhibit a stress drop similar to one observed for thinner cell wall topologies, which is associated to the local buckling of the cell walls (Fig. 3(a)). At a relatively large strain range, the progressive multiple stress drops become prominent. With the further decrease of the slenderness ratio of the triangular cell walls ($\rho/\rho_s = 0.55$), multiple sudden drops of stress peaks are present and the local buckling is not observed (Fig. 3(c)). Instead, one can observe a global catastrophic failure similar to the one shown by the regular honeycomb.

These analyses suggest that the slenderness ratio of the triangular lattice is critical to control the deformation behavior of the hierarchical

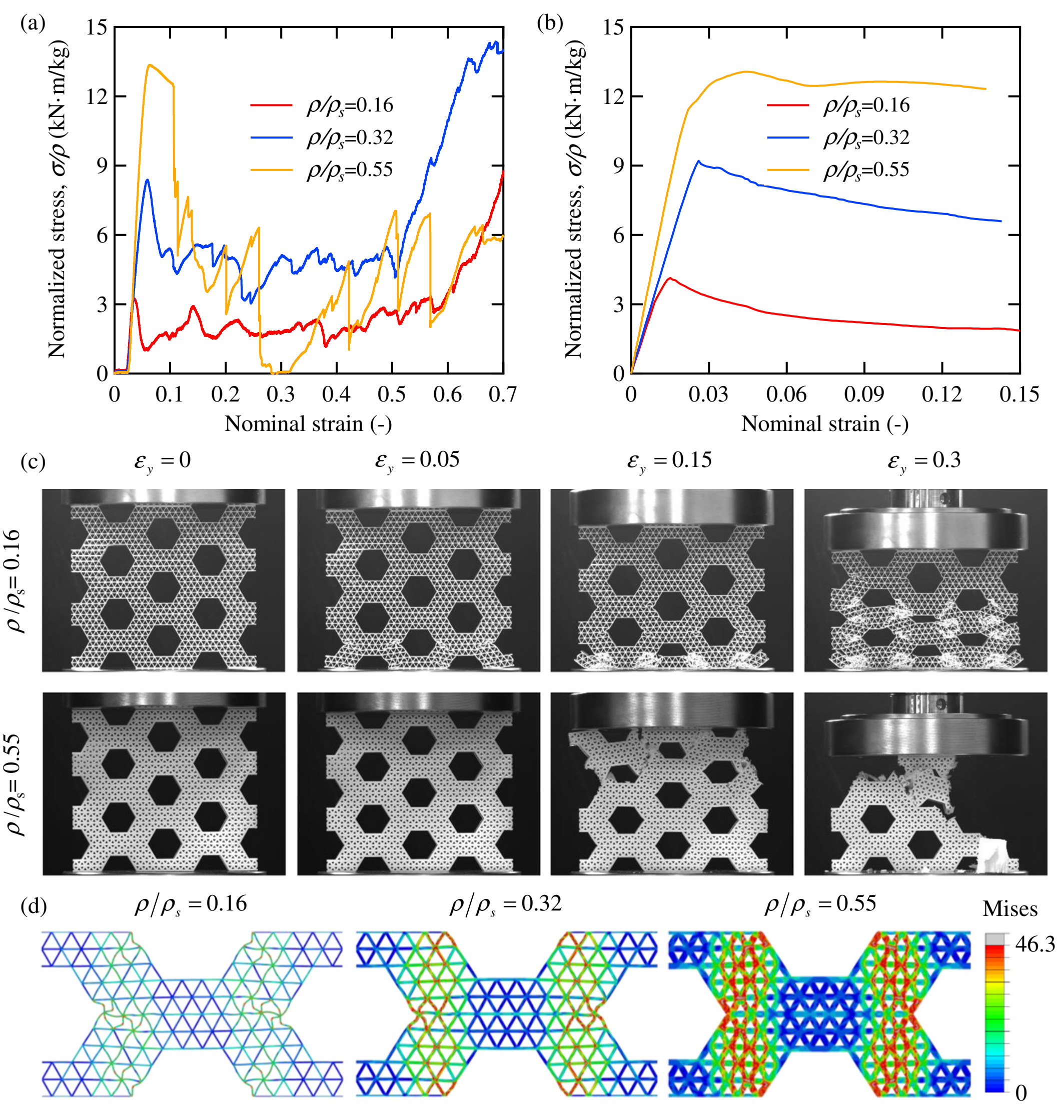


**Fig. 3.** The tunability of the mechanical response of hierarchical honeycombs. (a)–(b) Measured and simulated normalized stress-strain relations of hierarchical honeycombs with different relative densities, respectively. (c) Measured deformation pattern of hierarchical honeycombs with different relative densities. (d) Simulated deformation pattern of hierarchical honeycombs with different relative densities at a strain of $\varepsilon_y = -0.04$.Von Mises stress unit is MPa.

honeycombs. To understand this point from a quantitative perspective we have performed finite element analyses to evaluate the uniaxial compression behavior of the hierarchical honeycombs with different relative densities. The predicted normalized stress-strain relations are displayed in Fig. 3(b). Compared with the measured normalized stress-strain relations, one can observe a good agreement for compressive strains lower than 3%. At relative larger strain ranges our numerical model does not however capture completely the detailed deformation behaviors, because failure modes and contact friction are not included in our simulations. Yet, the numerical simulations still can provide some insightful information. As shown in Fig. 3(d), the local buckling of the triangular lattice can be predicted, which agrees well with the experimental observation. By contrast, the plastic deformation-induced failure is the dominant mechanism responsible for the sudden drop in the stress-strain relation of the hierarchical honeycomb with a relative density of $\rho/\rho_s = 0.55$. These experimental and numerical results suggest that the transition between progressive and catastrophic failure modes in the hierarchical honeycombs can be indeed tuned by tailoring the cell wall slenderness ratio.

### 3.2. Mechanical behavior under cyclic loading

We have shown so far results related to uniaxial compression tests, and have highlighted the mechanism behind their unusual mechanical behavior. We now proceed to examine the mechanical response of the hierarchical honeycombs under cyclic loading conditions. Fig. 4(a) displays the loading ($\varepsilon_y = 0\sim-0.6$) and unloading ($\varepsilon_y = -0.6\sim 0$) behavior of the hierarchical honeycombs with a relative density of $\rho/\rho_s = 0.16$. Upon unloading, the densified hierarchical honeycomb recovers to a strain of $\varepsilon_y = -0.3$ (refer to Video 3 in the supporting information for details). This is due to the release of the stored elastic energy in the buckled cell walls of the triangular lattice. The remaining (and incomplete) compression recovers very slowly, and it took about 72 h to fully restore the honeycomb to its initial form. The slow and incomplete recovery is due to irreversible effects, such as cell wall fracture and plastic deformation.

To accelerate the cyclic tests, each compressed sample after the first cycle was put in a chamber at 75 °C and heated for 10 min. This temperature is higher than the nominal 48 °C of the $T_g$ of the polymer, and after this treatment, the samples can recover their initial form due to shape memory effect. We then performed cyclic tests and shape memory assisted recovery on samples under maximum compressible strains of $\varepsilon_y = -0.2$ and , respectively. The measured loading-unloading stress-strain relations are plotted in Fig. 4(b)–(d). The effective stiffness, maximum stress, and loss factors are calculated from these measured stress-strain relations, as shown in Fig. 4(e)–(g). The effective stiffness of each hierarchical honeycombs under compressive cyclic strains decreases rapidly after each cycle because the above-cited non-recoverable effects

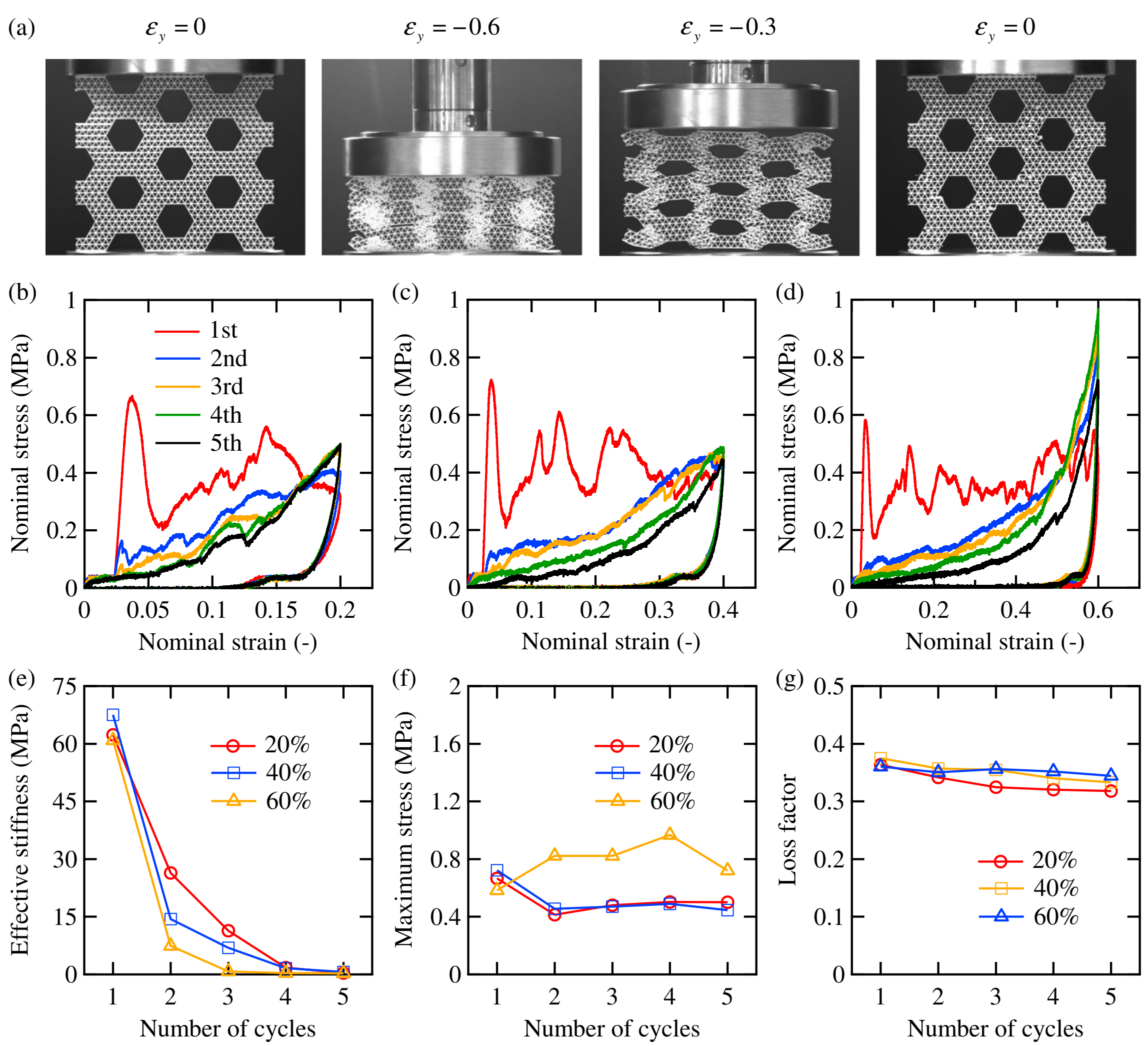


**Fig. 4.** Mechanical response of hierarchical honeycombs under cyclic loading. (a) Loading-unloading of the hierarchical honeycomb. (b)–(d) Stress-strain relations of loading-unloading cycles with compression strains up to $\varepsilon_y = -0.2$, $-0.4$, and $-0.6$, respectively. (e)–(g) Effective stiffness, maximum stress, and energy loss ratio of hierarchical honeycombs under different cycles. Here the relative density of each hierarchical honeycomb is $\rho/\rho_s = 0.16$.

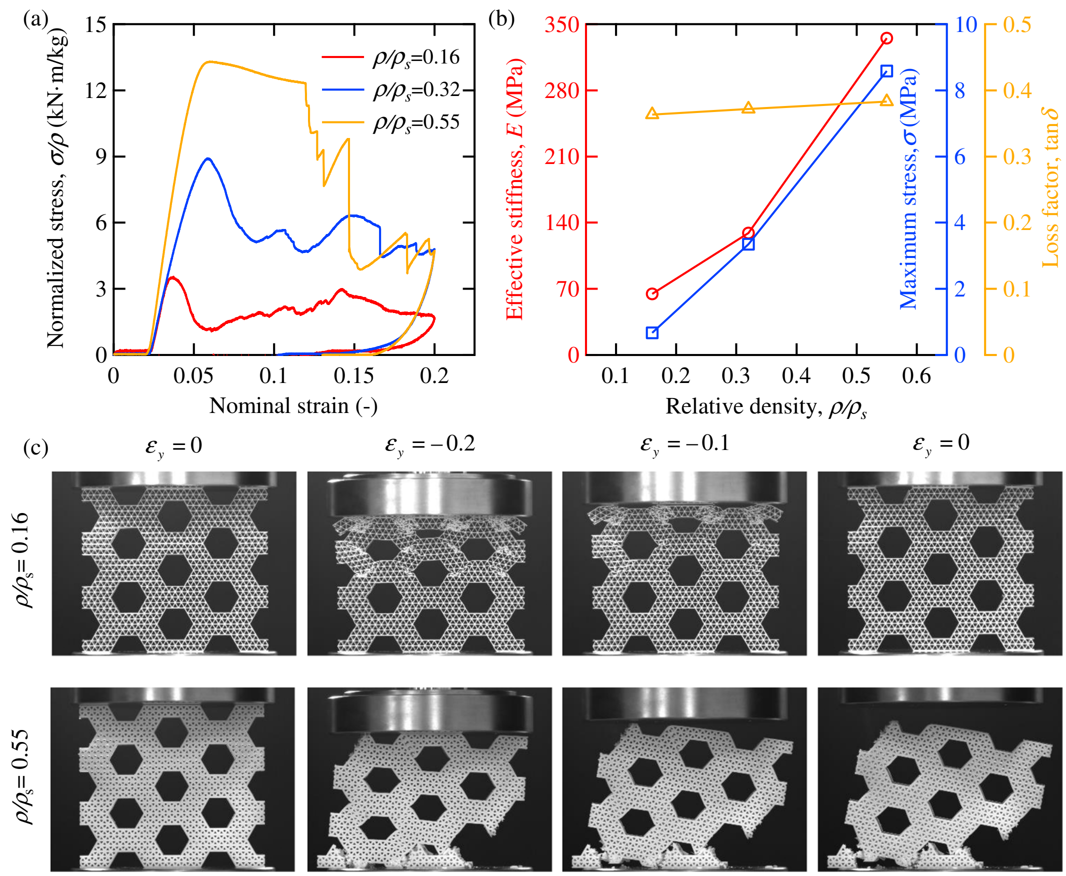


**Fig. 5.** The structural integrity of the hierarchical honeycombs. (a) The cyclic response of hierarchical honeycombs with different relative densities. (b) Effective stiffness, maximum stress, and energy loss ratio of hierarchical honeycombs with different relative densities. (c) Deformation pattern of hierarchical honeycombs with different relative densities. Here each hierarchical honeycomb is compressed up to a strain of $\varepsilon_y = -0.2$.

are a detriment to the initial loading carry capacity. In contrast to the significant decrease of effective stiffness, the maximum stress of the hierarchical honeycombs with a compressive strain of $\varepsilon_y = -0.2$ and $\varepsilon_y = -0.4$ has a 35% decrease after the first cycle (Fig. 4(f)), but tends then to stabilize. Quite remarkably, the maximum stresses in the hierarchical honeycombs with a compressive strain of $\varepsilon_y = -0.6$ tend to increase by 35% after the first cycle. The decreased and subsequent constant maximum stresses at smaller compressive strains are due to the failure and plastic deformation in the cell walls. On the other hand, the increased maximum stresses at larger compressive strains are attributed to the contact, slip-stick, and densification of the triangular lattice. These different deformation mechanisms are also responsible for the energy dissipation behavior of the hierarchical honeycombs. As can be seen in Fig. 4(g), after the first cycle all hierarchical honeycomb exhibit a high average loss factor of 0.34. Aside from the contact friction and plastic deformation, the intrinsic material damping also contributes

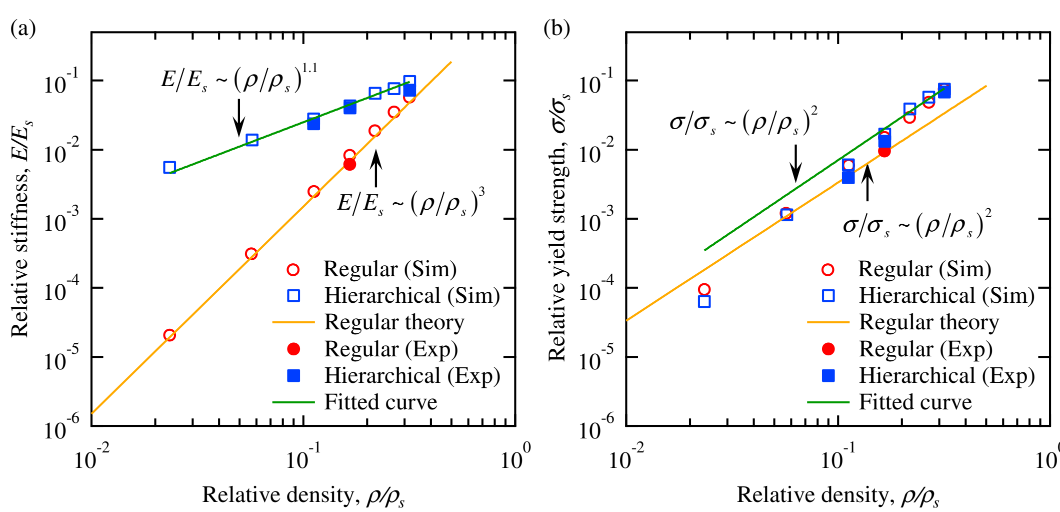


**Fig. 6.** Relative stiffness and yield strength as a function of relative density. (a) Relative stiffness and (b) relative yield strength. See main text for the discussion of theoretical solutions of regular honeycombs.

to the energy dissipation of the hierarchical honeycombs (Fig. S2). Yet, structural damping induced by the unique failure mode of the hierarchical honeycombs is still dominant.

We have shown that the proposed hierarchical honeycombs may exhibit some excellent structural and shape integrity under large compression strains, which is aided by the release of the stored elastic energy in the buckled cell walls. The structural integrity is also correlated to the slenderness ratio of the triangular lattice. To confirm this feature, we have run cyclic loading tests up to a strain of $\varepsilon_y = -0.2$ for hierarchical honeycombs with relative densities of $\rho/\rho_s = 0.16$, 0.32, and 0.55. The measured loading-unloading normalized stress-strain relations are plotted in Fig. 5(a) and extracted mechanical responses are summarized in Fig. 5(b). As expected, the specific effective stiffness and maximum stress increase with the relative density, while a very slight increase of the loss factor (tan$\delta$) is observed. It is also evident that shape memory treatment adopted allows recovering the shape of the hierarchical honeycombs, albeit to a different degree depending on the slenderness ratio. For a smaller slenderness ratio ($\rho/\rho_s = 0.16$), the compressed hierarchical honeycomb tends to conserve its overall shape with no visible damage or crack within its structure. No shape integrity can be however observed in the hierarchical honeycombs with a large slenderness ratio ($\rho/\rho_s = 0.55$) (Fig. 5(c)). These observations further confirm that the excellent structural integrity and shape preserving characteristics are intrinsically governed by the amount of stored elastic energy in the buckled cell walls.

### 3.3. Scaling laws of the hierarchical honeycombs

The simulated and experimental stiffness and strength of the hierarchical honeycombs with relative densities from 0.02 to 0.32 are plotted in Fig. 6(a) and (b). For the sake of comparison, the stiffness and strength as a function of the relative density for the regular honeycombs case are also displayed. One has to notice that the theoretical predictions for regular honeycombs are derived from the use of the Euler beam theory for truss structures, for which the shear deformation is neglected. Euler-Bernoulli beam theory is only valid for honeycombs with thin cell walls (relative density smaller than 0.25) [20]. In the case of thick cell walls, the analytical solutions for stiffness based on Timoshenko beam theory should be adopted [57,58]. One can notice that the simulated stiffness and strength of the regular honeycombs with relative densities smaller than 0.25 agree well with the linear-elastic and the brittle failure solutions [20]. In the case of regular honeycombs with a relative density of 0.55, the finite element analysis provides however an effective stiffness of 0.13, which agrees very well with the Timoshenko beam theory-based solution (0.12). These results indicate that our numerical modeling can predict the main mechanical response metrics of regular honeycombs.

Compared with regular honeycombs, the hierarchical honeycombs show a remarkably improved stiffness, and a comparable strength to the one of regular honeycombs. At a relatively low density of 0.02, the hierarchical honeycombs exhibit a ~160 times higher stiffness than the regular honeycombs. We have fit the relative stiffness and strength as a function of relative density using a scaling law, $E/E_s = B(\rho/\rho_s)^m$ and $\sigma/\sigma_s = C(\rho/\rho_s)^n$, where $E$ and $\sigma$ are the effective stiffness and strength of the hierarchical honeycombs, respectively. The parameters $B$ and $C$ are geometry-dependent proportionality constants, $m$ and $n$ are the scaling exponents. Here we have $B = 0.37$, $C = 0.83$, $m = 1.1$, and $n = 2$. One can notice a nearly linear relation between the non-dimensional stiffness and the relative density for hierarchical honeycombs. Compared with stochastic materials having a poorly scaling relation between stiffness and relative density, generally following $E/E_s \sim (\rho/\rho_s)^3$ [20], the proposed hierarchical honeycombs demonstrate great potential to be designed as ultralight mechanical metamaterials. This finding also suggests that the introduction of an inner triangular lattice enables us to change the bending-dominated behavior into a stretching-dominated one. The relative density dependency of the compressive strength for the hierarchical and the regular honeycombs is almost the same, with the hierarchical honeycombs exhibiting a slightly higher relative strength.

## 4. Conclusions

We have presented a new type of hierarchical honeycomb structure that exhibits a nearly linear relation between stiffness and relative density. At large uniaxial compression and cyclic loading conditions, these hierarchical honeycombs show improved energy absorption and recoverability. These anomalous mechanical behaviors are enabled by introducing a structural hierarchy into regular honeycombs, thereby resulting in the coexistence of local buckling, cell wall fracture, and plastic deformation. The recoverability is also aided by a specific heating process that activates the shape memory effect of the core polymer used in the 3D printing of these cellular structures. Slender cell walls can be critical in structural engineering applications for the potential onset of mechanical instability and catastrophic failure. Here we have however demonstrated that slender cell walls can be efficiently used to tailor the energy absorption and further provide recoverability. We have given evidence, both from an experimental and numerical standpoint that the slenderness ratio of the hierarchical triangular lattice is critical to tune the mechanical responses. The thermal heating applied to the compressed honeycomb above the $T_g$ of the constituent polymer allows preserving the shape of the hierarchical structures under large deformations, at avoiding visible cracks or damages inside the honeycombs.

The hierarchical honeycombs structures are fabricated using commercially available 3D printers and a brittle plastic polymer. It should be pointed out that other advanced manufacturing techniques, such as two-photon lithography can also be employed to create hierarchical cellular solids at a smaller length scale and with a broad choice of materials. As such, the mechanical response of the as-fabricated multiscale hierarchical honeycombs under different conditions, such as low velocity impact and shock wave loadings, can be investigated accordingly. The findings in this work not only provide a better understanding of the critical interplay between material architectures and mechanical properties, but also offer new opportunities to create new mechanical metamaterials with anomalous behaviors for a wide range of potential applications.

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