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ISOLATION OF SURFACE WAVE-INDUCED VIBRATION USING PERIODICALLY MODULATED PILES

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The isolation of surface wave-induced vibration using periodically modulated piles in soil is investigated. We demonstrate through simulations the dependence of complete bandgaps on the lattice symmetries, geometric parameters of the piles and material properties of the soil. The simulated results suggest that the piles modulated with square and hexagonal lattices are much more favorable for the formation of complete bandgaps than those modulated with honeycomb lattice. The height of the piles also plays a significant role in governing the evolution of complete bandgaps. Besides, complete bandgaps can be tuned by tailoring the volume fraction of the piles and the geometries of the pile cross section. Our results indicate that the contrast in the Young's modulus and the density is vital for the evolution of complete bandgaps and the viscosity of the soil should be considered as well. The analysis of surface wave propagation in a finite number of piles confirms the simulated complete bandgaps and also reveals that the complete bandgaps stem from Bragg interferences. This paper not only demonstrates the promising application of periodically modulated piles as wave barriers but also provides design guidelines for civil engineers.

Keywords: Isolation; vibration; surface wave; piles; periodic composites; complete bandgaps.

1. Introduction

The vibration induced by traffic, construction, industrial activities, blasting or natural earthquake not only damages civil infrastructures and affects high precision facilities, but also lowers the comfort level of residents. Therefore, vibration isolation is of great concern for both researchers and civil engineers. To decrease or eliminate the effects of vibration on infrastructures and facilities, two types of vibration isolation methods have been proposed, which are active vibration isolation and passive vibration isolation. Regarding the passive vibration isolation, open or filled trenches and piles are employed as wave barriers [Beskos *et al.*, 1986; Liao and

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Sangrey, 1978]. Trenches are often used to isolate vibrations for shallow structures, while piles are much more widely utilized due to their capabilities to be exempted from the influence of high groundwater level and to enhance the capacity of ground simultaneously.

The effectiveness of piles as wave barriers has been investigated intensively in the past decades. Liao and Sangrey [1978] experimentally studied the effects of pile barriers to reduce the ground vibrations and they found that the impedance mismatch between the piles and the soil has a great impact. However, full scale experiments of piles as wave barriers are limited due to the trade-off between costs and effectiveness. Therefore, many analytical and numerical methods have been proposed to solve the problem of pile barriers. Avilés and Sánchez-Sesma [1988] analytically investigated the two-dimensional (2D) and three-dimensional (3D) problem of pile barriers for the foundation and they concluded that stiff piles have better screening effectiveness than flexible piles. Takemiya [2004] studied the effectiveness of honeycomb wave impeding barriers using a 3D finite element method (FEM) and a dramatic reduction effect was achieved. The 3D boundary element method (BEM) has also been implemented to study the vibration isolation of piles as wave barriers against that of trenches [Gao *et al.*, 2006; Kattis *et al.*, 1999; Tsai *et al.*, 2008]. A 3D BEM–FEM coupling model for the time-harmonic dynamic analysis of piles and pile groups in soil was proposed by Padrón *et al.* [2007]. In their model, the piles were modeled using finite elements, while the soil was modeled using boundary elements. They concluded that this method was much more efficient than other numerical methods. Most of these theoretical studies are focused on formulating different analytical and numerical methods to analyze the effectiveness of piles as wave barriers. However, only single or several rows of piles were investigated for incident wave propagation along specific directions due to the complexity of wave propagation in piles and soil.

Recently, periodic composites have attracted much interest owning to their capabilities to tailor the propagation of mechanical waves. One fundamental property of these periodic composites is the existence of complete bandgaps (CBGs) — frequency ranges where mechanical waves are reflected or trapped inside regardless of the incident angle [Sigalas and Economou, 1993; Vasseur *et al.*, 2007]. Inspired by this, Huang and Shi [2011] considered the pile-soil system as a 2D periodic composite and numerically studied the dynamic attenuation behavior of pile barriers. Xiang *et al.* [2012] fabricated a scaled model frame and a periodic foundation, on which the shake table test was conducted, and they found that the vibration can be attenuated significantly. More recently, seismic metamaterial has been proposed to reduce the amplitude of seismic waves at the free surface and good agreement between the numerical and experimental results was observed [Brûlé *et al.*, 2013]. It should be pointed out that the aforementioned researchers assume that the height of piles is infinite in their simulation. However, the height of piles is finite and highly correlated with the elevation of the structures to be isolated in engineering practice.

In this paper, we investigate the surface wave propagation along the plane perpendicular to the pile axis using the finite element method (FEM). Piles with finite height periodically modulated in soil are regarded as a 2D periodic composite analogously. However, surface wave propagation in this periodic composite is indeed a 3D problem. We report the dependence of CBGs on different lattice symmetries, geometric parameters of piles and material properties of soil. Furthermore, we confirm the simulated bandgaps and also reveal the mechanism of CBG formation through analyzing surface wave propagation in the periodic composite with a finite number of piles. This paper not only demonstrates the promising application of periodically modulated piles as surface wave barriers but also provides design guidelines for civil engineers.

2. Numerical Modeling

2.1. *Governing equation and material behavior*

The governing equation of surface wave propagation in inhomogeneous materials is given by

$$\nabla \cdot \boldsymbol{\sigma}(\mathbf{u}) = -\rho\omega^2\mathbf{u}, \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy's stress tensor, $\mathbf{u}$ is the displacement vector, ω is the angular frequency and ρ is the mass density of constituent material.

For a linear elastic material, the constitutive relations of elasticity among the stress, the strain and the displacement fields are given by

$$\boldsymbol{\sigma} = \mathbf{C} : \varepsilon, \tag{2}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}[(\nabla\mathbf{u})^T + \nabla\boldsymbol{u}], \tag{3}$$

where $\mathbf{C}$ is the fourth-order elasticity tensor, and $\boldsymbol{\varepsilon}$ is the strain tensor.

For a linear viscoelastic material, the generalized Maxwell model is employed to approximate the relaxation function, which is given by

$$\Gamma(t) = G + \sum_{m=1}^{N} G_m \exp\left(-\frac{t}{\tau_m}\right), \tag{4}$$

where t is the time, G_m represents the stiffness of the spring in branch m, and τ_m is the relaxation time constant of the spring-dashpot pairs in the same branch. A commonly used parameter, the elastoviscosity, is defined as follows for each branch in terms of the relaxation time and shear modulus

$$\eta_m = G_m\tau_m. \tag{5}$$

Then, the constitutive relation of elasticity for a linear viscoelastic material can be written in an eigenfrequency analysis

$$\boldsymbol{\sigma} = K\, trace(\boldsymbol{\varepsilon})\mathbf{I} + 2(G' + jG'')\left(\boldsymbol{\varepsilon} - \frac{1}{3}trace(\boldsymbol{\varepsilon})\mathbf{I}\right), \tag{6}$$

$$G' = G + \sum_{m=1}^{N} G_m \frac{(\omega\tau_m)^2}{1+(\omega\tau_m)^2}, \tag{7}$$

$$G'' = \sum_{m=1}^{N} G_m \frac{\omega\tau_m}{1+(\omega\tau_m)^2}, \tag{8}$$

where K is bulk modulus, $\mathbf{I}$ is the unit tensor, G' and G'' are the storage modulus and the loss modulus, respectively. Note that, only the frequency-dependent storage modulus needs to be considered for the investigation of CBGs.

2.2. *Boundary condition using Bloch's theorem*

The characteristic of surface wave propagation in periodic composites can be captured by analyzing the representative volume element (RVE) according to Bloch's theorem. In this scenario, Floquet periodic boundary conditions are applied along x and y directions of the RVE such that [Khelif *et al.*, 2006]

$$\mathbf{u}(\mathbf{r}+\mathbf{R}) = e^{i\mathbf{k}\cdot\mathbf{R}}\mathbf{u}(\mathbf{r}), \tag{9}$$

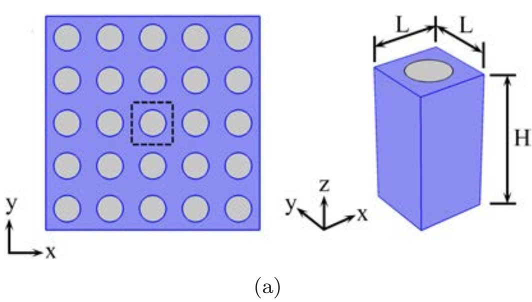
where $\mathbf{r}$ is the location vector, $\mathbf{R}$ is the lattice translation vector, $\mathbf{k}$ is the wave vector, and $i = \sqrt{-1}$. More details on the applications of Bloch's theorem to periodic composites can be found in the works by Gonella and Ruzzene [2008], Huang [2011] and Phani *et al.* [2006].

The eigenfrequency problem of the RVE is solved by FEM. The governing equation, combined with Eq. (9), leads to the standard eigenvalue problem

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{u} = 0, \tag{10}$$

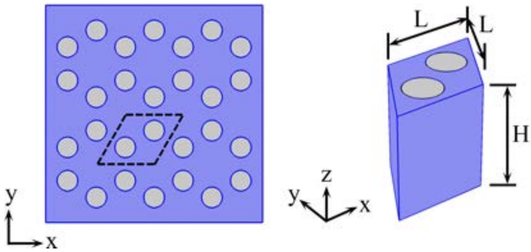
where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and mass matrices assembled using standard FEM procedure.

Figure 1 illustrates the piles that are periodically modulated in the soil with different lattice symmetries and the corresponding RVEs. The RVEs are discretized

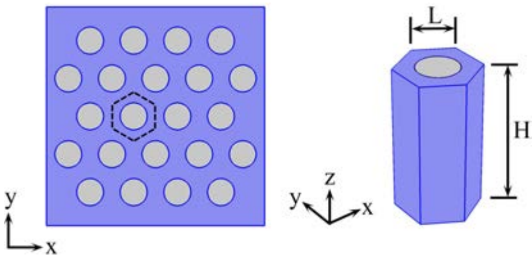


(a)

Fig. 1. Schematic illustration of the periodic composites consisting of piles and soil with a (a) square lattice, (b) honeycomb lattice and (c) hexagonal lattice. The edge lengths of the RVEs are $L = a$, $L = \sqrt{3}a$ and $L = \sqrt{3}a/3$, respectively. a is the nearest center-to-center distance between two nearest piles and H is the height of piles.



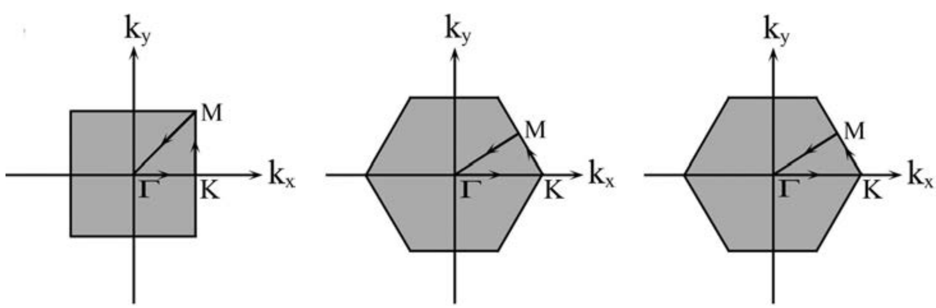
(b)



(c)

Fig. 1. (*Continued*)

using 10-node tetrahedron elements. Equation (10) is solved by imposing two components of wave vectors and hence yields the corresponding eigenfrequencies. The band structures are obtained by scanning of the wave vectors in the first irreducible Brillouin zone (FIBZ). The FBIZs are constructed by calculating the associated reciprocal lattices of the periodic composites. The shapes of FIBZs for different lattice symmetries are illustrated in Fig. 2 [Maldovan and Thomas, 2009].



(a) (b) (c)

Fig. 2. The FIBZs for the periodic composites with different lattice symmetries. (a) Square lattice [M $= \pi(1,1)/a$, $\Gamma = (0,0)$, X $= \pi(1,0)/a$], (b) honeycomb lattice [M $= 2\pi(1/2, 1/2\sqrt{3})/\sqrt{3}a$, $\Gamma = (0,0)$, X $= 2\pi(2/3,0)/\sqrt{3}a$] and (c) hexagonal lattice [M $= \pi(1, 1/\sqrt{3})/a$, $\Gamma = (0,0)$, X $= \pi(4/3,0)/a$]. a is the nearest center-to-center distance between two nearest piles.

3. Results and Discussion

Here we assume that the materials in the soil and the piles (made of concrete and or steel) are all homogeneous and isotropic. The pile materials are assumed to be linear elastic, while the soil is assumed to be linear elastic or viscoelastic. The material parameters used in the simulations are assigned as follows unless other specified: the density of concrete $\rho_c = 2500\,\text{kg/m}^3$, the Young's modulus $E_c = 40\,\text{GPa}$, and Poisson's ratio $\mu_c = 0.20$. The density of soil $\rho_s = 1800\,\text{kg/m}^3$, the Young's modulus $E_s = 30\,\text{MPa}$, and Poisson's ratio $\mu_s = 0.30$. The density of steel $\rho_{st} = 7850\,\text{kg/m}^3$, the Young's modulus $E_{st} = 210\,\text{GPa}$, and Poisson's ratio $\mu_{st} = 0.30$. The center-to-center distance between two nearest piles is $a = 2\,\text{m}$.

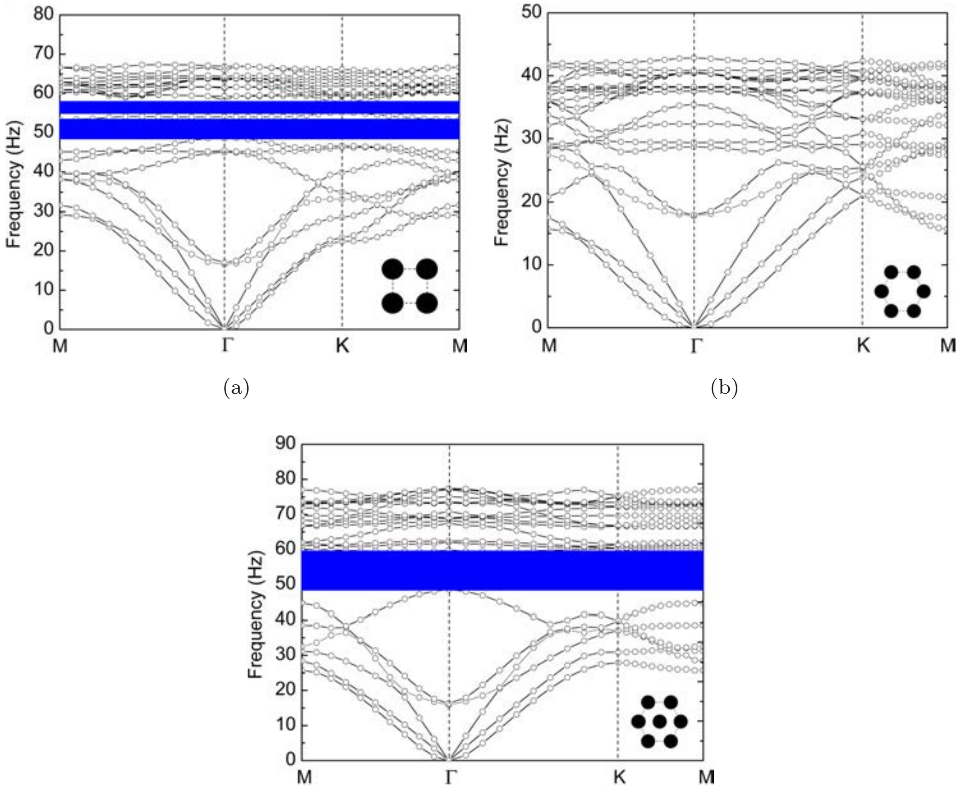
3.1. *Effects of lattice symmetry*

Previous studies show that lattice symmetries of periodic composites have a great impact on the evolution of CBGs resulting from Bragg interferences [Goffaux and Vigneron, 2001; Hsieh *et al.*, 2006; Sigalas and Economou, 1994; Kuang *et al.*, 2004]. Here we consider the periodic composites with three types of lattice symmetries, namely, square lattice, honeycomb lattice and hexagonal lattice. For comparison purposes, the height and the volume fraction of the piles (made of circular solid concrete) in the RVEs are set as 4 m and 0.50, respectively.

Figures 3(a)–3(c) shows the band structures of periodic composites with a square lattice, a honeycomb lattice and a hexagonal lattice, respectively. Two CBGs appear in the band structure of the periodic composite with a square lattice. The first CBG (between 48.67 Hz and 53.39 Hz) with a width of 4.72 Hz appears between the eighth and ninth bands and the second CBG (between 55.19 Hz and 57.91 Hz) with a width of 2.72 Hz lies between the ninth and tenth bands. However, no CBGs appear in the band structure of the periodic composite with a honeycomb lattice. In the band structure of the periodic composite with a hexagonal lattice, a large CBG appears between the sixth and seventh bands (between 48.56 Hz and 59.53 Hz) with a width of 10.97 Hz, which is twice that of the largest width of the periodic composite with a square lattice. Although it has been reported that piles modulated with honeycomb lattice in soil can reduce vibration effectively [Takemiya, 2004], our results suggest that the piles periodically modulated in soil with a square and a hexagonal lattice are much more favorable for the formation of CBGs than those with a honeycomb lattice. This is consistent with numerical studies by other researchers [Kuang *et al.*, 2004] on 2D phononic crystals, indicating the smallest bandgap appears in the case of honeycomb lattice.

3.2. *Effects of geometric parameters*

It is known that the topologies of periodic composites are vital for the formation of CBGs. Designing well-defined topologies for periodic composites with desired CBGs is of great concern for engineers [Dahl *et al.*, 2008; Kafesaki *et al.*, 1995; Liu *et al.*,



(a) (b) (c)

Fig. 3. Band structures of the periodic composites with different lattice symmetries. (a) Square lattice, (b) honeycomb lattice and (c) hexagonal lattice. The height and the volume fraction of the circular solid concrete piles in each RVE are 4 m and 0.50, respectively.

2002]. To further understand the effects of topologies on the evolution of CBGs and propose design guidelines for engineers to isolate the surface wave-induced vibration, the geometric parameters of the piles, namely, the height, the volume fraction and the geometry of the cross section, are selected to perform parametric studies.

3.2.1. *Height of piles*

Here we consider the periodic composites with a square lattice and a hexagonal lattice and the volume fraction of circular solid concrete piles in the RVEs are set as 0.50 for both cases. Figures 4(a) and 4(b) show the evolution of CBGs as a function of height of the piles for the periodic composites with a square lattice and a hexagonal lattice, respectively. In the periodic composite with a square lattice, the width of the first CBG increases as the pile height increases from 2 m to 3 m and then shrinks rapidly as the pile height increases from 3 m to 6 m. This CBG nearly diminishes as the pile height increases from 6 m to 8 m. Notably, another CBG arises

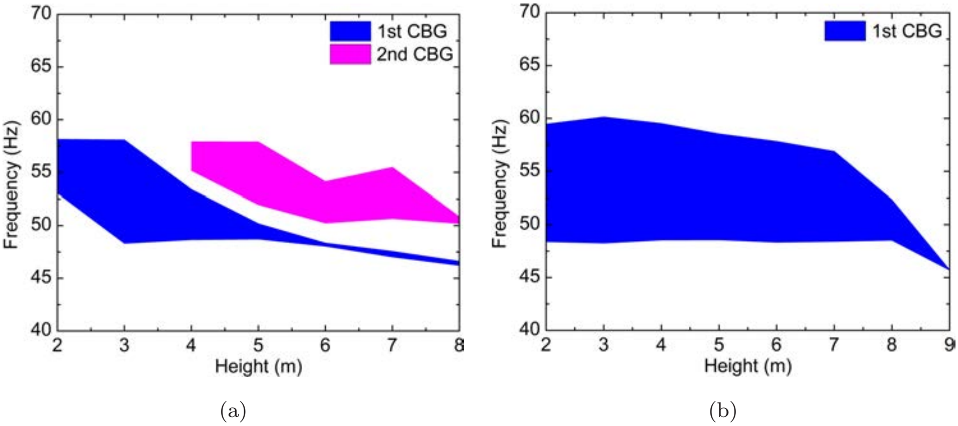


Fig. 4. Evolution of CBGs in the periodic composites with different lattice symmetries as a function of the pile height. (a) Square lattice and (b) hexagonal lattice. The volume fraction of the circular solid concrete piles in each RVE is 0.50.

when the pile height reaches 4 m. The width of the second CBG fluctuates as the pile height increases from 4 m to 8 m. The evolution trend of the CBG of the periodic composite with a hexagonal lattice exhibits a similar behavior as the first CBG of the periodic composite with a square lattice. However, the maximum width of the CBG of the periodic composite with a hexagonal lattice is about 1.2 times that of the periodic composite with a square lattice. These results indicate that the height of piles is an important geometric parameter governing the evolution of CBGs. In addition, our results further support the conclusion that the periodic composite with a hexagonal lattice is much favorable for the formation of CBGs.

3.2.2. *Volume fraction of piles*

We extend our study to the effect of the volume fraction of piles on the evolution of CBGs in the periodic composites with a square lattice and a hexagonal lattice. The height of the circular solid concrete piles is set as 4 m for comparison purposes. Figures 5(a) and 5(b) show the evolution of CBGs as a function of the volume fraction of the piles for the periodic composites with a square lattice and a hexagonal lattice, respectively. Two CBGs appear in the periodic composite with a square lattice. The first and second CBGs open at the volume fractions of 0.30 and 0.40, respectively. The width of the first CBG enlarges as the volume fraction increases from 0.30 to 0.60 and then decreases and keeps constant as the volume fraction increases from 0.60 to 0.70, while the width of the second CBG increases steadily with the increase of volume fraction from 0.40 to 0.70. In the periodic composite with a hexagonal lattice, we only observe one CBG, which increases rapidly as the volume fraction increases from 0.30 to 0.80. However, the largest width of the CBG in the periodic composite with a hexagonal lattice is larger than that of any CBG in the periodic composite with a square lattice. The evolution trend of CBGs in the

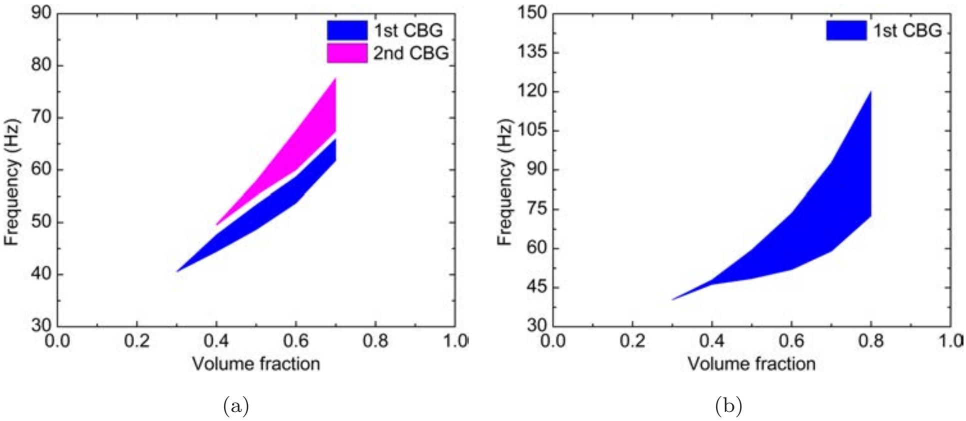


Fig. 5. Evolution of CBGs in the periodic composites with different lattice symmetries as a function of the volume fraction of the piles. (a) Square lattice and (b) hexagonal lattice. The height of the circular solid concrete piles in each RVE is 4 m.

periodic composite with a square lattice and a hexagonal lattice demonstrates that the width of CBGs in the latter composite is much more sensitive to the volume fraction. These results also suggest that we can achieve desired CBGs by tailoring the volume fraction of the piles in the periodic composites.

3.2.3. *Geometry of pile cross-section*

In civil engineering design, different types of piles are utilized according to the ground capacity requirement and soil properties. Here we select five types of piles to investigate the effect of geometries of pile cross-section on the evolution of CBGs. In addition to circular solid concrete (C) piles, the other four common types of piles in civil engineering are steel pipe (SP) piles, concrete pipe (CP) piles, concrete-filled steel tubes (CST) piles, and solid concrete square (S) piles. In the following, we only consider the periodic composite with a hexagonal lattice. The height and the volume fraction of each type of aforementioned piles in the RVEs are set as 4 m and 0.50, respectively (the geometric parameters for each type of piles are described in the caption of Fig. 6). Figures 6(a)–6(d) show the band structures of periodic composites composed by the piles with the aforementioned cross-sections. Among the five types of cross-sections, the largest CBG appears in the periodic composite with inclusions of CST piles. The second largest CBG appears in the periodic composite with inclusions of C piles, and the width of the CBG in the periodic composite with inclusions of CST piles is 1.5 times larger than that of the C pile composite. CBG also arises in the periodic composite with inclusions of S piles with a width of 7.76 Hz, which is smaller than that of the C and CST cases. It is interesting to note that the width of the CBGs in the periodic composite with inclusions of S piles can be tuned by rotating the pile along its z-axis. As shown in Fig. 6(e), the width of the CBG enlarges when the cross-section is rotated from 0° to 45°. When the rotation angle reaches 45° (indicated by SA), the largest CBG is achieved with a width of 10.14 Hz, which is comparable with that of the periodic composite with inclusions of C piles (10.97 Hz). The largest CBGs in the periodic composites with inclusions of each type of piles are summarized in Fig. 6(f). Note that, no CBG is observed in the periodic composite with inclusions of SP piles. These results indicate that the largest CBGs can be achieved by selecting the CST piles as inclusions in the periodic composites. In addition, periodic composites with C piles and SA piles exhibit equivalently considerable CBGs, although both of which are smaller than that in the periodic composite with CST piles.

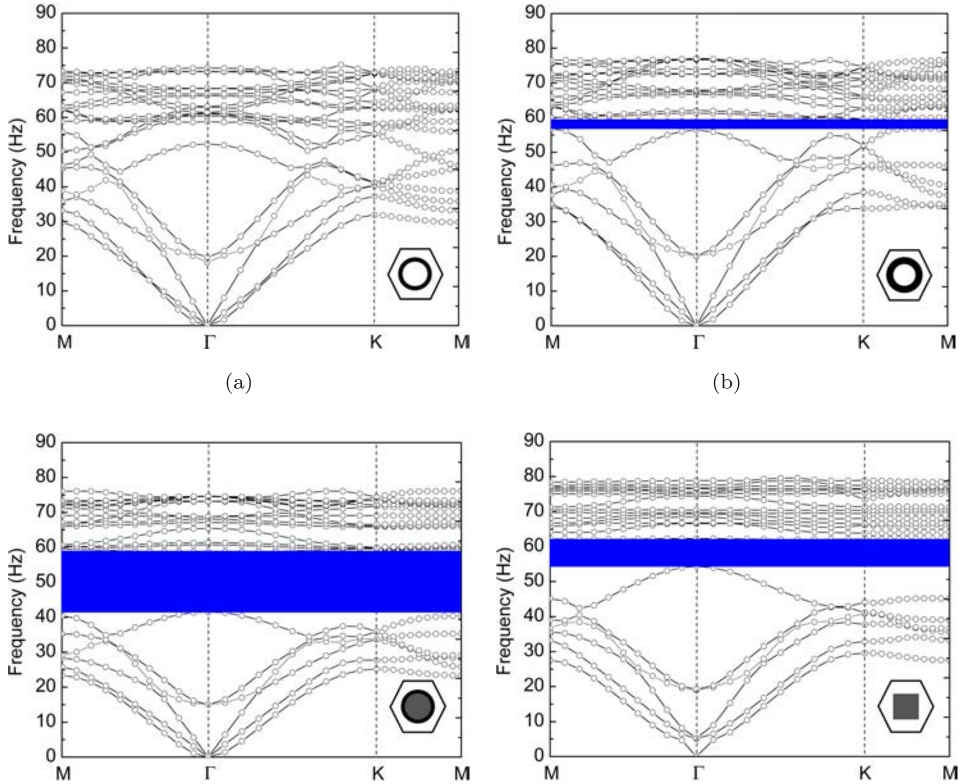
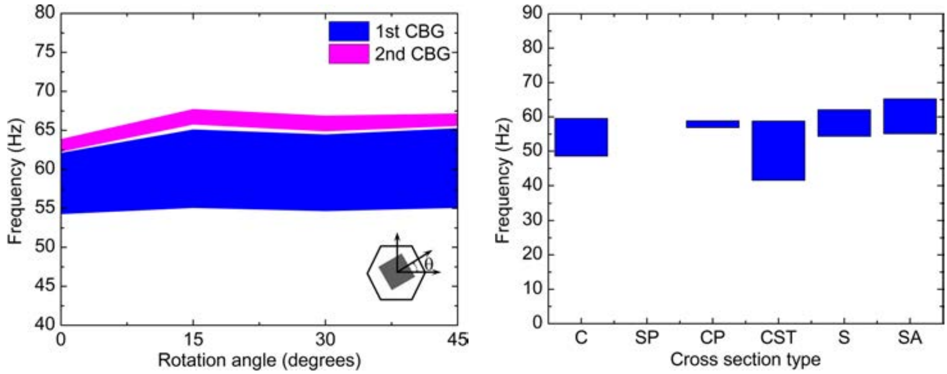


Fig. 6. Effect of the geometry of pile cross section on the evolution of CBGs in periodic composites with a hexagonal lattice. (a) Band structure for steel pipe (SP) piles: the outer radius is 0.74 m and the thickness of the steel is 0.05 m. (b) Band structure for concrete pipe (CP) piles: the outer radius is 0.74 m and the thickness of the concrete is 0.15 m. (c) Band structure for concrete-filled steel tube (CST) piles: the outer radius is 0.74 m and the thickness of the steel is 0.05 m. (d) Band structure for solid concrete square (S) piles: the edge length is 1.316 m. (e) Evolution of CBGs in the periodic composite with inclusions of S piles as a funcation of the rotation angle, and (f) comparation of CBGs for different types of pile cross sections, where C indicates circular solid concrete piles and SA indicates square pile with 45° rotation angle.



(e) (f)

Fig. 6. (*Continued*)

3.3. *Effect of material properties*

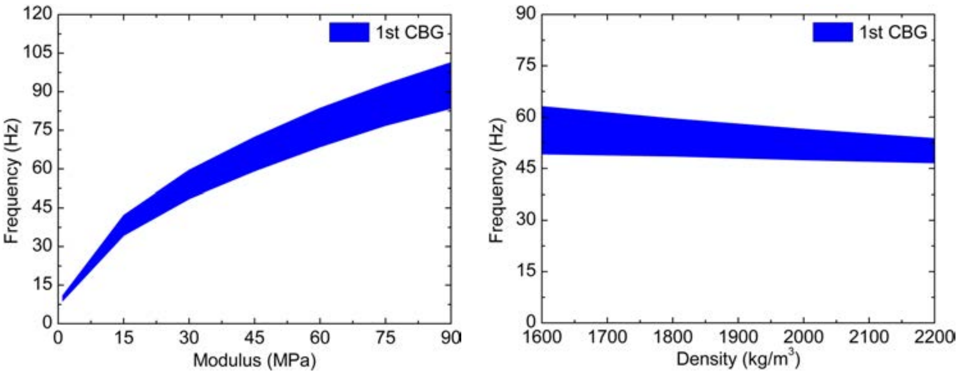
It has been demonstrated that the contrast in elastic constants, wave velocities and densities of constituents are of critical importance for governing the width and location of CBGs [Kafesaki *et al.*, 1995]. The contrast in aforementioned properties of the piles and the soil also exists, because the physical properties of the piles are relatively stable and can be tailored, while the properties of the soil may vary from site to site. Here we focus on the Young's modulus, the density and the viscosity of the soil to investigate the effects of these properties on the evolution of CBGs. In the following, we consider the periodic composite with a hexagonal lattice and the height and the volume fraction of the circular solid concrete piles are set as 4 m and 0.50, respectively.

3.3.1. *Young's modulus and density of soil*

The Young's modulus and the density of the soil vary in the range of 1–90 MPa and 1600–2200 kg/m^3, respectively. Figures 7(a) and 7(b) show the evolution of CBGs as a function of the Young's modulus and the density of the soil, respectively. When the modulus of the soil increases from 1 MPa to 90 MPa, the width of the CBG enlarges rapidly and shifts toward higher frequency range. In contrast, when the density of the soil increases from 1600 kg/m^3 to 2200 kg/m^3, the width of the CBGs shrinks steadily and shifts to lower frequency range. The evolution trend of these CBGs suggests that the Young's modulus of the soil has a greater impact on the evolution of CBGs. These results also indicate that we need take into account of the Young's modulus and the density of the soil at each site to achieve desired CBGs.

3.3.2. *Viscosity of soil*

It has been experimentally observed that soil exhibits time dependent behavior under constant loading, known as viscosity [Karmakar and Kushwaha, 2007;



(a) (b)

Fig. 7. Evolution of CBGs as a function of the Young's modulus (a) and the density (b) of the soil. The height and the volume fraction of the circular solid concrete piles in the RVE are 4 m and 0.50, respectively.

Widjaja and Hsien-Heng Lee, 2013]. Meanwhile, recent studies in phononic crystals show that viscosity has a great impact on the width and the location of CBGs [Liu *et al.*, 2008; Merheb *et al.*, 2008; Wei and Zhao, 2010]. These studies inspire us to investigate the effect of the viscosity of the soil on the evolution of CBGs in the considered periodic composites. Here, the dynamic modulus of the soil is modeled using the generalized Maxwell model, which captures the viscoelastic behavior of the soil. The viscosity observed from experimental studies are discrete, hence we consider the viscosity at the range of 0–300 kPa·s [Karmakar and Kushwaha, 2007]. Two relaxation times, $\tau_m = 0.01\,\mathrm{s}$ and $\tau_m = 0.05\,\mathrm{s}$, are employed in the generalized Maxwell model, respectively. Figures 8(a) and 8(b) show the evolution of CBGs as a function of the viscosity in the periodic composite with inclusions of circular piles. Compared with the CBG without considering viscosity, the width of the CBG enlarges slightly and shifts toward high frequency range as the viscosity increases for both relaxation times. This phenomenon can be interpreted by the characteristic of frequency-dependent storage modulus. As shown in Fig. 8(c), the coefficient of storage modulus, $\phi = (\omega\tau_m)^2/(1+(\omega\tau_m)^2)$, increases with the frequency for a given relaxation time and hence results in an increasing storage modulus. The results demonstrate that the viscosity of the soil is an important parameter that should be carefully considered when designing the periodic composite as surface wave barriers.

3.4. *Analysis of the periodic composite with finite RVEs*

In previous sections, we assume that the periodic composite is infinite along both x and y directions. However, this is not suitable in engineering practice. Here, we analyze the periodic composite with finite periodicities along x and y directions. The goals of this analysis are (i) to calculate the transmission coefficient of surface

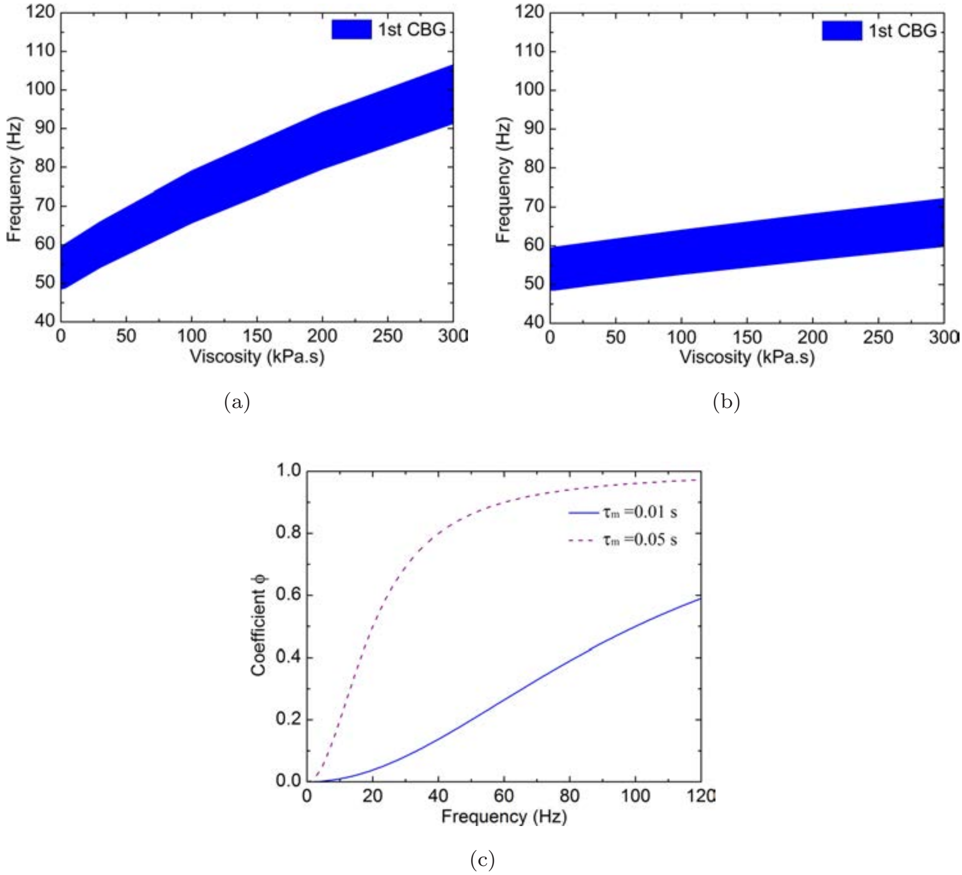


Fig. 8. Evolution of the CBG as a function of the viscocity for (a) relaxation time $\tau_m = 0.01\,\text{s}$, and (b) relaxation time $\tau_m = 0.05\,\text{s}$. (c) Coefficient of the storage modulus as a function of the frequency. The height and the volume fraction of the circular solid concrete piles in the RVE are 4 m and 0.50, respectively.

wave propagation to verify the simulated CBGs and (ii) to observe the behavior of the deformation in the periodic composite to reveal the mechanism of CBGs formation.

The considered periodic composite with a square lattice consists of an assembly of 8×6 RVEs with inclusions of circular solid concrete piles and sandwiched by two homogeneous parts of the soil. Perfectly matched layers (PMLs) are applied at the two ends of the homogeneous parts to prevent reflections by the scattering waves from the domain boundaries [Khelif *et al.*, 2010]. In addition, periodic boundary conditions are applied along y-direction of the periodic composite. The height and the volume fraction of the circular solid concrete piles in the system are set as 3 m and 0.50, respectively. To determine the transmission coefficient along ΓK-direction, an incident surface wave with polarizations along x- and y-directions is modeled by applying harmonic displacements with the amplitude of $u_0 = 0.01\,\text{m}$ and $v_0 = 0.01\,\text{m}$ at the interface between the PML and the homogeneous part on the left-hand side. The transmission coefficient is defined as $\phi = 20\log(d_i/d)$, where d_i and d are total averaged displacements collected along the interface between the PML and the homogeneous part on the right-hand side and the left-hand side, respectively.

The frequency domain analysis sweeping from 0 Hz to 75 Hz is performed and the simulated transmission coefficient as a function of frequency is plotted in Fig. 9(b). The strong attenuation zones agree well with the bandgaps along ΓK-direction in the band structure of the periodic composite (Fig. 9(a)). This result confirms that the simulated CBGs using Bloch theorem is reliable and also demonstrates the potential application of a finite number of piles as wave barriers.

To reveal the mechanism of CBGs formation, we further present the total displacement fields for the incident surface wave with frequency below and within the CBGs. Figures 10(a) and 10(b) show the total displacement fields of the periodic composite at the frequency of 35 Hz and 50 Hz, respectively. At the frequency of 35 Hz, the incident surface wave is partly reflected and partly passes through the periodic composite. In contrast, the incident surface wave is almost totally reflected and confined in the left homogeneous part when the frequency of the incident wave (50 Hz) lies within the CBG. This phenomenon is similar to the experimental observation in a periodic array of pillars when the frequency of incident surface elastic wave lies in a Bragg-type bandgap [Achaoui *et al.*, 2011]. We therefore deem that

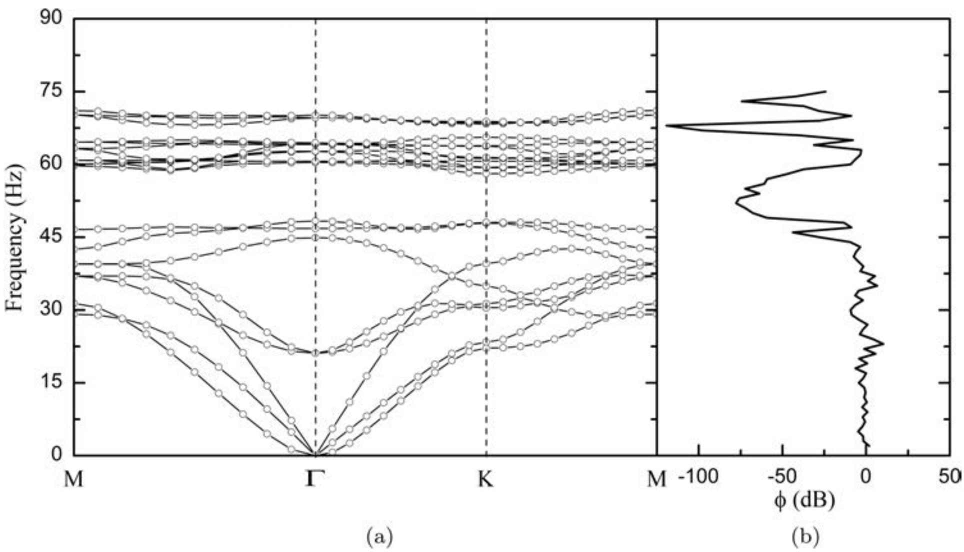


Fig. 9. (a) Band structure of the periodic composite with a square lattice and (b) the transmission coefficient along ΓK direction. The height and the volume fraction of the circular solid concrete piles in the RVE are 3 m and 0.50, respectively.

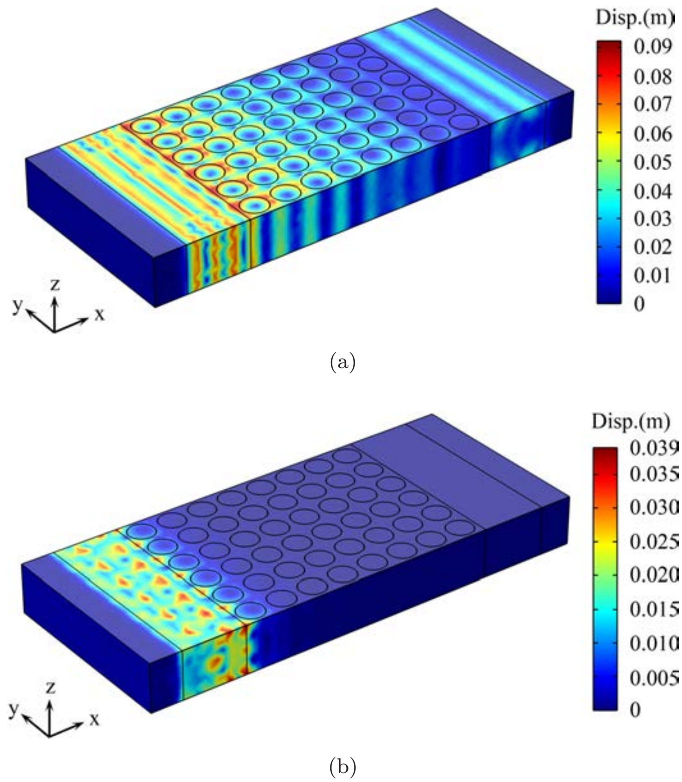


Fig. 10. Total displacement fields for surface wave propagation in the considered periodic composite with excitation frequencies (a) 35 Hz, below the CBG and (b) 50 Hz, within the CBG.

the simulated bandgaps are derived from Bragg interferences. It is worth noting that the frequency ranges of the simulated CBGs are scalable because the periodicity of the piles must be of the same order as the wavelength in Bragg-type CBGs [Liu *et al.*, 2000].

4. Conclusion

In summary, we have numerically investigated the isolation of surface wave-induced vibration using periodically modulated piles in soil. We observe that the periodic composites with a square lattice and a hexagonal lattice are much favorable for the formation of CBGs. The height, the shape, and the material properties of the piles play a significant role in governing the evolution of CBGs. Furthermore, we demonstrate that the width and the location of CBGs can be tuned by tailoring the contrast in the Young's modulus and the density between the piles and the soil. The material property of the soil, such as the viscosity, should also be considered. Finally, the analysis of surface wave propagation in the periodic composite with a finite number of piles confirms the simulated CBGs and also demonstrates the potential application of periodically modulated piles in soil as wave barriers. This work will provide design guidelines for civil engineers to isolate surface wave induced vibration.

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References

Achaoui, Y., Khelif, A., Benchabane, S., Robert, L. and Laude, V. [2011] "Experimental observation of locally-resonant and Bragg band gaps for surface guided waves in a phononic crystal of pillars," *Physical Review B* **83**, 104201.

Avilés, J. and Sánchez-Sesma, F. J. [1988] "Foundation isolation from vibrations using piles as barriers," *Journal of Engineering Mechanics* **114**(11), 1854–1870.

Beskos, D., Dasgupta, B. and Vardoulakis, I. [1986] "Vibration isolation using open or filled trenches," *Computational Mechanics* **1**, 43–63.

Brûlé, S., Javelaud, E., Enoch, S. and Guenneau, S. [2013] "Seismic metamaterial: How to shake friends and influence waves?" arXiv:1301.7642.

Dahl, J., Jensen, J. S. and Sigmund, O. [2008] "Topology optimization for transient wave propagation problems in one dimension," *Structural and Multidisciplinary Optimization* **36**, 585–595.

Gao, G., Li, Z., Qiu, C. and Yue, Z. [2006] "Three-dimensional analysis of rows of piles as passive barriers for ground vibration isolation," *Soil Dynamics and Earthquake Engineering* **26**(11), 1015–1027.

Goffaux, C. and Vigneron, J. [2001] "Theoretical study of a tunable phononic band gap system," *Physical Review B* **64**, 075118.

Gonella, S. and Ruzzene, M. [2008] "Analysis of in-plane wave propagation in hexagonal and re-entrant lattices," *Journal of Sound and Vibration* **312**(1/2), 125–139.

Hsieh, P.-F., Wu, T.-T. and Sun, J.-H. [2006] "Three-dimensional phononic band gap calculations using the FDTD method and a PC cluster system," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* **53**, 148–158.

Huang, J. and Shi, Z. [2011] "Application of Periodic Theory to rows of piles for horizontal vibration attenuation," *International Journal of Geomechanics* **13**(2), 132–142.

Huang, Z.-G. [2011] "Silicon-based filters, resonators and acoustic channels with phononic crystal structures," *Journal of Physics D: Applied Physics* **44**, 245406.

Kafesaki, M., Sigalas, M. and Economou, E. [1995] "Elastic wave band gaps in 3-D periodic polymer matrix composites," *Solid State Communications* **96**(5), 285–289.

Karmakar, S. and Kushwaha, R. [2007] "Development and laboratory evaluation of a rheometer for soil visco-plastic parameters," *Journal of Terramechanics* **44**(2), 197–204.

Kattis, S., Polyzos, D. and Beskos, D. [1999] "Vibration isolation by a row of piles using a 3-D frequency domain BEM," *International Journal for Numerical Methods in Engineering* **46**(5), 713–728.

Khelif, A., Achaoui, Y., Benchabane, S., Laude, V. and Aoubiza, B. [2010] "Locally resonant surface acoustic wave band gaps in a two-dimensional phononic crystal of pillars on a surface," *Physical Review B* **81**, 214303.

Kuang, W. M., Hou, Z. L. and Liu, Y. Y. [2004] "The effects of shapes and symmetries of scatterers on the phononic band gap in 2D phononic crystals," *Physics Letters A* **332**, 481–490.

Khelif, A., Aoubiza, B., Mohammadi, S., Adibi, A. and Laude, V. [2006] "Complete band gaps in two-dimensional phononic crystal slabs," *Physical Review E* **74**, 046610.

Liao, S. and Sangrey, D. A. [1978] "Use of piles as isolation barriers," *Journal of the Geotechnical Engineering Division* **104**, 1139–1152.

Liu, Z., Chan, C. and Sheng, P. [2002] "Three-component elastic wave band-gap material," *Physical Review B* **65**, 165116.

Liu, Y., Yu, D., Zhao, H., Wen, J. and Wen, X. [2008] "Theoretical study of two-dimensional phononic crystals with viscoelasticity based on fractional derivative models," *Journal of Physics D*: *Applied Physics* **41**, 065503.

Liu, Z., Zhang, X., Mao, Y., Zhu, Y., Yang, Z., Chan, C. and Sheng, P. [2000] "Locally resonant sonic materials," *Science* **289**, 1734–1736.

Maldovan, M. and Thomas, E. L. [2009] *Periodic Materials and Interference Lithography for Photonics*, *Phononics and Mechanics* (Wiely-VCH Verlag GmbH & Co. KGaA, Weinheim).

Merheb, B., Deymier, P., Jain, M., Aloshyna-Lesuffleur, M., Mohanty, S., Berker, A. and Greger, R. [2008] "Elastic and viscoelastic effects in rubber/air acoustic band gap structures: A theoretical and experimental study," *Journal of Applied Physics* **104**, 064913–064919.

Padrón, L., Aznárez, J. and Maeso, O. [2007] "BEM–FEM coupling model for the dynamic analysis of piles and pile groups," *Engineering Analysis with Boundary Elements* **31**(6), 473–484.

Phani, A. S., Woodhouse, J. and Fleck, N. [2006] "Wave propagation in two-dimensional periodic lattices," *The Journal of the Acoustical Society of America* **119**, 1995.

Sigalas, M. and Economou, E. [1993] "Band structure of elastic waves in two-dimensional systems," *Solid State Communications* **86**(3), 141–143.

Sigalas, M. and Economou, E. [1994] "Elastic waves in plates with periodically placed inclusions," *Journal of Applied Physics* **75**, 2845–2850.

Takemiya, H. [2004] "Field vibration mitigation by honeycomb WIB for pile foundations of a high-speed train viaduct," *Soil Dynamics and Earthquake Engineering* **24**(1), 69–87.

Tsai, P. H., Feng, Z. Y. and Jen, T. I. [2008] "Three-dimensional analysis of the screening effectiveness of hollow pile barriers for foundation-induced vertical vibration," *Computers and Geotechnics* **35**(3), 489–499.

Vasseur, J., Hladky-Hennion, A.-C., Djafari-Rouhani, B., Duval, F., Dubus, B., Pennec, Y. and Deymier, P. [2007] "Waveguiding in two-dimensional piezoelectric phononic crystal plates," *Journal of Applied Physics* **101**, 114904.

Wei, P. and Zhao, Y. [2010] "The influence of viscosity on band gaps of 2D phononic crystal," *Mechanics of Advanced Materials and Structures* **17**(6), 383–392.

Widjaja, B. and Hsien-Heng Lee, S. [2013] "Flow box test for viscosity of soil in plastic and viscous liquid states," *Soils and Foundations* **53**(1), 35–46.

Xiang, H., Shi, Z., Wang, S. and Mo, Y. [2012] "Periodic materials-based vibration attenuation in layered foundations: Experimental validation," *Smart Materials and Structures* **21**, 112003.