MEC 316:

Instrumentation and Solid Mechanics Laboratory

Guide To Report Writing and Instructions for Experiments

Fall 2023

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TABLE OF CONTENTS

TABLE OF CONTENTS 2
1 LAB 1: Determination of Young's Modulus and Poisson's Ratio 7
1.1 Objectives 7
1.2 Equipment 7
1.3 Background Information 7
1.3.1 Stress and Strain 7
1.3.2 Strain Gauge and Strain Indicator 8
1.3.2.1 Strain Gauges 8
1.3.2.2 Wheatstone Bridge 9
1.3.2.3 Strain Indicators 10
1.3.2.4 Balance Units 12
1.3.3 Determination of Young's Modulus and Poisson's Ratio 13
1.3.4 Experimentation 14
1.4 Experimental Procedure 15
1.4.1.1 Determination of Young's Modulus, E, and Poisson's Ratio, v 15
1.4.1.2 Tensile test for the stress-strain diagram. 16
1.5 Requirements for Report 16
1.6 References 17
2 LAB 2: Natural Vibration Modes of a Cantilever Beam 18
2.1 Objectives 18
2.2 Equipment 18
2.3 Background Knowledge 18
2.3.1 Deriving the Equation of Motion and its Solution for a Uniform Cantilever Beam 18
2.3.2 Specification of End Conditions 21
2.3.3 The Natural Frequencies of a Cantilever 22
2.3.4 Determination of the Natural Frequencies of a Uniform Cantilever Beam 24
2.4 Experimental Procedure 24
3 LAB 3: LabVIEW Based Instrumentation to Calibrate a Linear Variable Differential Transformer, and DC voltage and AC Signal Measurements 27
3.1 Objectives 27
3.2 Equipment 27
3.3 Background Knowledge 27
3.3.1 LVDT Principles of operation 27
3.3.2 Calibration of LVDT 29
3.3.3 DC Voltage Resolution 30
3.3.4 AC Signal Sampling Rate 30
3.4 Experimental Procedures 31
3.4.1 LVDT Calibration and measurement 31
3.4.2 DC Voltage Measurement 32
3.4.3 AC signal measurement 32

3.5 Data Analysis 33
4 LAB 4: Pholoelastic Stress Analysis of Beams 34
4.1 Objectives 34
4.2 Specimens and Instrumentations 34
4.3 Background Knowledge 34
4.3.1 Double Refraction and Stress Optical Law 34
4.3.2 Polariscope 35
4.3.3 Determination of material stress fringe value 'f_σ' for the given material. 37
4.3.4 The theoretical prediction for pure bending beam. 37
4.3.5 Determination of the material stress fringe value f_σ by pure bending beam. 41
4.4 Experimental Procedure: 42
4.4.1 Observation of the isochromatic fringe pattern. 42
4.4.2 Determination of the fringe order. 42
4.4.3 Determination of the material stress fringe value f_σ. 42
4.5 Requirement: 43
4.6 Part 2. Determination of the fiber stresses along the top and bottom edges of a beam subjected to three-point bending. 43
4.7 Testing procedure: 44
4.8 Requirement: 44
5 Lab 5: Shadow Moiré Method for Deflection Measurement, Shape Measurement, and Optical Metrology 45
5.1 Objective 45
5.2 Equipment 45
5.3 Background Knowledge 45
5.3.1 Shadow Moiré Method 45
5.4 Experimental Procedures 47
5.4.1 Testing Procedure: 47
5.4.2 Analysis Procedure: 47
5.5 Requirements: 47
6 LAB 6: Determination of Shear Modulus and Metal Fatigue 48
6.1 Objectives 48
6.2 Equipment 48
6.3 Background Knowledge 48
6.3.1 Determination of Shear Modulus. 48
6.3.2 Experimentation. 52
6.4 Experimental Procedure 52
6.5 Requirement 55
6.6 Fatigue and determination of S-N curve 55
6.6.1 Background Knowledge 55
6.7 Experimental Procedure (RBF-200 fatigue testing machine) 57
6.8 Testing Procedure 58
6.9 Requirement 60
7 Lab 7: Structural Instability 61
7.1 Purpose of Experiment 61
7.2 Equipment 61
7.3 Background knowledge 61

7.3.1 Determination of load-deflection curves and critical loads for buckling of straight columns with various end conditions. 61
7.3.1.1 Struts Subject To Axial Load 61
7.3.2 Buckling test of straight columns. 66
7.4 Experimental Procedure: WP 120 Vertical Buckling Test Device 67
7.5 Requirement 68
8 LAB 8: Straightness Measurement of Linear Motion 72
8.1 Objectives 72
8.2 Equipment 72
8.3 Background Information 72
8.3.1 Straightness Measurement Method 73
8.4 Experimental Procedure 75
9 Lab 9: Digital Image Correlation/Digital Speckle Photography Techniques for Deformation Analysis 78
9.1 Objective 78
9.2 Equipment 78
9.3 Background Knowledge. 78
9.3.1 Determination of tensile strain ε using the speckle method 78
9.4 Experimental Procedure 79
9.4.1 Testing Procedure 79
9.4.2 Analysis procedure 80
9.5 Requirements 81
10 Lab 10: Photoelasiticity for Stress Concentration Analysis 82
10.1 Objectives 82
10.2 Specimens and Instrumentations 82
10.3 Background Knowledge 82
10.3.1 Double Refraction and Stress Optical Law 82
10.3.2 Polariscope 83
10.4 Determination of Stress concentration in a perforated sheet undergoing tensile load. 85
10.5 Testing procedure: 87
10.6 Requirement: 88
1 WRITING LAB REPORTS FOR MEC 316 90
1.1 Introduction 90
1.2 Lab Report Format 90
1.3 Grading of the Lab Reports 91
1.4 Before Beginning Each Lab 92
1.5 Deadlines and Late Lab Reports 92
1.6 General Comments on Lab Report Writing 92
1.7 Figures 93
2 TIME SAVING TIPS 95
2.1 Report Writing 95
2.2 Analysis 95
2.3 Graphs and Figures 96
3 ERROR ANALYSIS 97
3.1 Introduction 97
3.2 Accuracy and Precision 97

► Accuracy 97
► Precision 98
3.3 Different Types of Measurement Error 98
► Systematic or bias error 98
► Precision or random error 99
► Illegitimate errors 99
3.4 Determining Measurement Uncertainty 99
3.5 Determining the Uncertainty of Input Variables 100
► Instrument uncertainty 100
► Sample measurement uncertainty 101
► Uncertainty of the Mean 103
► Total Uncertainty 104
3.6 Error Propagation 104
3.7 Absolute Uncertainty 105
3.8 Root Sum Square Uncertainty 105
3.9 Practical Matters 106
3.10 Some Useful Relationships 106
3.11 Further Reading 107
4 THE UNCERTAINTY TREE: TOWARDS A MORE ENJOYABLE ERROR ANALYSIS 108
4.1 Introduction 108
4.2 Constructing the Uncertainty Tree 109

Part I:
Laboratory Instruction Manual

LAB 1: DETERMINATION OF YOUNG'S MODULUS AND POISSON'S RATIO

1.1 Objectives

1. Familiarization with the transducers used in strain and displacement measurements.
2. Perform a tensile test to understand the operation of the strain indicator and balance unit, and the strain measurement technique.
3. Determination of material properties and observation of material response at different stages of loading.
4. Familiarization with strain gage and digital strain indicator, and their use for strain measurement.
5. Familiarization with the operation of Tinius Olsen 1000 Universal Digital Testing Machine.
6. Familiarization with using LVDT system for displacement measurement.

1.2 Equipment

- P3 Digital Strain Indicator.
- Computer with P3 software.
- Tinius Olsen 1000 Universal Digital Testing Machine.
- LVDT and displacement display unit.
- Aluminum and steel tensile specimen with four strain gages already mounted.
- Aluminum tensile specimen for destructive testing.

1.3 Background Information

1.3.1 Stress and Strain

Consider a bar subjected to the axial load *T*. Under no-load conditions the length of the bar is *L* and the diameter is *D*. The cross-sectional area of the bar is designated by *A*. If the load is applied such that the stress does not exceed the elastic limit of the material, the axial strain is given by

$$\epsilon_a = \frac{\left(\frac{T}{A}\right)}{E} = \frac{\sigma_a}{E} \tag{1.1}$$

where σ_a is the axial stress and *E* is Young's modulus for the material. The unit axial strain, ε_a, defined by the following relation:

$$\epsilon_a = \frac{\delta L}{L}. \tag{1.2}$$

1.3.2 Strain Gauge and Strain Indicator

1.3.2.1 Strain Gauges

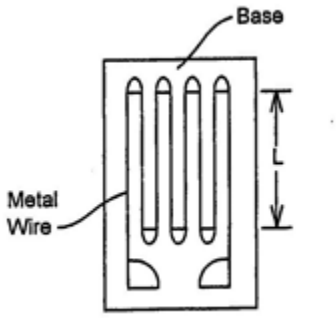


Figure 1.1: Strain Gage

Among all experimental methods for the measurement of small strains, the strain gauge method is one of the most accurate and popular. It is a point by point measurement technique. A strain gauge is made of a fine metal wire fixed on a base as shown in Figure. 1.1. The resistance *R* of the wire is calculated by

$$R = \frac{\rho l}{S} \tag{1.3}$$

where *l* is the wire length, *S* is the wire cross-sectional area, and ρ is the specific resistance, which is a material constant. A strain gauge is, therefore, a resistor. Its resistance depends on the length, cross-sectional area, and the material of the fine metal wire in the gauge. When a strain gauge is mounted on the surface of a specimen and the specimen is extended by a tensile force, the strain gauge will be extended too. A change of the resistance of the gauge will be caused by the change of the length of the fine metal wire fixed on the base of the gauge. The relationship between them is

$$\frac{\Delta R}{R} = K\frac{\Delta L}{L} \tag{1.4}$$

where *L* is the gauge length, *R* is the resistance of the gauge, ΔL is the extension of the gauge, and *K* is the gauge factor, which is supplied by the manufacturer. A substitution of $\Delta L/L = \varepsilon$ allows for a relationship of resistance of the gauge and the strain to be produced, as follows:

$$\frac{\Delta R}{R} = K\epsilon \tag{1.5}$$

1.3.2.2 Wheatstone Bridge

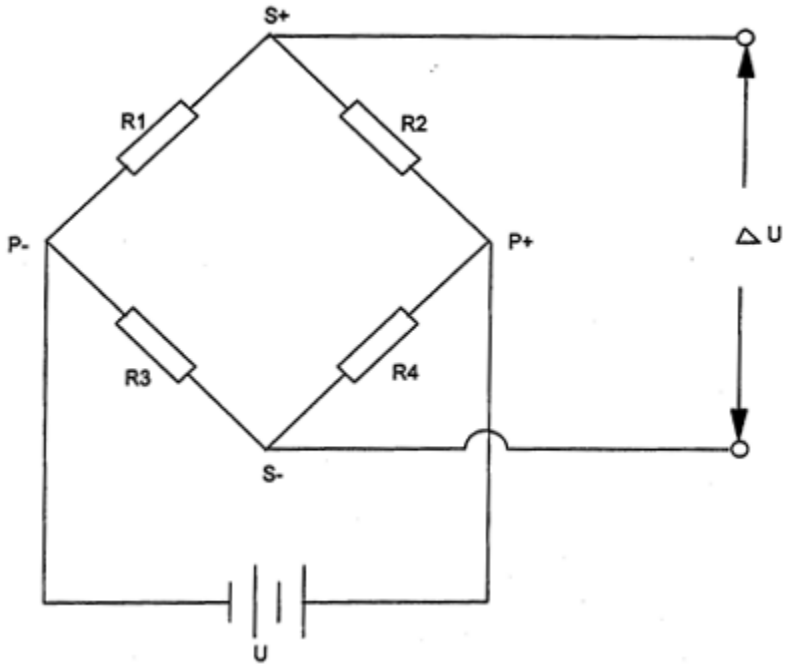


Figure 1.2: Wheatstone Bridge

The change of the resistance of a resistor can be easily measured by a Wheatstone bridge as shown in Figure. 1.2. We know that the bridge balance condition ($\Delta U = 0$) is

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \tag{1.6}$$

U is the excitation voltage. If $R_1 = R_2 = R_3 = R_4$ then $\Delta U = 0$. Now R_1 is a strain gauge that is already mounted on a specimen. Its resistance is R. Before the specimen has any deformation, the potential between S_+ and P_- is

$$\Delta U_{(S_+P_-)} = \frac{(R_1 U)}{R_1 + R_2} = \frac{U}{2}, \tag{1.7}$$

and the potential between S_- and P_- is

$$\Delta U_{(S_-P_-)} = \frac{(R_3 U)}{R_3 + R_4} = \frac{U}{2}. \tag{1.8}$$

When the specimen is loaded the resistance of the strain gage will be changed. The potential between S_+ and P_- is

$$\Delta U'_{(S_+P_-)} = \frac{[(R_1 + \Delta R_1)U]}{R_1 + \Delta R_1 + R_2} \tag{1.9}$$

The change of the potential is the output of the Wheatstone bridge

$$\Delta U'_{(S_+S_-)} = \frac{[(R_1 + \Delta R_1)U]}{R_1 + \Delta R_1 + R_2} - \frac{R_3 U}{R_3 + R_4} \tag{1.10}$$

Because $R_1 = R_2 = R_3 = R_4$then

$$\Delta U_{(S_+S_-)} = \frac{\Delta R_1 U}{4R + 2\Delta R} = \frac{K\epsilon U}{4 + 2K\epsilon} \tag{1.11}$$

The gauge factor *K* is usually around *2* and the strain within the elasticity range is very small. The second term of the denominator compared with the first term can be neglected. Therefore,

$$\Delta U_{(S_+S_-)} = \frac{K\epsilon U}{4} \tag{1.12}$$

Since both *K* and *U* are known, the relationship between the bridge output and the strain is as follows:

$$\Delta U = C\epsilon \tag{1.13}$$

where *C* is a constant that depends on the gauge factor and excitation voltage of the bridge. Thus, a physical measurement of ε is transformed into an electrical measurement by the Wheatstone Bridge. If R_1, R_2, R_3, and R_4 all are strain gauges, the strain of these gages are ε_1, ε_2, ε_3, and ε_4 respectively, the total output of the Wheatstone Bridge will be

$$\Delta U_{(S_+S_-)} = \frac{[K(\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4)U]}{4} \tag{1.14}$$

1.3.2.3 Strain Indicators

The output of the Wheatstone bridge is

$$\Delta U_{(S_+S_-)} = \frac{K\epsilon U}{4} \tag{1.15}$$

If R_1 is a strain gauge, the excitation voltage U cannot be too high, because of the higher the excitation voltage, the higher the gauge current. This will cause a higher temperature effect (i.e., the strain-gage wire will heat up and its resistance will change) and ΔU will drift. K is always around *2* and ε is very small. Therefore, ΔU will be a small fraction of U. For instance, if $K = 2$, $U = 6V$, and $\varepsilon = 333$ microstrain, then ΔU will be only about *4 mV*. For higher measurement sensitivity, a DC amplifier is necessary. A digital strain indicator is shown in Figure. 1.3. The left part is a Wheatstone bridge. The output from the bridge is connected to a DC amplifier. The output from the DC amplifier, the amplified output of the Wheatstone bridge, is connected to the digital display unit. An analog output also can be picked up from the output. The Amp. zero adjustment can be used to adjust the DC amplifier output to zero when the input is zero. The balance control of the bridge is a variable potentiometer across P_+ and P_-. Terminal b connects to S_+. R_1 is a strain gauge. If the resistance of R_1 has some difference with R ($R_2 = R_3 = R_4 = R$) adjust b until

$$r_1' = \frac{r_1 R_1}{r_1 + R_1}, \qquad r_2' = \frac{r_2 R_2}{r_2 + R_2}, \qquad \text{and} \qquad \frac{r_1'}{r_1} = \frac{R_3}{R_4}, \tag{1.16}$$

then the bridge will be in balance. If r >> R, the formula $\Delta U = KU\varepsilon_1/4$ still can be used.

The gage factor adjustment is a variable potentiometer too (a change in the position of terminal b, changes the excitation voltage). The strain indicator was designed for use with gages for which K = 2. If K = K' ≠ 2, the output ΔU will be changed to $\Delta U' = K'U\varepsilon/4$. Therefore, some correction must be made. Since the strain does not change, the reading ΔU' has some error. The correct reading should be ΔU. Since

$$\Delta U = \frac{KU\varepsilon}{4} \quad \text{and} \quad \Delta U' = \frac{K'U\varepsilon'}{4}, \tag{1.17}$$

the condition to keep the output the same as before is

$$K\varepsilon = K'\varepsilon'. \tag{1.18}$$

Therefore, the correct reading ε is

$$\varepsilon = \frac{K'\varepsilon'}{K} \tag{1.19}$$

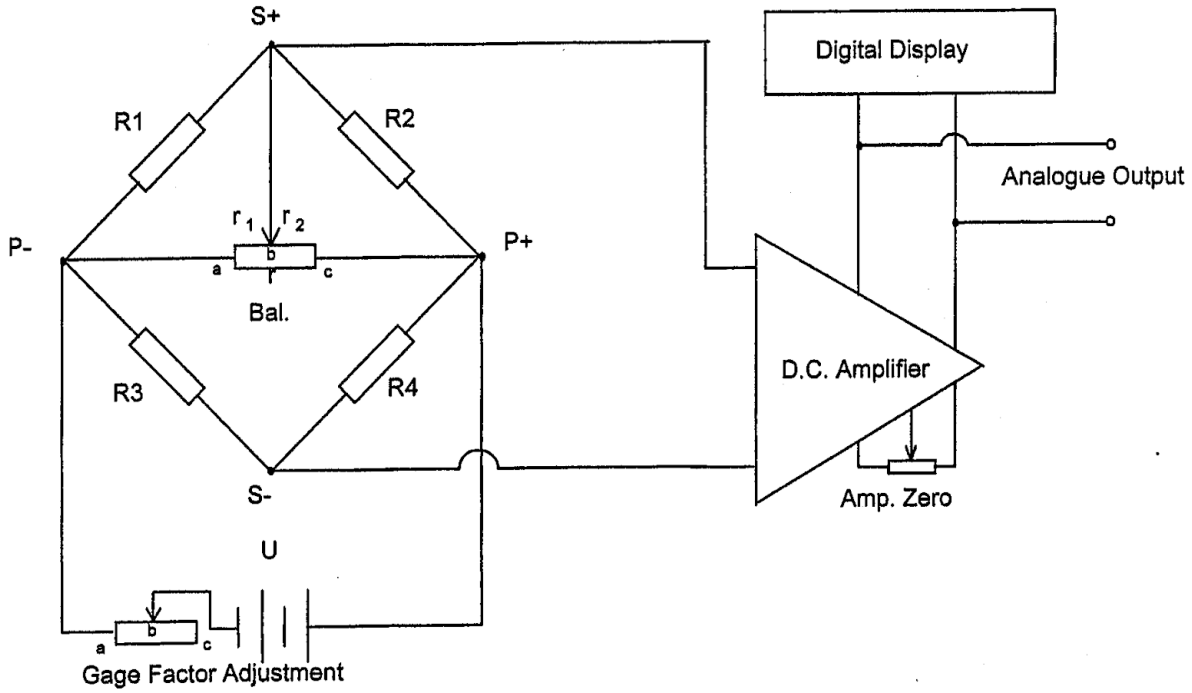


Figure 1.3: Digital Strain Indicator

1.3.2.4 Balance Units

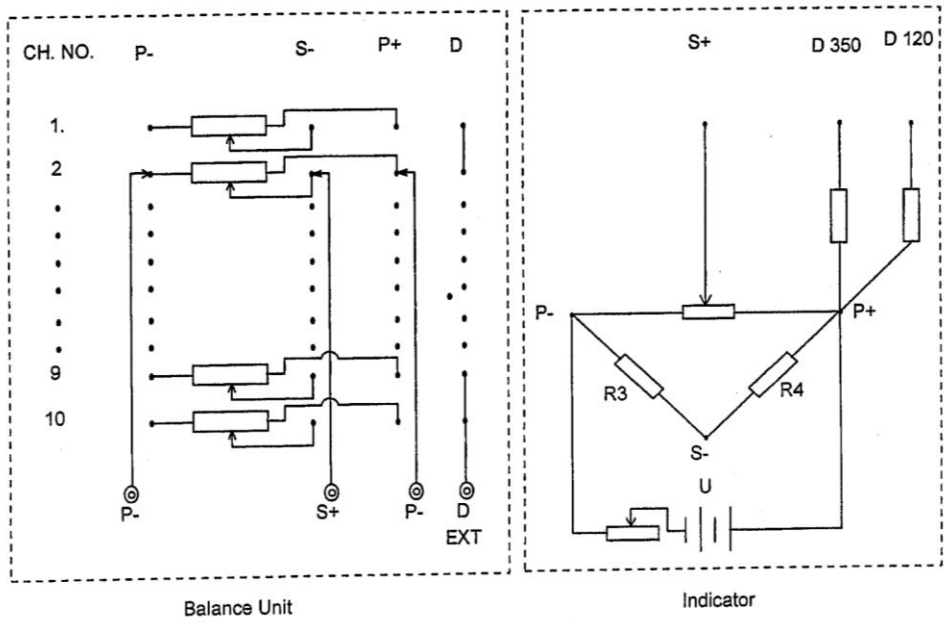


Figure 1.4: Balance Unit

For multiple channel measurement, a balance unit must be used. A balance unit is just like a switch. When a signal from channel 1 is being measured, you switch to channel 1. When a signal from channel 2 is being measured, you switch to channel 2 and so on. The only thing you must do is to make correct connections. Figure. 1.4 shows the balance unit and part of the strain indicator.

One balance unit can be used to switch up to 10 separate channels. For a quarter bridge (only R_1 is a strain gauge and the others are dummy resistors) measurement the binding posts P_+, P_-, S_+, S_-, and DEXT on the front panel of the balance unit must be connected to the corresponding front panel binding posts of the strain indicator. For 120 strain gauge, DEXT must be connected to D 120 of the strain indicator. The strain gauge must connect to the corresponding channel terminal P_- and S_+ and a group must be made across S_+ and P_- terminals. The balance control on the front panel of the strain indicator must be set at mid-way. Now separate channels have their own balance controls on the panel of the balance unit.

1.3.3 Determination of Young's Modulus and Poisson's Ratio

Experiment on the extension of prismatic bars under tensile load has shown that within certain limits, the elongation of the bar is proportional to the tensile force applied. For many structural materials, this simple linear relationship between the force and the elongation it produces was first formulated by the English scientist Robert Hooke in 1678 and bears his name. Using the notation:

P = the force-producing an extension of the bar
L = length of the bar
A = the cross-sectional area of the bar
δ = total elongation of the bar
E = the elastic constant of the material, called modulus of elasticity or Young's modulus.

From the above variables Hooke's law may be given by the following equation:

$$\delta = \frac{PL}{AE} \tag{1.20}$$

In a unit axial tension test, the stress σ in the prismatic bar is the force per unit of cross-sectional area, i.e.:

$$\sigma = \frac{P}{A} \tag{1.21}$$

Meanwhile, the axial strain is the elongation per unit length, is determined by the equation

$$\epsilon = \frac{\delta}{L} \tag{1.22}$$

Using equations (1.20), (1.21) and (1.22), Hooke's law may also be written in the following form:

$$\sigma = E\epsilon \tag{1.23}$$

It is observed that an axial elongation is always accompanied by lateral contraction of the bar and this ratio is named the Poisson's ratio, ν, defined as:

$$\nu = \frac{-(unit\ lateral\ contraction)}{(unit\ axial\ elongation)} \tag{1.24}$$

In the elastic limit ν is a constant for a given material. This constant is denoted by and is named after the name of French mathematician who determined this ratio analytically by using the molecular theory of the structure of the material.

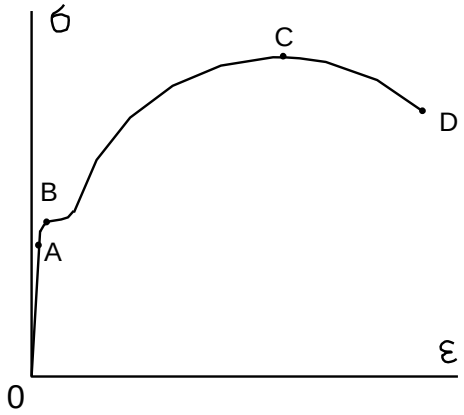


Figure 1.5: Stress-strain relationship of a structural steel

The proportionality between the tensile force and the corresponding elongation holds only up to a certain limiting value of the tensile stress, called the proportional limit, which is a material property. Beyond this limit, the relationship between stress and strain becomes more complicated. Figure 1.5 shows a typical tensile test diagram of structural steel, which gives the relationship between stress and strain. From Point *O* to *A*, the stress and the strain are proportional. Beyond Point *A*, Hooke's law no longer applies. Hence, the stress at Point *A* is the proportional limit. When loaded beyond this limit, the strain increases more rapidly for a given increment of stress. At Point *B*, a sudden additional elongation of the bar takes place without an appreciable increase in the tensile force. This phenomenon is called the yielding of the material, this typically occurs at a strain of 0.2%. The stress corresponding to Point *B* is called the yield stress. Upon further stretching of the bar, the material recovers its resistance and, as is seen from the diagram, the tensile force increases with the corresponding increase in elongation. At Point *C*, where the force attains its maximum, the state of stress is called the ultimate strength of the material. Beyond Point *C*, elongation of the bar takes place with decreasing force. Finally, at Point *D*, fracture of the bar occurs.

1.3.4 Experimentation

A flat specimen with two strain gauges on each side is mounted at the center along *x* and *y*-axes, respectively shown in Figure. 1.6. The strain gages are connected to a switch balance unit combine with a strain indicator to measure the strain. The specimen is to be loaded by a digital universal testing machine. The digital display of the testing machine will display the load.

Denoting the strain readings for gage 1 and 2 by ε_{xx} and ε_{yy}, respectively, and noting that

$$\sigma_{xx} = \frac{P}{A}, \tag{1.25}$$

where *P* is the loading force and *A* is the section area of the specimen, Young's modulus, *E,* can be calculated by

$$E = \frac{\sigma_{xx}}{\epsilon_{xx}} \tag{1.26}$$

And Poisson's ratio, *v*, can be calculated by

$$v = -\frac{\epsilon_{yy}}{\epsilon_{xx}} \tag{1.27}$$

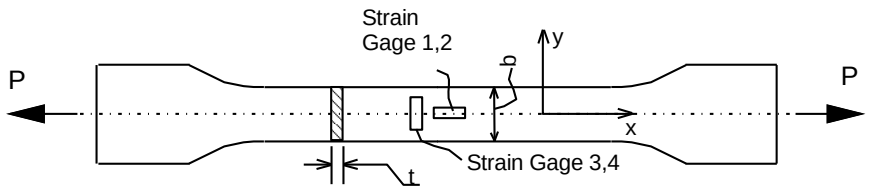


Figure 1.6: Diagram of a prismatic bar with strain gages applied

1.4 Experimental Procedure

1.4.1.1 Determination of Young's Modulus, E, and Poisson's Ratio, v

There are four strain gages on each of the testing specimen, two on each side, to avoid error from the flatness of the specimen. Familiarize yourself with their location, orientation, and purpose.

1) Take some time to familiarize yourself with the operation software, the P3 Digital strain indicator unit, and the Tinius Olsen digital universal testing machine before running the test.
2) Measure the width and the thickness of the specimen to find out the section area of the specimens.
3) Run the P3 software to start the setup procedure.
4) Select CHANNEL: check the channel/channels are being used. Select BRIDGE: check the quarter bridge option. Select GAGE FACTOR: input 2.05 as the gage factor for each channel.
5) Select BAL: select auto balance and check ZERO to set all strain gages initially zero.
6) Set the load and displacement readings on the front panel of the digital testing machine to zero.

7) Clamp the specimen into the upper grip of the testing machine, Use the faster compression or tension loading speed to move the lower grip to fit the length of the specimen. Tighten the two grips. Use the slowest tensile loading speed to give the specimen an initial load (about 20 lb.).
8) Open a folder on the Desktop of the computer using your own group name for saving your testing data files. Back to the P3 software and select RECORD. When the record panel comes up select "save in the computer", the testing data files will save in the hard drive.
9) The load must be applied in steps of 50 lb. The final load for steel specimen is to be 500 lb. and the aluminum specimen is to be 300 lb. (before the yield point). At each step take the readings of all strain gages. Run the test three times for each specimen (steel and aluminum).
10) Record the exact initial load and the load readings at each increment manually. At the same time check the "record" option on the front panel to begin recording the strain readings. When one trial of the test is finished, check "save" on the recording panel. The software will ask for file name and location for the testing file. Give a file name and the location of your own group folder and check "record". The testing data file with the given filename will be saved.

1.4.1.2 Tensile test for the stress-strain diagram.

1) An aluminum specimen is provided for the tensile test to find out the stress-strain diagram. Measure the width and the thickness of the specimen to find out the cross-sectional area of the specimen.
2) The upper and a lower fixture used to mount the LVDT are provided, and a digital displacement indicator is provided for displacement measurements.
3) Mount the upper and the lower fixture to the specimen and make the distance to 4 and a half inches. Mount the LVDT into the upper fixture and the core of the LVDT rests on the lower fixture.
4) Power on the Digital Testing Machine and the Digital Displacement Indicator Unit.
5) Adjust the position of the LVDT on the upper fixture to make the reading on the digital indicator close to zero and tighten the screw on the upper fixture. Take this zero reading as the zero position of the LVDT.
6) Set the load range of the testing machine at 100%(1000 lb. range). Push the LOAD button until the load display "0".
7) Insert a known thickness block (0.5 in) between the LVDT and the lower fixture. Take this reading and subtract from the zero reading as the calibration reading of -0.5 inches of displacement.
8) Load the specimen with an appropriate loading speed. Take the load and displacement readings at the same time until the specimen failure.

1.5 Requirements for Report

1) Calculate the Young's modulus E and Poisson's ratio ν of the steel and aluminum specimen using the linear regression method.
2) Plot out the stress vs. strain curve of the tensile test of the aluminum specimen. Give the proportional limit, yield point, and ultimate strength of the material.

3) Find out the uncertainties of the experimental results and the uncertainty tree.

1.6 References

"An Introduction to the Mechanics of Solids", 2nd Edition by Stephen H. Crandall, Norman C. Dahl, and Thomas J. Lardner, Published by McGraw-Hill Book Company, 1976.

"Experimental Stress Analysis", 2nd Edition, by J.W. Dally and W.F. Riley, Published by McGraw-Hill Book Company. 1978.

"The Dynamical Behavior of Structures", 2nd Edition by G.B. Warburton, Published by Pergammon Press Ltd., 1976.

LAB 2: NATURAL VIBRATION MODES OF A CANTILEVER BEAM

2.1 Objectives

1. Familiarization with the operation of the instruments used to generate and study vibrations.
2. Understand the dynamical behavior of a vibrating cantilever.

2.2 Equipment

- Digital Function Generator
- Power Amplifier
- Shaker
- Piezoelectric Sensor
- Oscilloscope
- Strobe Light.

2.3 Background Knowledge

Techniques used to model and study vibrations are vital to successful mechanical design. When an object is excited at one of its natural frequencies it causes a mechanical resonance. This means the system will be responding at a greater amplitude to an induced vibration or oscillatory load. Often this will lead to violent, and potentially catastrophic, failure of the system. The goal then is to design a system such that operational conditions do not cause these resonance effects.

2.3.1 Deriving the Equation of Motion and its Solution for a Uniform Cantilever Beam

In deriving the equation governing free undamped vibration in flexure of beams it is assumed that vibration occurs in one of the principle planes of the beam. The effects of rotatory inertia and of transverse shear deformation are neglected. Gravitational forces will also be neglected by measuring the displacement from the position of static equilibrium of the beam.

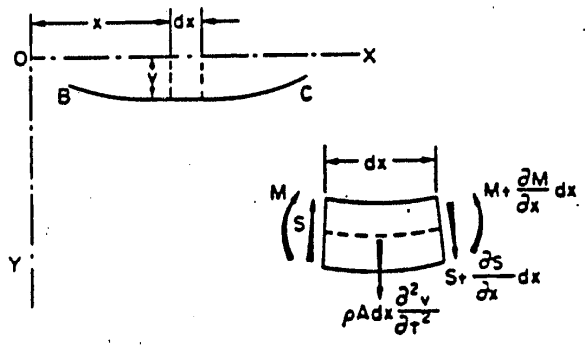


Figure 2.1: Free Body Diagram of a differential element of a beam experiencing vibrational flexure

In Figure 2.1, the line BC represents the centerline of the beam during vibration; the displacement at any section *x* at time *t* is denoted by v. The forces and the moments on an element of length *dx* are shown also in Figure 1; *S* and *M* are the shear force and bending moment respectively at section x; the inertial force on the element is

$$\rho A dx \frac{\partial^2 v}{\partial t^2} \tag{2.1}$$

Where ρ is the density of the material of the beam and A is its cross-sectional area.

Taking moments about the center line of the element (neglecting products of small quantities), and resolving for forces in the Y-direction we obtain the moment equation

$$Sdx + M - \left(M + \frac{\partial M}{\partial x} dx\right) = 0 \tag{2.2}$$

or

$$S = \frac{\partial M}{\partial x} \tag{2.3}$$

And the force equation,

$$\frac{\partial S}{\partial x} = \rho A \frac{\partial^2 v}{\partial t^2} \tag{2.4}$$

From the relation between bending moment and curvature and the approximate curvature-displacement relation, used in determining static deflections of beams, we have

$$M = -EI\frac{\partial^2 v}{\partial x^2}, \tag{2.5}$$

where E is Young's modulus and I is the relevant second moment of area of the cross-section (in this case: $\frac{bh^3}{12}$). Combining equations (2.3) to (2.5) we obtain

$$\frac{\partial^2}{\partial x^2}\left(-EI\frac{\partial^2 v}{\partial x^2}\right) = \rho A\frac{\partial^2 v}{\partial t^2} \tag{2.6}$$

Equation (2.4) can be used for uniform and non-uniform beams; for the latter the flexural rigidity (EI) and the mass per unit length (ρA) are functions of the coordinate x. For a beam of uniform cross-section, Equation (2.6) reduces to

$$EI\frac{\partial^4 v}{\partial x^4} + \rho A\frac{\partial^2 v}{\partial t^2} = 0 \tag{2.7}$$

Which is the equation of motion for the beam. For free vibration, $v(x,t)$ must be a harmonic function of time such that

$$v(x,t) = V(x)\,sin(\omega t + \alpha) \tag{2.8}$$

Substituting equation (8) in (7), we obtain

$$\frac{d^4V}{dx^4} - \frac{\rho A\omega^2}{EI}V = 0. \tag{2.9}$$

A solution of Equation (9) of the form

$$V = Be^{\lambda_o x} \tag{2.10}$$

is satisfactory, if

$$\lambda_0^4 = \frac{\rho A\omega^2}{EI}. \tag{2.11}$$

This has four roots of

$$\lambda_0 = \pm\lambda$$

And

$$\lambda_0 = \pm i\lambda$$

Where

$$\lambda = \left(\frac{\rho A \omega^2}{EI}\right)^{\frac{1}{4}} \tag{2.12}$$

Of which the general solution is

$$V = B_1 \sin(\lambda x) + B_2 \cos(\lambda x) + B_3 \sinh(\lambda x) + B_4 \cosh(\lambda x) \tag{2.13}$$

2.3.2 Specification of End Conditions

The four constants are determined from the end conditions; the standard end conditions are:

a) Simply supported or pinned, for which the displacement is zero and the bending moment is zero as there is no rotational constraint

b) Fixed or clamped for which the displacement and slope are zero

c) Free, for which the bending moment and shear force are zero.

In terms of function $V(x)$ these conditions for a uniform beam are:

a) Simply supported:

$$V = 0 \; and \; \frac{d^2V}{dx^2} = 0 \tag{2.14}$$

b) Clamped:

$$V = 0 \; and \; \frac{dV}{dx} = 0 \tag{2.15}$$

c) Free:

$$\frac{d^2V}{dx^2} = 0 \; and \; \frac{d^3V}{dx^3} = 0 \tag{2.16}$$

2.3.3 The Natural Frequencies of a Cantilever

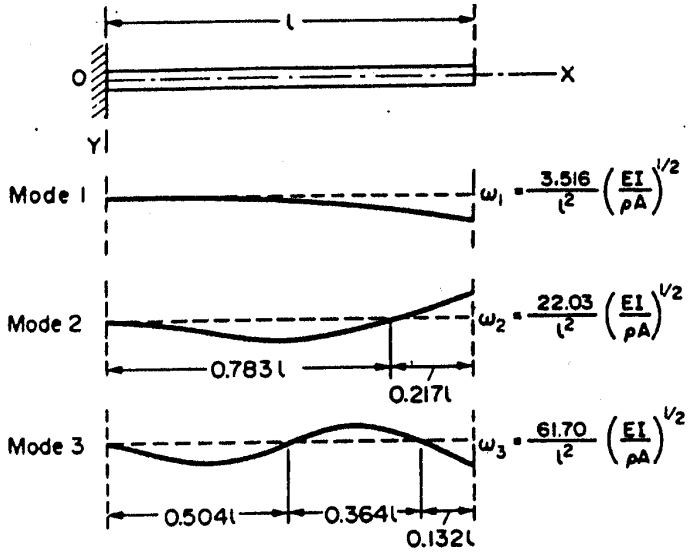


Figure 2.2: First Three Vibrational Modes and Natural Frequencies of a Cantilever Beam

With the origin at the fixed end, as in Figure 2.2, and using Equations (2.15) and (2.16), the end conditions are:

a) At x = 0 , V = 0, or $0 = B_2 + B_4$.

b) At x = 0 , $\frac{dV}{dx}$ = 0, or $0 = \lambda B_1 + \lambda B_3$.

c) At x = L, $\frac{d^2V}{dx^2}$ = 0, or: $0 = \lambda^2\left(-B_1 \sin(\lambda L) - B_2 \cos(\lambda L) + B_3 \sinh(\lambda L) + B_4 \cosh(\lambda L)\right)$.

d) At x = l, $\frac{d^3V}{dx^3}$ = 0, or: $0 = \lambda^3\left(-B_1 \cos(\lambda L) + B_2 \sin(\lambda L) + B_3 \cosh(\lambda L) + B_4 \sinh(\lambda L)\right)$

Hence

$$B_3 = -B_1 \tag{2.17}$$

And

$$B_4 = -B_2 \tag{2.18}$$

And

$$B_1(\sin(\lambda L) + \sinh(\lambda L)) + B_2\left(\cos(\lambda L) + \cosh(\lambda L)\right) = 0 \tag{2.19}$$

And

$$-B_1(\cos(\lambda L) + \cosh(\lambda L)) + B_2\big(\sin(\lambda L) - \sinh(\lambda L)\big) = 0 \tag{2.20}$$

Using Equations (2.19) and (2.20), a matrix is formed to eliminate B_1 and B_2 as follows:

$$\begin{bmatrix} (\sin(\lambda L) + \sinh(\lambda L)) & \big(\cos(\lambda L) + \cosh(\lambda L)\big) \\ -(\cos(\lambda L) + \cosh(\lambda L)) & \big(\sin(\lambda L) - \sinh(\lambda L)\big) \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2.21}$$

To remove B_1 and B_2 the determinate of the matrix is used.

$$(\sin^2(\lambda L) - \sinh^2(\lambda L)) + \big(\cos(\lambda L) + \cosh(\lambda L)\big)^2 = 0 \tag{2.22}$$

or

$$\sin^2(\lambda L) + \cos^2(\lambda L) + 2\cos(\lambda L)\cosh(\lambda L) + \cosh^2(\lambda L) - \sinh^2(\lambda L) = 0 \tag{2.23}$$

or

$$\cos(\lambda L)\cosh(\lambda L) = -1 \tag{2.24}$$

The successive roots, λ_1, λ_2, λ_3, … of Equation (2.24), from which the natural frequencies can be obtained, are given by

$$\lambda_1 = \frac{1.875}{L}, \qquad \lambda_2 = \frac{4.694}{L}, \qquad \lambda_3 = \frac{7.855}{L}, \qquad \lambda_r \approx \frac{\left(r - \frac{1}{2}\right)\pi}{L} \; for \; r \geq 4 \tag{2.25}$$

and the natural frequency can be determined with Equation (2.12) substituting values from Equation (2.25) will allow the natural frequencies to be expressed in terms of material properities.

$$\omega_n = (\lambda_n)^2 \sqrt{\frac{EI}{\rho A}} \tag{2.26}$$

The shape of the r^{th} mode, in terms of a single arbitrary constant, is

$$\mathrm{Vr(x)} \; = \; \mathrm{B_r}\,[\cosh(\lambda_r x) - \cos(\lambda_r x) + \eta(\sinh(\lambda_r x) - \sin(\lambda_r x))] \tag{2.27}$$

where

$$\eta = \frac{\cosh(\lambda_r x) + \cos(\lambda_r x)}{\sinh(\lambda_r x) + \sin(\lambda_r x)} \tag{2.28}$$

2.3.4 Determination of the Natural Frequencies of a Uniform Cantilever Beam

This experiment is designed to give students a physical view of the nature of the dynamical behavior of a vibrating cantilever by measuring the natural frequencies and observing the vibration mode shapes of the cantilever beam using both an electronic and a visual technique.

A digital function generator, a power amplifier, a shaker, a cantilever beam, a piezoelectric sensor, an oscilloscope, and a strobe light will be used in this experiment.

A digital function generator is used to create a digital signal at a specified, and adjustable, amplitude and frequency. The sinusoidal electric signal produced by the digital function generator is sent to the power amplifier. The signal from the function generator is then amplified such that it is powerful enough to activate the shaker. The shaker is excited by the amplified input signal and will create an oscillatory vibration in phase with it. The vibration tip of the shaker is centered about the entire beam as show in Figure 3 and defines the fixed end of the cantilever experiment. The pulses provided by the shaker will excite the cantilever beam such that the material vibrates based the input signal to the shaker, and the materials properties.

A piezoelectric sensor shall be used to empirically determine the vibration characteristics of the cantilever beam. The piezoelectric sensor contains a special material, often a ceramic, which generates an electric charge in response to mechanical stress. When the sensor is under dynamic loading, it will produce electricity, and if excited by an AC signal it can be used to measure dynamic force. The output of the piezoelectric is connected to an oscilloscope. Since the deflection of the cantilever is always higher when the beam is vibrating at one of the natural frequencies the output from the piezoelectric is also higher at these frequencies. Due to this, as one modulates the frequency provided by the function generator such that it approaches and then passes through one of natural frequencies, they will see a sine wave that increases in size to a peak and then rapidly begins to decline in size. This behavior shall repeat for each natural frequency.

A strobe light shall be used to provide a visual check of the frequencies determined used the oscilloscope. The strobe light is a frequency adjustable flashing light. When the cantilever is excited at one of its natural frequencies, the strobe can adjusted to flicker at a multiple of the natural frequency of the cantilever, the vibration modes can be seen clearly. However, due to the active length of the specimen higher order natural frequencies may be difficult (or impossible) to detect visually.

2.4 Experimental Procedure

1. Review the experimental setup as shown in Figure 2.3. Ensure that all components are powered off.

2. Open the Natural Frequencies LabVIEW application. Take note of the Input and Output sections and what information is contained within them.
3. If the Cantilever beam is in the beam clamp, remove it. After which take the following measurements of the beam using the provided inspection equipment.
    a. Measure the Total Beam Length.
    b. Measure the Beam Width.
    c. Measure the Beam Height.
    d. Measure the Beam Weight.
    e. Look up the Young's modulus for the material(ask the instructor what the material is).
4. Replace the specimen in the Beam Clamp. Ensure that the specimen is well clamped.
5. Measure the Active Beam Length (See Figure 2.3). The Active Beam Length should be about 12 inches.
6. Run the Natural Frequencies application. Take note of the natural frequencies. These are your theoretical values.
7. Ensure all connections are made properly. Power on all of the equipment.
8. Set the frequency on the function generator to a value 10% less than to the natural frequency. Ask your instructor for assistance using the function generator as the equipment may be different from group to group.
9. Slowly turn up the gain on the power amplifier until the beam begins to move. ***Do not over excite the beam, this can damage and break it.***
10. Turn up the frequency on the function generator using coarse adjustment while watching the oscilloscope (Note: You will have to make changes to the voltage and time settings to keep the waveform with the area of the screen). Continue to do this until you see the wave form begin to decrease in size. Confirm this behavior with your instructor.
11. When you have empirically determined a natural frequency, set the strobe light appropriately and then shine it at the beam. Take note of the oscillatory behavior of the beam.
12. Repeat Steps 8 – 11 for each natural frequency.

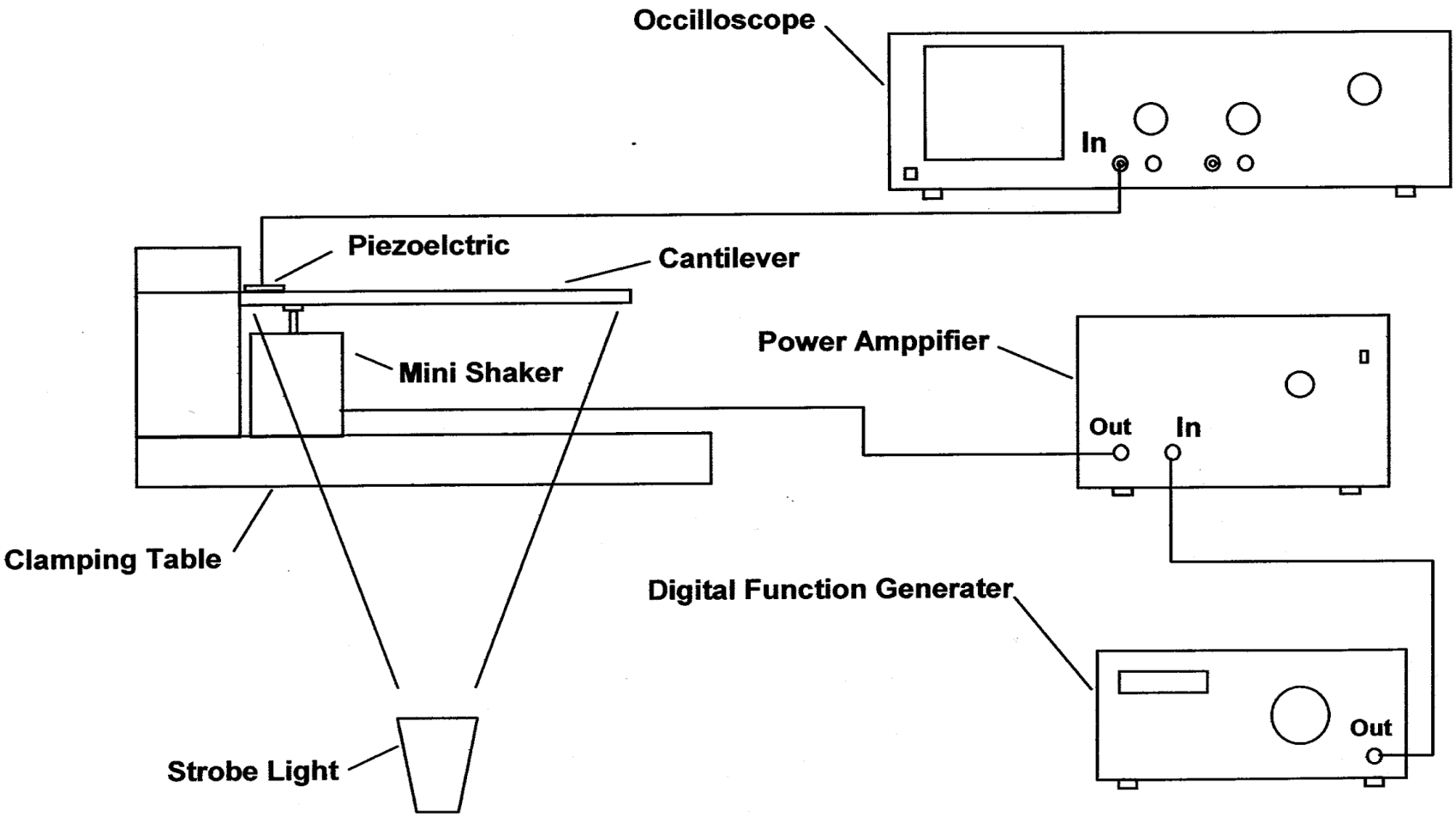


Figure 2.3: Experimental setup for Vibration Test

LAB 3: LABVIEW BASED INSTRUMENTATION TO CALIBRATE A LINEAR VARIABLE DIFFERENTIAL TRANSFORMER, AND DC VOLTAGE AND AC SIGNAL MEASUREMENTS

3.1 Objectives

Through a data acquisition (DAQ) computer system create a LabVIEW program to find out the Displacement-Output Voltage relationship of a LVDT, understand the basic principle of a LVDT, and create programs using LabVIEW to measure DC voltage and AC signal.

3.2 Equipment

- Stand with a micrometer
- LVDT
- Power for LVDT
- DC Power Supply
- Digital Function Generator
- NI USB 6008 DAQ Box
- Laptop with NI DAQmax and LabVIEW software

3.3 Background Knowledge

3.3.1 LVDT Principles of operation

The LVDT is frequently used to measure displacements and produces an analog signal output. It can be used with computer aided data acquisition systems.

A schematic diagram of the differential transformer is shown in Figure. 3.1. Three coils are placed in a linear arrangement as shown with a magnetic core which may move freely inside the coils. The construction of the device is indicated in Figure. 3.2. An alternating input voltage is impressed in the center coil, and the output voltage from the two end coils depends on the magnetic coupling between the core and the coils. This coupling, in turn, is dependent on the position of the core. Thus, the output voltage of the device is an indication of displacement of the core.

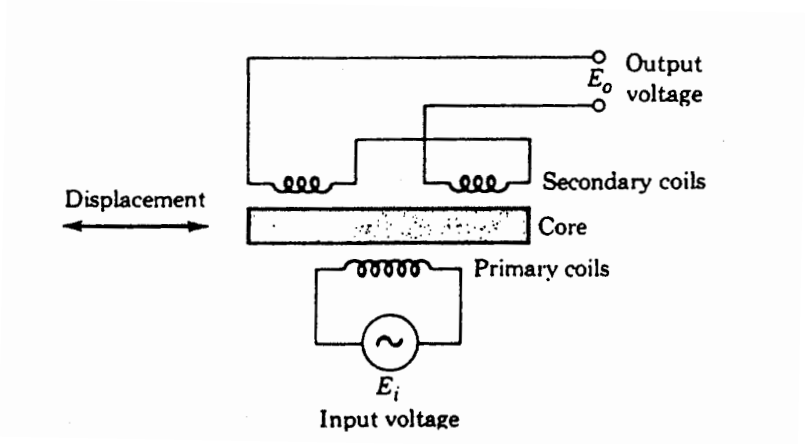


Figure 3.1: Schematic diagram of differential transformer

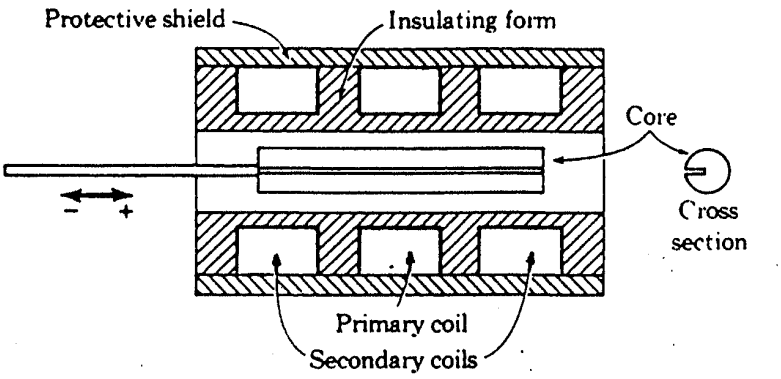


Figure 3.2: Construction of differential transformer device

The excitation of such devices is normally a sinusoidal voltage of 3 to 15 V RMS amplitude and frequency of 60 to 20,000 Hz. The two identical secondary coils have induced in them sinusoidal voltages of the same frequency as the excitation; however, the amplitude varies with the position of the iron core. When the secondary coils are connected in series opposition, a null position exists at which the net output E_0 is essentially zero. Motion of the core from null then causes a larger mutual inductance (coupling) for one coil and a smaller mutual inductance for the other, and the amplitude of E_0 becomes a nearly linear function of core position for a considerable range either side of null. The voltage E_0 undergoes a 180^o phase shift in going through null.

These instruments record the actual waveform of the output as an amplitude-modulated sine wave, which is usually undesirable. What is desired is an output-voltage record that looks like the mechanical motion being measured. To achieve the desired results, demodulation and filtering must be performed; if it is necessary to detect unambiguously the motions on both sides of null, the demodulation must be phase-sensitive.

Figure. 3.3 shows the circuit arrangement for phase-sensitive demodulation using semiconductor diodes. Ideally, these pass current only in one direction; thus, when f is positive and e is negative, the current path is *efgcdhe*, while when f is negative and e positive, the path is *ehedgfe.* The current through R is therefore always from c to d. A similar situation exists in the lower diode bridge. It is then necessary to connect e_0 of Figure. 3.3 to the input of a low-pass filter

which will pass the frequencies present in x_i but reject all those (higher) frequencies produced by the modulation process. The design of such a filter is eased by making the LVDT excitation frequency much higher than the x_i frequencies.

3.3.2 Calibration of LVDT

The output from an LVDT is an analog signal proportional to the displacement. The relationship between the displacement and the output voltage must be found through the calibration procedure so that the displacement can be determined from the calibration curve afterward.

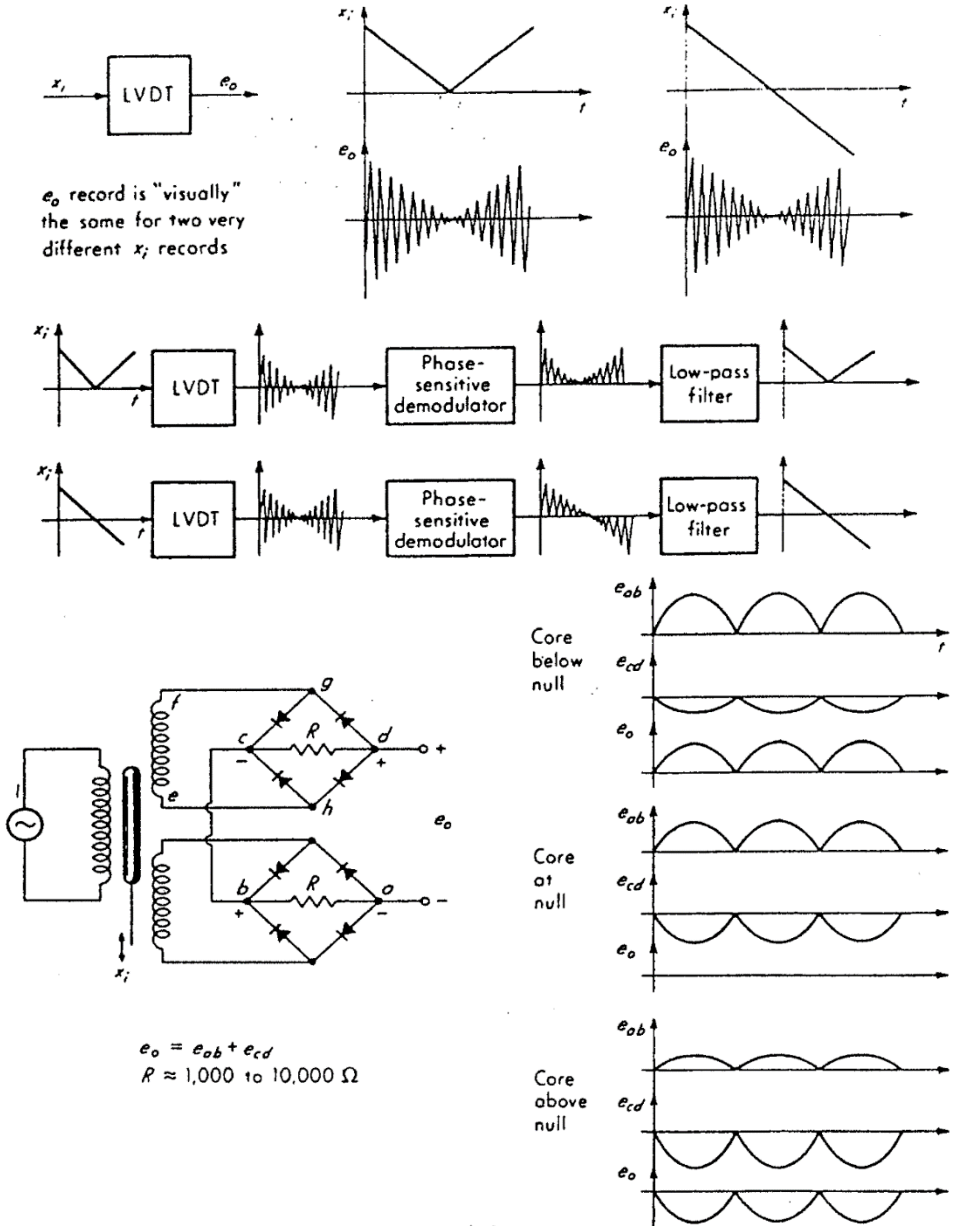


Figure 3.3: circuit arrangement for phase-sensitive demodulation by semiconductor diodes

3.3.3 DC Voltage Resolution

$$V_{cw} = \frac{range}{2^{resolution}}, \tag{3.1}$$

Where the resolution is given in bits. For example, a 12-bit DAQ board with a 0 to 10 V range detects a 2.4 mV change. This is calculated as follows:

$$V_{cw} = \frac{range}{2^{resolution}} = \frac{10V}{2^{12}} = 2.4mV, \tag{3.2}$$

While the same board with a -10 to 10V range detects only a change of 4.8 mV:

$$V_{cw} = \frac{range}{2^{resolution}} = \frac{20V}{2^{12}} = 4.8mV, \tag{3.3}$$

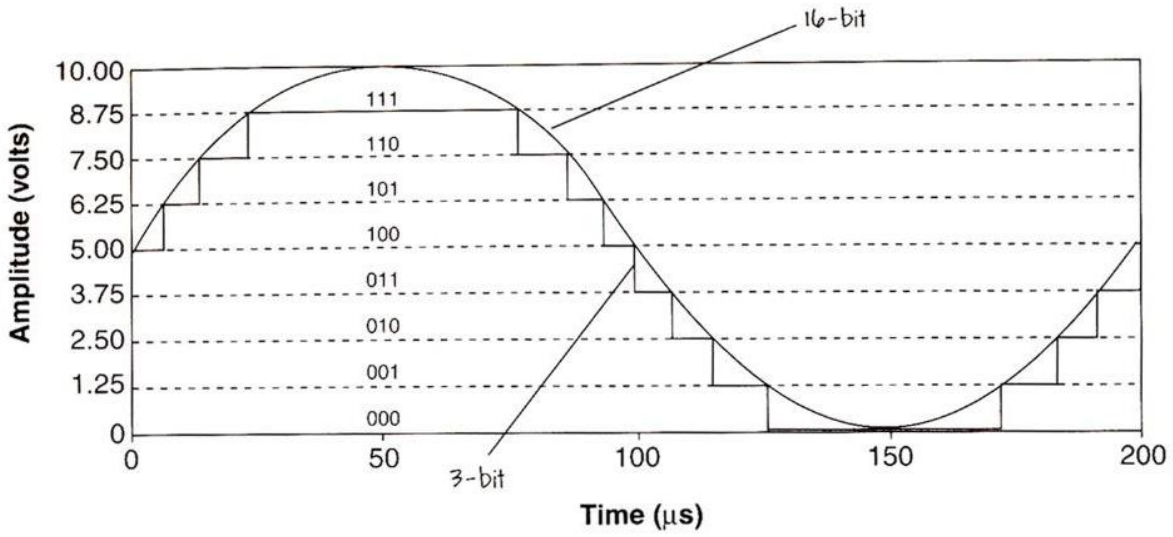


Figure 3.4: Comparison of resolution (16-bit vs. 3-bit for 5 kHz sine wave)

3.3.4 AC Signal Sampling Rate

Sampling rate must be sufficiently higher than the measuring frequency. Usually, Sampling rate is selected as 30 times of measuring frequency.

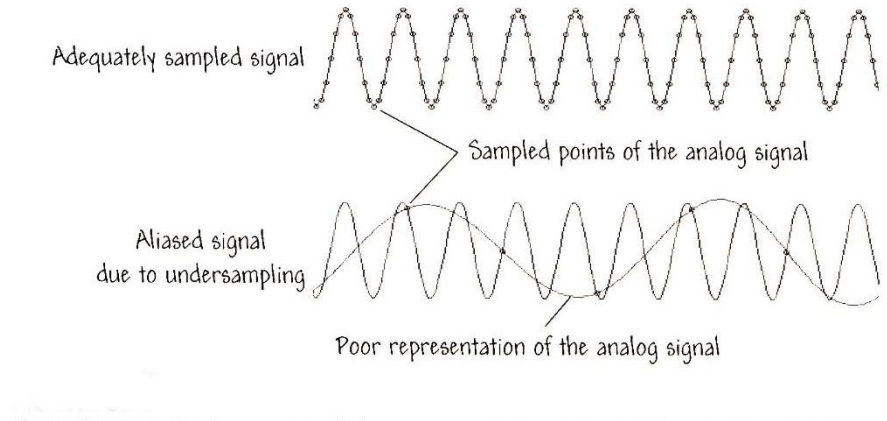


Figure 3.5: Effect of sampling rate in obtaining the analog signal

3.4 Experimental Procedures

3.4.1 LVDT Calibration and measurement

Insert the LVDT into the calibration unit.

1. Connect the black and red wires from the LVDT to the power supply unit (12V). Connect the output wires from the LVDT to a specific input channel of NI USB 6008 DAQ module.
2. It is recommended to open the NI MAX software to obtain the device number of NI USB 6008 DAQ module and input channel connected.
3. Create a program using LabVIEW to do the calibration task.

- On Block Diagram from EXPRESS to INPUT get the DAQ assist Vi----Acquire Signal-----Analog Input----Voltage then check the device number should agree with the device number in "devices and interfaces" in NI MAX. Use the DAQ Vi to acquire data. Set sampling rate at 1000 and number of samples at 1000 too. From MATHMATICS to PROBOBILITY and STASTISTICS and get the MEAN Vi for average the data.
- From EXPRESS to OUTPUT get the WRITE TO MEASUREMENT FILE Vi and open the "properties".

  [Properties]

  Filename-----LVDT

  Action-----Save to one file

  If a file already exists-----Append to file

  File format-----Text (LVM)

  Segment headers-----No headers

  X value (time) columns-----Empty time columns

  Delimiter-----comma
- On Front Panel popup the Controls to get the Waveform Graph and Numeric Indicator display the acquire voltage. The Waveform Graph and Numeric Indicator Vis

will appear on the block diagram. Wire these Vis up correctly to complete the LabVIEW program. This program will be used to perform the LVDT calibration.

4. Push the LVDT all the way to the extreme position against the flat tip of the micrometer and screw tight the LVDT.
5. Use the created LabVIEW program to read the LVDT output voltage.
6. Start to turn the micrometer in steps, each step should be 0.01 in or 0.5 mm, and take a reading of the LVDT output until 0.3 in or 6 mm.
7. Use MS-EXCEL program to open the data file to access the calibration data.

3.4.2 DC Voltage Measurement

1. Connect the DC voltage output to NI USB 6008 DAQ box AI0 input.
2. Connect the NI USB 6008 DAQ box through the USB connecter of the laptop.
3. Turn on the laptop and the DC power supply.
4. Run the LabVIEW program and create a Blank VI.
5. On the Block Diagram pop up the Functions Palette.
6. Get the DAQ assist VI from Express-Input.
7. Run the Measurement & Automation, from Devices and Interfaces find out the NI USB 6008 device number x.
8. Configure the DAQ assist VI to Acquire Signals-Analog Input-Voltage-Dev x –AI0. Terminal Configuration should be RSE (reference single end). Signal input range should be 10V to -10V.
9. On the Front Panel popup the Control Palette, open the Graph Indicators and get the Graph. Get two Numeric Controls for number of sample and sampling rate. Get a numeric indicator to display voltage.
10. On the Block Diagram pop up the Function Palette. Get the Mean VI from Mathematic - Prob & Stat.
11. Wire up all these Vis to make a program for DC voltage measurement.
12. Get 3 or 4 different voltage from the power supply. Run the LabVIEW program you created and check the voltage compare with the power supply display.

3.4.3 AC signal measurement

1. Connect the AC signal output from the Digital Function Generator to the AI1 input of NI USB 6008 DAQ module.
2. Create a Blank VI.
3. On the Block Diagram, pop up the Function Palette to get the DAQ Assist VI from Express to Input, and configure the VI to Acquire Signal-Analog Input- Voltage-Dev x – AI1.
4. From Express to Output, get the Write to Measurement File VI and go to Properties and make the file format to Text. In Action, use Ask User To Choose File and check Ask Each Iteration. In If A File Already Exists check Rename Existing File. In X Value(Time)Columns, check One Column Only.
5. Point the Wire tool to Sample and Rate of the DAQ assist VI terminals, to create two digital controls for Sampling Rate and Number of Sample.

6. On the Front Panel pop up the Control Palette, open the Graph Indicator and get the Waveform Graph.
7. On the front Panel use the Wire tool to touch the Rate terminal of the DAQ assist Vi and pop up----create----control to get the sampling rate digital control. Pop up the sample terminal----create----control to get number of sample digital control. Wire up the data output from the DAQ assist VI to the signal input of the Write to the Measurement File VI and the Graph VI.
8. Chose a few frequencies from 30 Hz to 300 Hz.
9. The sampling rate must be 30X of the set frequency.
10. Number of samples used should be 100.
11. Set the frequency on the Digital Function Generator and run the LabVIEW program.
12. Specify the data file name and save properly.
13. Use MS-Excel program to open the data file (all file).
14. Find out the frequency from the stored data and compare with the frequency set in the function generator.

3.5 Data Analysis

1. Plot out the displacement-voltage output curve of the LVDT.
2. Use linear curve fitting to determine the calibration curve and associated uncertainties.
3. Compare the DC voltage value from the power supply, and the value acquired from the DAQ
4. Calculate the frequency of the AC signal measured with the DAQ, and compare with the output of the signal generator
5. Analyze the uncertainty of the above analysis, and show an uncertainty tree.

LAB 4: PHOLOELASTIC STRESS ANALYSIS OF BEAMS

4.1 Objectives

1. Familiarization with the operations of a polariscope
2. Understanding of the elementary photoelastic method for stress measurement
3. Determination of photoelastic material stress fringe value f_σ using a beam under pure bending
4. Develop understanding of the neutral axis of a beam under bending

4.2 Specimens and Instrumentations

1. Polariscope
2. HD digital camera and LCD TV
3. Photoelastic beam specimen for pure bending and three-point bending tests

4.3 Background Knowledge

4.3.1 Double Refraction and Stress Optical Law

The method of photoelasticity is based on the principle of double refraction observed in a certain class of transparent materials called photoelastic or birefringent materials. This double refraction is a temporary phenomenon associated with the mechanical stressing of the object. When the loads are removed, the optical property of the material returns to normal, which means it will be optically isotropic. Consider a ray of light *Ri* entering a birefringent medium *I* from free space *o* (see Figure. 4.1). Assuming that the object is free of stresses, one can observe the refraction of light based on Snell's Law. Let *Rr* be the refracted ray and *i* and γ be angles of incidence and refraction measured with respect to the surface normal. The ratio $\frac{\sin i}{\sin \gamma}$ is then the refractive index n_{10}.

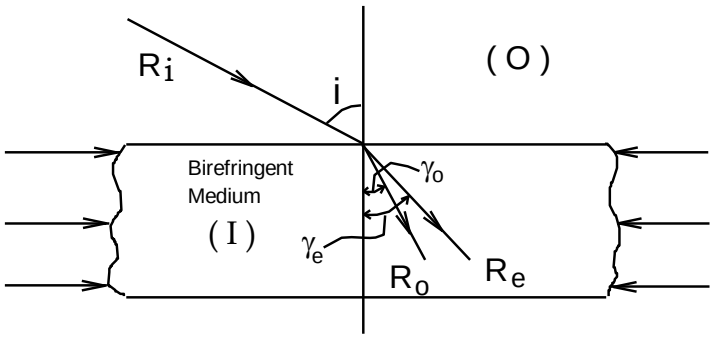


Figure 4.1: Behavior of light in a Birefringent Medium

Now, consider a birefringent medium subjected to external loads. For every incident ray R_i, double refraction gives rise to two refracted rays, R_o and R_e at different angles γ_0 and γ_e respectively with respect to the normal. These two rays propagate inside the medium with different velocities. When they exit from the medium, a phase difference Δ has occurred between the two waves. This phase difference is the result of stress in the medium and their relationship is the stress-optical law given below

$$\sigma_1 - \sigma_2 = \frac{\Delta}{2\pi}\frac{f_\sigma}{D} = \frac{N f_\sigma}{D} \tag{4.1}$$

where N is the fringe order, f_σ is the material stress fringe value and D the thickness of the birefringent material; σ_1, σ_2 are the two principal stresses, and in photoelasticity, it is always assumed that$\sigma_1 \geq \sigma_2$. Thus, once the fringe order is known, the principal stress difference $\sigma_1 - \sigma_2$ at any point can be determined.

4.3.2 Polariscope

The instrument that enables one to determine the stress-induced phase difference is called a polariscope. There are two types of polariscopes. One is called a plane polariscope. Its optical elements consist of one polarizer and one analyzer. A polarizer is an optical element that only allows a light vector to oscillate along a predetermined direction. An analyzer is also a polarizer that is used to analyze the polarization state of the impinging light. As shown in Figure. 4.2, the direction of polarization of polarizer *P* and analyzer *A* are perpendicular to each other.

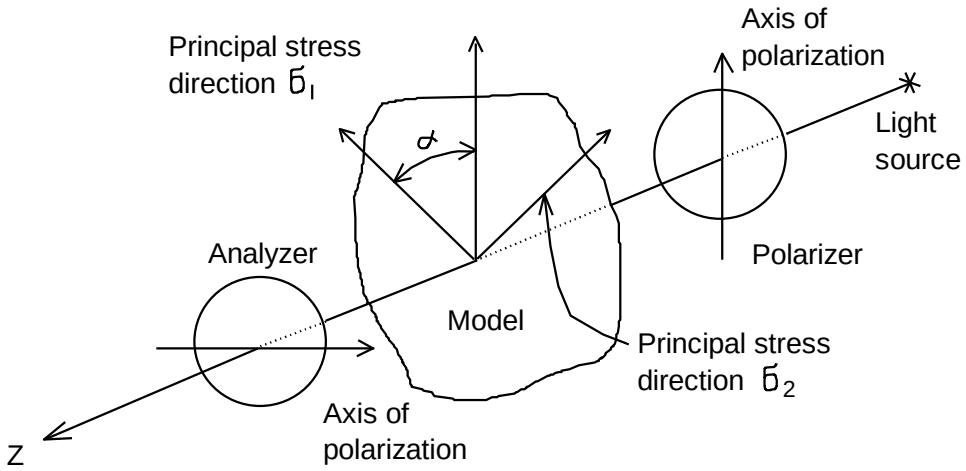


Figure 4.2: Optical arrangement of a plan polariscope

It can be shown that the intensity of light that immerges from the analyzer when a stressed photoelastic model is placed between them is given by the following equation,

$$I = K \sin^2 \alpha \times \sin^2 \frac{\Delta}{2} \tag{4.2}$$

where *K* is a constant, α the angle between σ_1 and the axis of the polarizer and Δ the stress-induced phase difference. Two type of dark fringes are observed. One is the result of

$$I = 0, \text{ when } \sin\alpha = 0, \alpha = n\pi; n = 0,1,2,3 \ldots$$

These fringes are called isoclinics, or locations of points of equal principal directions. The other type of dark fringes is the result of

$$I = 0, \text{ when } \sin\frac{\Delta}{2} = 0, \frac{\Delta}{2} = n\pi; n = 0,1,2,3 \ldots$$

since

$$N = \frac{\Delta}{2\pi} = n\,; n = 0,1,2,3 \ldots$$

is nothing but the fringe order given in the stress-optical law. These fringes are called isochromatics because they appear as colored in a white light illumination, except the zeroth order fringe which is always dark.

The second type of polariscope is the circular polariscope whose optical arrangement is as shown in Figure. 4.3. It has two more optical elements called quarter wave plates. The first quarter wave plate converts a plane-polarized light emerging from the polarizer with a circularly polarized light. The second quarter wave plate with its fast axis and slow axis orientation reversed cancels the effect of the first quarter wave plate. It can be shown that if a stressed photoelastic model is placed in between the two quarter-wave plates of a circular polariscope the presence of the isoclinics is eliminated. The resulting light intensity emerges from the analyzer is simply,

$$I = K' \sin^2\frac{\Delta}{2}, \tag{4.3}$$

where *K'* is a constant. It can also be shown that if the analyzer is turned $90°$ so that it is parallel to the axis of the polarizer the intensity of the light that emerges is given by

$$I = K' \cos^2\frac{\Delta}{2}. \tag{4.4}$$

Thus,

$$I = 0, \text{ when } \frac{\Delta}{2} = \frac{(2n+1)\pi}{2}; n = 0,1,2,3 \ldots$$

And the isochromatic fringe order *N* is

$$N = \frac{\Delta}{2\pi} = \frac{2n+1}{2} = n + \frac{1}{2}; n = 0,1,2,3 \ldots$$

These are the half order fringes.

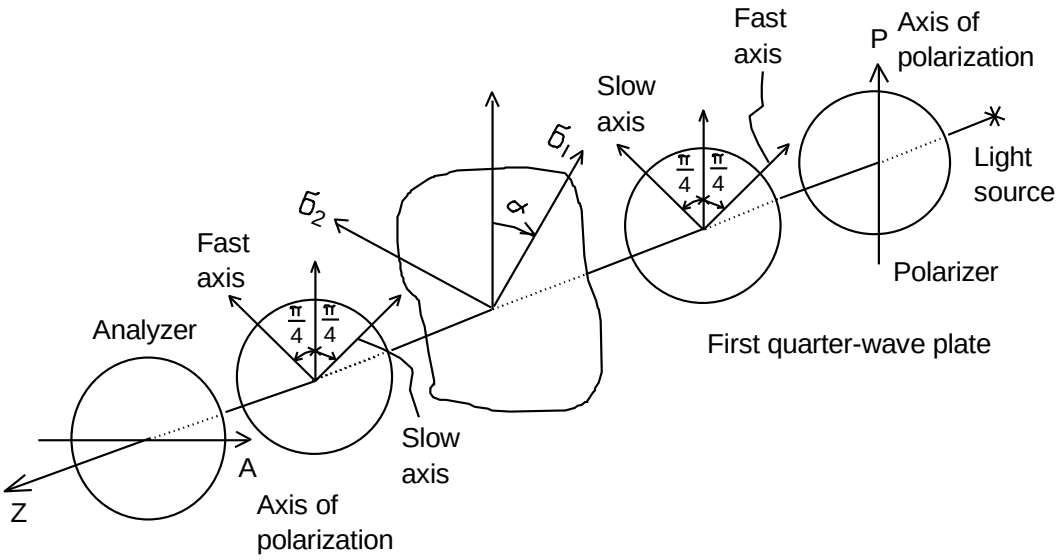


Figure 4.3: Optical arrangement of a circular polariscope

4.3.3 Determination of material stress fringe value 'fσ' for the given material.

The stress-optical law $\sigma_1 - \sigma_2 = \frac{Nf_\sigma}{D}$ is the basic equation for photoelasticity experiments. *N* is the fringe order that can be determined from the fringe pattern of the model. *D* is the thickness of the model. f_σ is the material stress fringe value. It is different for different photoelasticity materials. Therefore, the determination of the material stress fringe value f_σ for different photoelasticity materials in photoelasticity experiments is key to finding out the principal stresses of the model. Several experiments can be used to determine the material stress fringe value f_σ. The four-point bending experiment is the most common.

4.3.4 The theoretical prediction for pure bending beam.

When a beam is under pure bending, the shearing force is zero at every cross section. An example of such bending is shown in Figure.4.4. From the balance of forces, we conclude that the reactions in this case are equal to $\frac{P}{2}$. Considering the equilibrium of the portion of the beam to the left of cross section *mm*, it can be concluded that the internal forces which are distributed over the cross section *mm* and which represent the action of the removed right portion of the beam on the left portion must be statically equivalent to a couple equal and opposite to the bending moment $\frac{Pa}{2}$. To find the distribution of these internal forces over the cross section, the deformation of the beam must be considered. For the simple case of a beam having a longitudinal plane of symmetry with the external bending couples acting in this plane, bending will take place in this same plane. If the beam is of rectangular cross section and two adjacent vertical lines *mm* and *pp* are drawn on its sides, a direct experiment shows that these lines remain straight during bending and rotate so as to remain perpendicular to the longitudinal fibers of the beam (Figure. 4.5).

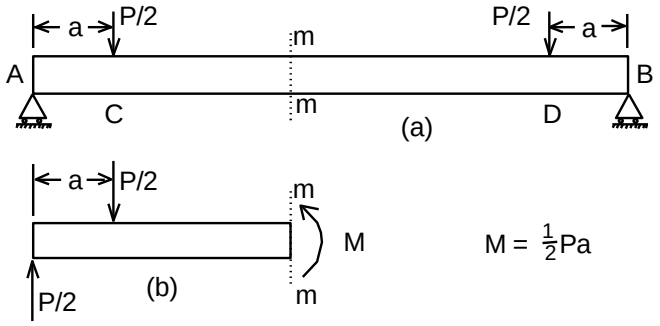


Figure 4.4: A beam under pure bending

The following theory of bending is based on the assumption that not only such lines as *mm* remain straight, but that the entire transverse section of the beam, originally plane, remains plane and normal to the longitudinal fibers of the beam after bending. Experiment shows that the theory based on this assumption gives very accurate results for the deflection of beams and the strain of longitudinal fibers.

From the above assumption, it follows that during bending cross sections *mm* and *pp* rotate with respect to each other about axes perpendicular to the plane of bending so that longitudinal fibers on the convex side suffer extension and those on the concave side suffer compression.

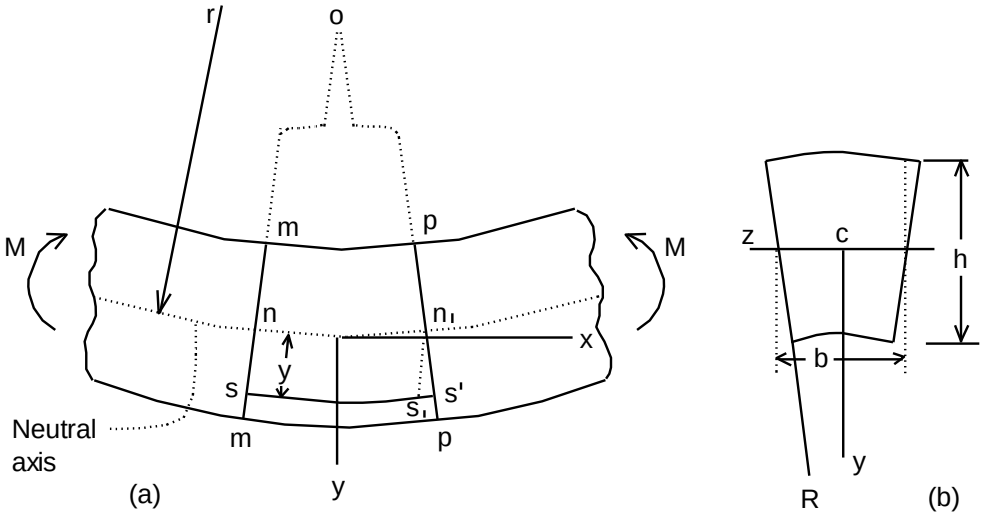


Figure 4.5: Bending deformation of a beam under pure bending

The line nn_1 is the trace of the surface in which the fibers do not undergo strain during bending. This surface is called the neutral surface, and its intersection with any cross-section is called the neutral axis. The elongation $s's_1$ of any fiber at distance *y* from the neutral surface is obtained by drawing the line n_1s_1 parallel to *mm* (Figure.4.5). Denoting by *r* the radius of curvature of the deflected axis of the beam and using the similarity of the triangles non_1 and s_1n_1s', the unit elongation of the fiber ss' is

$$\epsilon_{xx} = \frac{s's_1}{nn_1} = \frac{y}{r} \tag{4.5}$$

It can be seen that the strain of the longitudinal fibers, are proportional to the distance *y* from the neutral surface and inversely proportional to the radius of curvature. From the strains of the longitudinal fibers, the corresponding stresses follow from Hooke's law:

$$\sigma_{xx} = E\epsilon_{xx} \tag{4.6}$$

or

$$\sigma_{xx} = \frac{Ey}{r} \tag{4.7}$$

The distribution of these stresses is shown in Figure 4.6. The stress in any fiber is proportional to its distance from the neutral axis *nn*. The position of the neutral axis and the radius of curvature *r*, the two unknowns in Eq. (4.6) and Eq. (4.7), can now be determined from the condition that the forces distributed over any cross-section of the beam must give rise to a resisting couple *M* (Figure. 4.5).

Let *dA* denote an elemental area of the cross-section at distance *y* from the neutral axis (Figure. 4.6). The force acting on this elemental area is the product of the stress (Eq. 4.6 & Eq. 4.7) and the area *dA*, or

$$F = \frac{Ey}{r} dA. \tag{4.8}$$

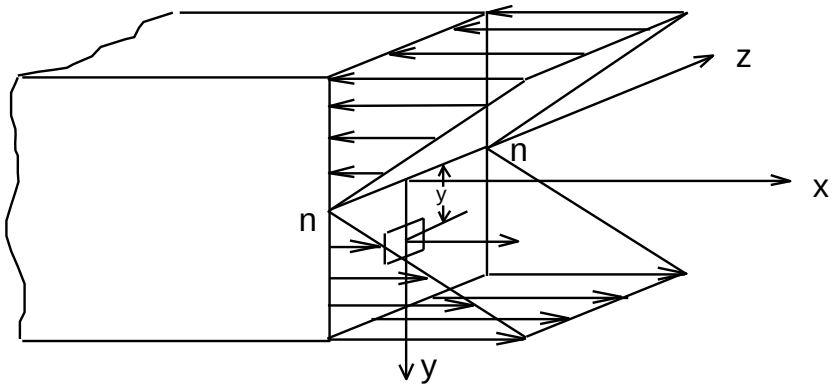


Figure 4.6: Stress distribution of a beam under pure bending

Due to the fact that all such forces distributed over the cross-section represent a system equivalent to a couple, the resultant of these forces in the x-direction must be equal to zero and we obtain

$$\int \frac{Ey}{r} dA = \frac{E}{r} \int y dA = 0 \tag{4.9}$$

Which is the moment of the area of the cross-section with respect to the neutral axis is equal to zero. Hence the neutral axis passes through the centroid of the section. The moment of the force acting on the element *dA* with respect to the neutral axis is

$$M = \frac{Ey}{r} * dA * y \tag{4.10}$$

Adding all such moments over the cross-section and putting the resultant equal to the moment *M* of the external forces, the following equation for determining the radius of curvature *r* is obtained:

$$\int \frac{E}{r} y^2 dA = \frac{EI_z}{r} = M \tag{4.11}$$

or

$$\frac{1}{r} = \frac{M}{EI_z} \tag{4.12}$$

In which

$$I_z = \int y^2 dA \tag{4.13}$$

is the moment of inertia of the cross-section with respect to the neutral axis *z*. From Eq.(4.9) it is seen that the curvature varies directly as the bending moment and inversely as the quantity *EIz*, which is called the flexural rigidity of the beam. Elimination of *r* from Eqs. (4.7) and (4.10) gives the following equation for the stresses:

$$\sigma_{xx} = \frac{My}{I_z}. \tag{4.14}$$

In this equation, *M* is positive when it produces a deflection of the bar convex down, as in Figure. 4.5; *y* is positive in the downward direction. In the case of a rectangular cross section we have:

$$I_z = \frac{bh^3}{12}. \tag{4.15}$$

For a circular cross-section of diameter *d*:

$$I_z = \frac{\pi d^4}{64} \tag{4.16}$$

4.3.5 Determination of the material stress fringe value f_σ by pure bending beam.

A four-point bending beam is shown in Figure. 4.7a. Let two $\frac{p}{2}$ be the symmetrically applied loads separated by a distance l_1 is shown. Let l_2 be the span between supports. A zone of constant bending moment *M* and zero shear force zone exist in the span l_1. Thus, the 2-D stress components in the span l_1 are $\sigma_{yy} = 0, \tau_{xy} = 0$, and $\sigma_{xx} = \frac{My}{I}$ where *I* is the moment of inertia of the beam, *y* is the *y* coordinate of any point under the consideration and

$$M = \frac{\left[\left(\frac{P}{2}\right)(l_2 - l_1)\right]}{2} = Constant \tag{4.17}$$

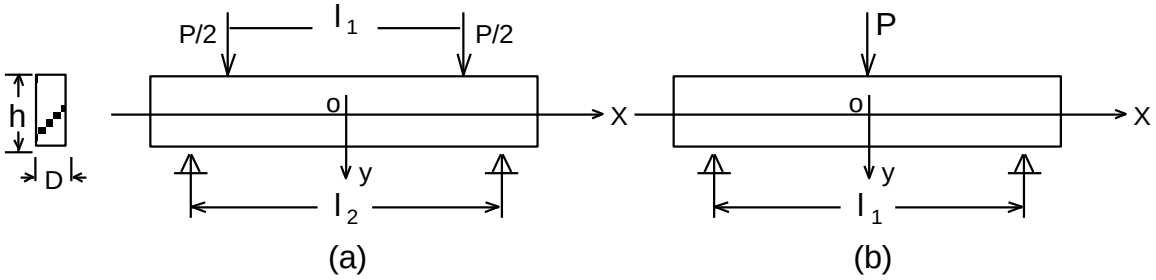


Figure 4.7: Specimen under bending (a) four-point pure bending, (b) three-point bending

Since the σ_{xx} is the only fiber stress, it is the principal stress σ_1 with σ_2 being zero. Thus,

$$\sigma_{xx} = \frac{My}{I} = \frac{Nf_\sigma}{D} \tag{4.18}$$

Or

$$f_\sigma = \frac{MD}{I}\frac{y}{N} \tag{4.19}$$

4.4 Experimental Procedure:

4.4.1 Observation of the isochromatic fringe pattern.

1. A load cell is mounted on the loading frame that connected to a strain indicator to indicate the load. The gage factor of the strain indicator should be set to 3.94 by pre-calibration to indicate the load in 'lb'.
2. Turn on the strain indicator, push down the Amplifier button, adjust the Amplifier Zero adjustment and zero the output reading.
3. Push down the Gage Factor button, adjust the gage factor turning knobs to set the gage factor to 3.94.
4. Push down the Run button, use the balance control to set the reading to zero again.
5. Turn on the light source. The optical arrangement should be set as shown in Figure. 4.3, crossed circular polariscope.
6. Measure the thickness and the height of the beam.
7. Measure the distance between the supports of the base fixture (the span). Measure the distance of the loading points of the upper loading fixture.
8. Place the photoelastic beam with the fixtures for four-point bending onto the loading frame. Adjust the height of the moving frame and the HD camera to obtain a clear image of the beam on the LCD TV screen.
9. Apply some load to the beam, the colored isochromatic fringes should appear. Adjust the fixtures to obtain a symmetric isochromatic fringe pattern.

4.4.2 Determination of the fringe order.

1) The fringe order can be determined from the theoretical prediction of a pure bending beam. Knowing that the neutral axis of the beam coincides with the x-axis, we can ascertain that the N = 0 fringe also coincides with the x-axis of the beam. Since σ_{xx} varies linearly from zero at y = 0 to a maximum at y = ± h/2. In a crossed circular polariscope the fringes in such a beam can then be ordered sequentially as *N = 0, 1, 2, 3, ...* start from the neutral axis. This information can be used to plot a straight line of *y vs N* and its slope $\frac{y}{N}$ can be used in Eq. (4.19) for the calculation of f_σ .
2) The fringe order also can be determined experimentally. Slowly change the load that applies to the beam. The colored isochromatic fringes will move when the load is changing, but a dark fringe at the center of the beam that coincides with the x-axis will not move. Also the free corner areas of the beam will always in dark. These are the zero stress areas that the fringe order *N=0*. Watch the color sequence of the colored isochromatic fringes from the center of the beam to the top and bottom edges of the beam, the color sequence is yellow-red-green-yellow-red-green…. These are the fringe order increasing directions. Vice versa the color sequence of the colored ischromatic fringes is green-red-yellow-green-red-yellow…, it is the decreasing direction of the fringe order. Therefore, the highest fringe order of the isochromatic fringes is always at $y = \pm\frac{h}{2}$,the top and bottom edge of the beam.

4.4.3 Determination of the material stress fringe value f_σ.

1. Put a filter in front of the camcorder to change to mono-color fringes. The colored isochromatic fringes will change to a much sharper dark fringes.

2. Increase the load slowly until the N^{th} order isochromatic fringe just shows at the top edge of the beam.
3. Record the load from the strain indicator display.
4. f_σ can be determined by the following equation:

$$f_\sigma = \frac{3P(L_2 - L_1)}{2Nh^2} \tag{4.20}$$

where

P: The load applied on the beam.
N: The fringe order at the bottom edge of the beam.
h: The height of the beam.
L_1: The distance between two loading point.
L_2: The distance between two supports of the beam.

5. Run the experiment three times.
6. Use the digital camcorder to take color and black and white images pictures and save into the memory stick.

4.5 Requirement:

1. Plot y vs N relation and fit it with a straight line. Use the slope y/N for the calculation of $f_{\sigma.}$
2. Find out the uncertainty of the experimental results, and show an uncertainty tree.

4.6 Part 2. Determination of the fiber stresses along the top and bottom edges of a beam subjected to three-point bending.

In this experiment, the beam is subjected to a central load *P* and supported symmetrically by the two supports spaced l_1 apart (see Figure. 4.7b). The magnitude of bending moment at each section within $l_1$is

$$M(x) = \frac{P}{2}\left(\frac{l_1}{2} - |x|\right) \tag{4.21}$$

The normal stress at the outermost fiber (i.e. $y = \pm\frac{h}{2}$) is again the only stress. Thus,

$$\sigma_1\left(x, \pm\frac{h}{2}\right) = \sigma_{xx}\left(x, \pm\frac{h}{2}\right) = \frac{M(x)\left(\pm\frac{h}{2}\right)}{I} = \frac{N\left(x, \pm\frac{h}{2}\right) f_\sigma}{D} \tag{4.22}$$

Using the material stress fringe value, f_σ, calculated from the previous experiment, one can calculate the fiber stress $\sigma_{xx}(y = \pm\frac{h}{2})$ at various cross-sections.

4.7 Testing procedure:

1. Change the upper loading fixture for three-point bending.
2. Apply some load on the beam and adjust the locations of the fixtures to obtain a symmetric isochromatic fringe pattern.
3. Using white light source with dark background, the four free corners of the beam must be dark because of the zero stress. Start from these corners using the color sequence of the colored isochromatic fringe to determine the fringe orders on the top and bottom edges of the beam. The highest order fringe will locate at the middle of the bottom edge.
4. Place the filter at the front of the camera the color isochromatic fringe change to dark and sharpen fringes. Apply a load to obtain a clear fringe pattern. Record the load from the strain indicator display.
5. Find out the fringe order of the isochromatic fringes and the fringe locations along the top and the bottom edges of the beam.
6. Run the experiment three times.
7. Use the camera to take the isochromatic fringe pattern images.

4.8 Requirement:

1. Compare the two different calculated values of the material stress fringe value, f_σ with the known value of 40 lbf/in.
2. Plot the experimental and theoretical values of σ_{xx} vs x for (y = ± h/2), compare the result with that of the experiment of Three-Point Bending using photoelasticity model. Explain the difference, if any, between the two results.

LAB 5: SHADOW MOIRÉ METHOD FOR DEFLECTION MEASUREMENT, SHAPE MEASUREMENT, AND OPTICAL METROLOGY

5.1 Objective

Using moiré fringe for metrological studies is a powerful tool utilized in both academic institutions and industry. It is a powerful tool that gives rise to full-field information. Unlike photoelasticity where a birefringent material must be employed, the moiré technique can be applied to almost any engineering material with deformation ranging from elastic, viscoelastic to plastic. The purpose of this experiment is to expose students to this powerful modern tool. Due to the time limitation, only shadow moiré methods will be introduced.

5.2 Equipment

1). Moiré grating: 20 lines/inch for shadow moiré.
2). Cylindrical specimen.
3). Mannequin head.
4). Light source.
5). DSLR Camera.

5.3 Background Knowledge

5.3.1 Shadow Moiré Method

Shadow moiré is for measuring out-of-plane deformation, as well. Instead of two, only a single grating is used. The grating's shadow under illumination acts as the specimen grating. A general optical arrangement is as shown in Figure. 5.1.

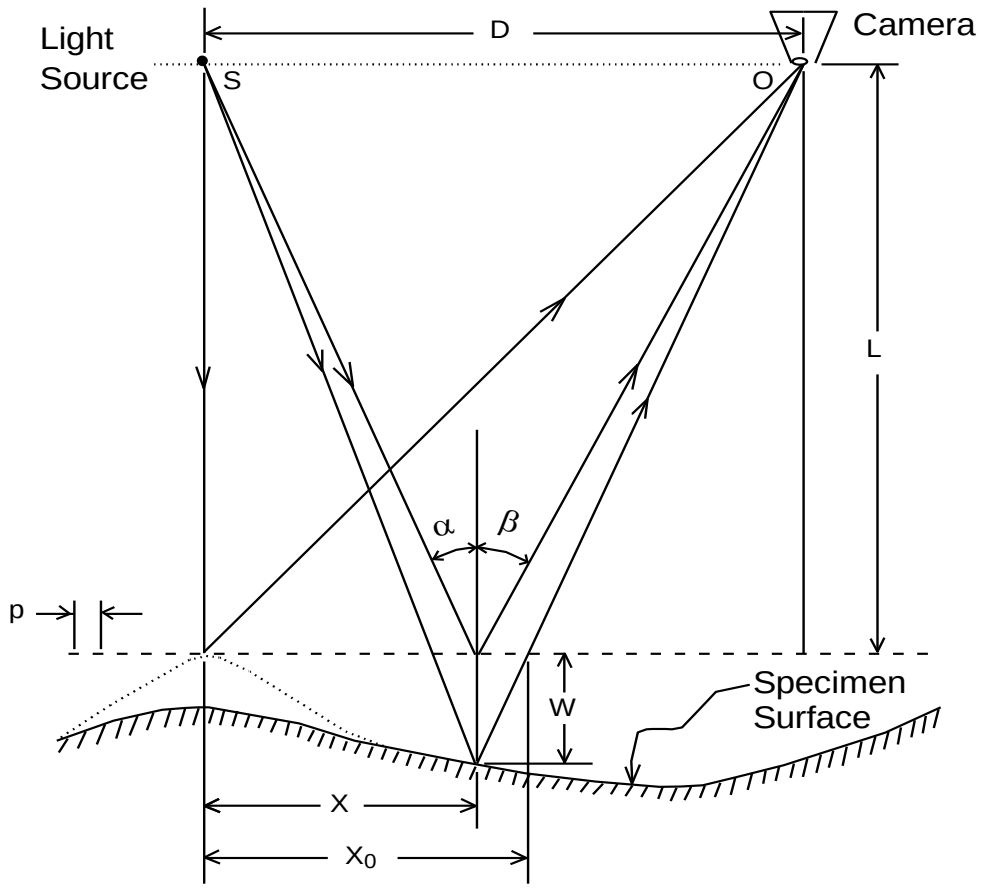


Figure. 5.1 Shadow-moiré method with point illumination and point receiving

When a specimen is placed behind the moiré grating and illuminated the distance *w* from the specimen to the grating can be calculated using the following equation:

$$w = \frac{Np}{tan\,\alpha + tan\,\beta} \tag{5.1}$$

where *N* is the fringe order; *p* is the grating pitch, α and β are the illuminating and receiving angles from the normal to the grating plane. In the special case where both the light source and recording camera are at infinity and β=0, the above equation is reduced to

$$w = \frac{Np}{tan\,\alpha} \tag{5.2}$$

And this is the equation to be used in this experiment. A more detailed description of the shadow moiré method can be provided in the form of a published chapter on the topic, should you want it.

5.4 Experimental Procedures

5.4.1 Testing Procedure:

1) Measure the diameter of the cylinder.
2) Pointing the optical axis of the recording camcorder perpendicular towards the grating plane.
3) Place the cylinder in slight contact with the grating lines.
4) Adjust the projection light such that its optical axis is pointing directly towards the center of the specimen.
5) Measure the angle between the optical axes of the camcorder and the projecting light.
6) Record the moiré pattern thus obtained using the camera.
7) Replace the cylinder with the mannequin head.
8) Record the moiré pattern thus obtained using the camera.

5.4.2 Analysis Procedure:

1) Using Eq. (5.2) calculate the shape of the specimens.
2) Compute the grating constant p using the knowledge that the moiré grating is 20 lines/in.
3) Order the moiré fringe orders as follows: N=0 for the first fringe at the center point of the specimen; the next one N=1, and then N=2, and so on.
4) Calculate w using the equation.

5.5 Requirements:

1) Bring a flash drive or similar storage device to bring the images collected home.
2) Plot *w* as a function of position along a diametrical section for the cylindrical specimen.
3) Compare the result with the theoretical value calculated from the diameter for the cylindrical specimen.
4) Plot *w* as a function of position along a horizontal section that crosses the nose of the mannequin head.

LAB 6: DETERMINATION OF SHEAR MODULUS AND METAL FATIGUE

6.1 Objectives

1. Determination of material properties and observation of material response at different stages of loading.
2. Familiarization with the operation of WP 500 Torsion Testing Machine.
3. Familiarization with the operation of RBF-200 fatigue testing machine and understand how to determine the fatigue S-N curve.

6.2 Equipment

- WP 500 Torsion Tester
- Torsion Specimen
- Torsiometer
- RBF-200 fatigue testing machine.
- A steel fatigue specimen.

6.3 Background Knowledge

6.3.1 Determination of Shear Modulus.

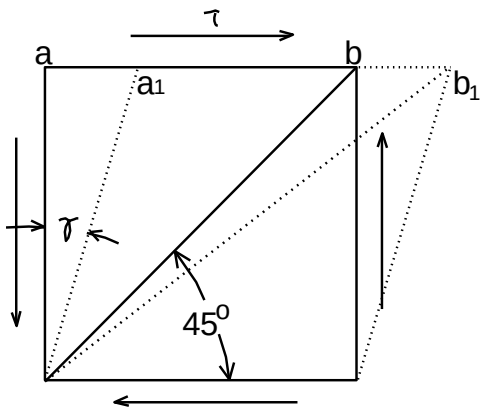


Figure 6.1: A state of pure shear deformation

Consider a state of pure shear as shown in Figure. 6.1. After the distortion produced by the shearing stress τ, and assuming that the material obeys Hooke's law, the shearing strain γ is proportional to the shearing stress τ and we can express the relation between them by the equation

$$\gamma = \frac{\tau}{G} \tag{6.1}$$

in which *G* is a material constant. Eq. (6.1) is analogous to Eq. (1.23), and the constant *G* is called the modulus of elasticity in shear, or sampling shear modulus. Let us consider a circular shaft fixed at the upper end and twisted by a couple applied to the lower end (Figure. 6.2). If the angle of twist is small, it may be assumed that the circular cross-sections of the shaft remain circular during the twist and that their diameters and the distances between them do not change.

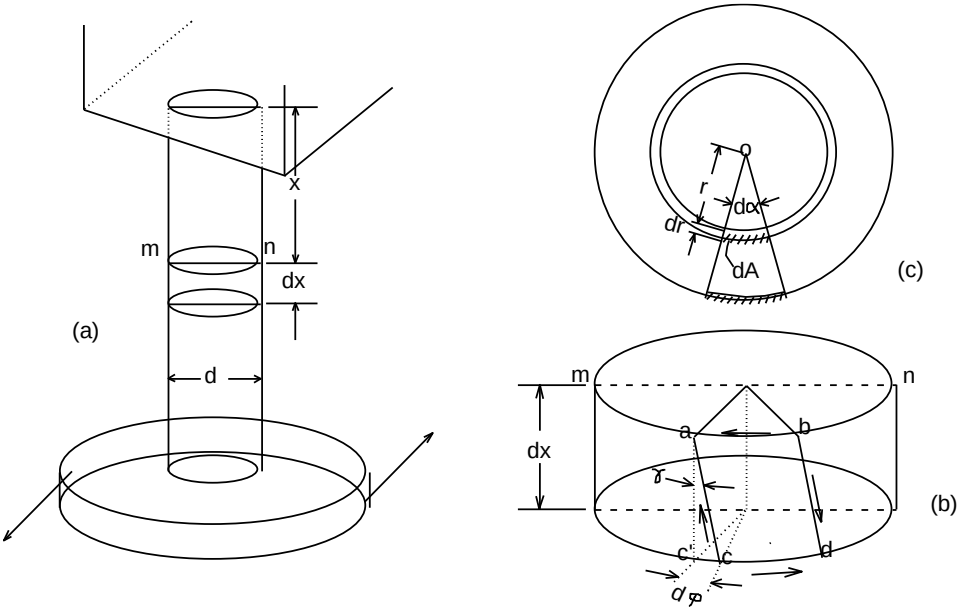


Figure 6.2: Shear deformation of a long circular cylinder

A disc isolated from the shaft as shown in Figure. 6.2b is in the following state of strain: there is a rotation of its lower cross-section with respect to its top cross-section through an angle $d\varphi$, where φ measures the rotation of the section *mn* with respect to the built-in fixed end. The rectangular element *abcd* off the lateral surface of the disc takes the form shown in Figure. (6.2b). The lengths of the sides remain essentially the same and only the angles at the corners change. The element is in a state of pure shear and the magnitude of the shearing strain γ is found from the small triangle *cac'*:

$$\gamma = \frac{c'c}{ac'} \tag{6.2}$$

Since *c'c* is the small arc of radius $\frac{d}{2}$ corresponding to the difference $d\varphi$ in the angle of rotation of the two adjacent cross-sections, $c'c = \left(\frac{d}{2}\right) d\phi$, we obtain

$$\gamma = \frac{1}{2}\frac{d\phi}{dx}d \tag{6.3}$$

For a shaft twisted by a torque at the end, the angle of twist is proportional to the length and $\frac{d\phi}{dx}$ is constant. This represents the angle of twist per unit length of the shaft and is denoted by θ. Thus, from Eq. (6.3),

$$\gamma = \frac{1}{2}\theta d \tag{6.4}$$

The shearing stresses which act on the sides of the element and produce the above shear have the directions as shown. The magnitude of which, from Eq. (6.1), is

$$\tau = \frac{1}{2}G\theta d \tag{6.5}$$

Thus, the state of stress of an element at the surface of the shaft is specified completely. For an element within, the shaft it is assumed the circular boundaries of the cross sections of the shaft remain plane and rotate as if absolutely rigid, for instance, every diameter of the cross-section remains straight and rotates through the same angle. Tests of circular shafts show that the theory developed on this assumption is in very good agreement with experimental results. This being the case, the discussion for the element *abcd* at the surface of the shaft (Figure. 6.2b) also holds for a similar element on the surface of an inner cylinder, whose radium replaces $\frac{d}{2}$ (Figure. 6.2c). The thickness, *dr*, of the element in the radial direction is considered as very small. Such elements are then also in a state of pure shear and the shearing stress on their sides is

$$\tau = Gr\theta \tag{6.6}$$

This states that the shearing stress varies directly as the distance *r* from the axis of the shaft. As in Figure. 6.3 which depicts this stress distribution. The maximum stress occurs in the surface layer of the shaft.

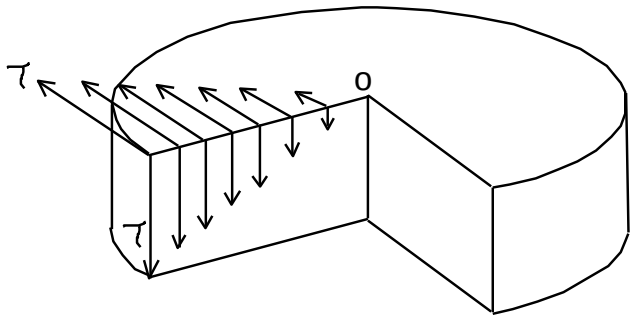


Figure 6.3: Shear stress distribution in a sectional plane

We seek now the relationship between the applied twisting couple *Mt* and the stress produced. From the equilibrium of the portion of the shaft between the bottom and the cross section *mn*

(Figure. 6.2a), we conclude that the shearing stresses distributed over the cross-section are statically equivalent to a couple equal and opposite to the torque *Mt*. For each element of area *dA* (Figure. 6.2c) the shearing force is τdA. The moment of this force about the axis of the shaft is:

$$(\tau dA)r = G\theta r^2 dA \tag{6.7}$$

from Eq. 6.6. The torque *M* is the summation, taken over the entire cross sectional area, of these moments,

$$M_t = \int_A G\theta r^2 dA = G\theta \int_A r^2 dA = G\theta I_p \tag{6.8}$$

where I_p is the polar moment of inertia of the circular cross-section. For a circle of diameter *d* we have

$$I = \frac{\pi d^4}{32} \tag{6.9}$$

and therefore

$$M_t = G\theta \frac{\pi d^4}{32} \tag{6.10}$$

and

$$\theta = \frac{M_t}{G}\frac{32}{\pi d^4} = \frac{M_t}{GI_p} \tag{6.11}$$

We see that θ, the angle of twist per unit length of the shaft, varies directly as the applied torque and inversely as the modulus of shear, *G*, and the fourth power of the diameter. If the shaft is of length *L*, the total angle of twist is

$$\phi = \theta L = \frac{M_t L}{GI_p} \tag{6.12}$$

It should be noted that experiments in torsion are commonly used for determining the shear modulus *G* for various materials. If the angle of twist produced in a given shaft by a given torque is measured, the magnitude of *G* can easily be obtained from Eq. (6.12).

6.3.2 Experimentation.

Shear modulus *G* can be determined from the experiment in torsion. The torque *M* can be read from the E101 torque meter and the angle of twist can be read from the Torsiometer that is installed on the specimen. The shear modulus *G* can be calculated from Eq. (6.12) as

$$G = \frac{M_t L}{\phi I_p} \tag{6.13}$$

Using the notation:

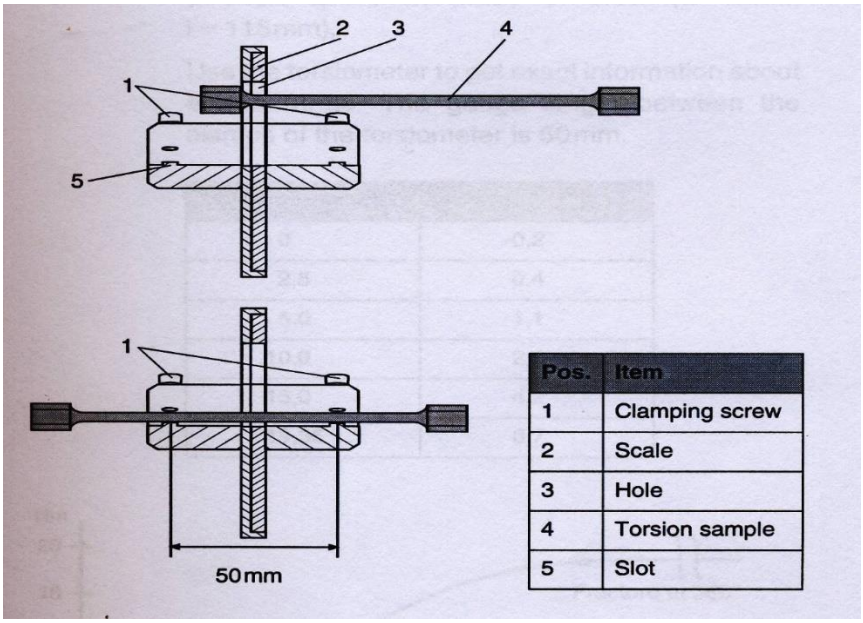
M_t = the applied torque
L = the measurement length of the torsiometer
Φ = the angle of twist read from torsiometer (in radians)
I_p = polar moment of inertia.

As before, the torque is to be applied in steps and a curve of (*MtL*) vs (*φIp*) is to be plotted. Theoretically, it should be a straight line and its slope is the shear modulus *G*. The torque is kept relatively small and must be read at every step. Two kinds of specimens are to be tested: carbon steel and aluminum specimens

6.4 Experimental Procedure

1) Measure the diameter (d) of the specimens.
2) Remove the two clamping screws (Figure. 6.4 Pos 1) until the loading slot is free. Set the scale (Figure. 6.4 Pos 2) to zero, so that the holes (Figure. 6.4 Pos 3) are aligned.
3) Take a torsion specimen (Figure. 6.4 Pos 4) and put it through the holes in the scale disc. Put the specimen exactly to the ground of the slot (Figure. 6.4 Pos 5).
4) Insert the specimen together with the torsiometer into the torsion tester.
5) Adjust the scale to zero and tighten the clamping screws (Figure. 6.4 Pos 1). Take care that the zero setting of the scale will not be readjusted.
6) Distance L of the two screws is 50mm.
7) Insert sockets (Figure. 6.7.1) into the square connections.
8) Release clamping lever (Figure. 6.7.2) on torque measurement unit (Figure. 6.7.3) and push it backwards.
9) Place the specimen (Figure. 6.7.4) with the torsiometer in the sockets and slide the torque measurement unit forwards again. Make sure that the moving driver (Figure. 6.7.5) is located in the center of its range of movement.
10) Fix the torque measurement unit in place with clamping levers.

11) Carefully pre-tension the specimen until there is no slack and the torque display begins to move.
12) On the torque display unit press and hold down the ▼ key and then press P. The display returns to zero.
13) Set the dial gauge (Figure. 6.7.10) on the compensation device to zero by rotating the scale ring.
14) Set the torque display to zero again.
15) Now the experiment is ready to run.
16) Turn the hand wheel (Figure. 6.7.8) to load the specimen, display in Nm on the digital display of the torque display unit.
17) Read the torque angle on the scale of the torsiometer (Figure. 6.5), the angel can be read to 0.1 degree accuracy using Nonius scale (Figure. 6.6).
18) Each 0.5 degree turn wheel (Figure. 6.7.11) until the dial gauge (Figure. 6.7.10) zero again, takes a reading of the torque until 2.5 degree, three trials for each specimen, aluminum, and steel.



Pos.	Item
1	Clamping screw
2	Scale
3	Hole
4	Torsion sample
5	Slot

Figure 6.4: Torsion machine

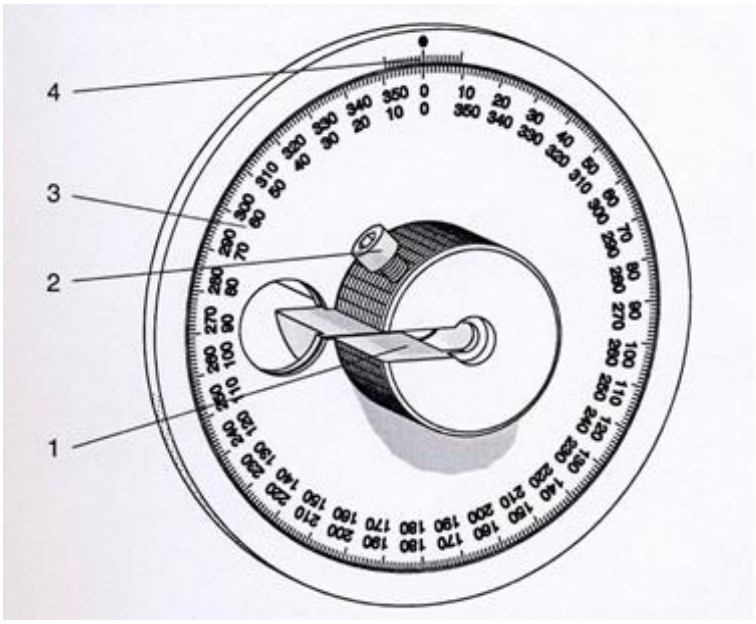


Figure 6.5: WP 500.90 Torsiometer

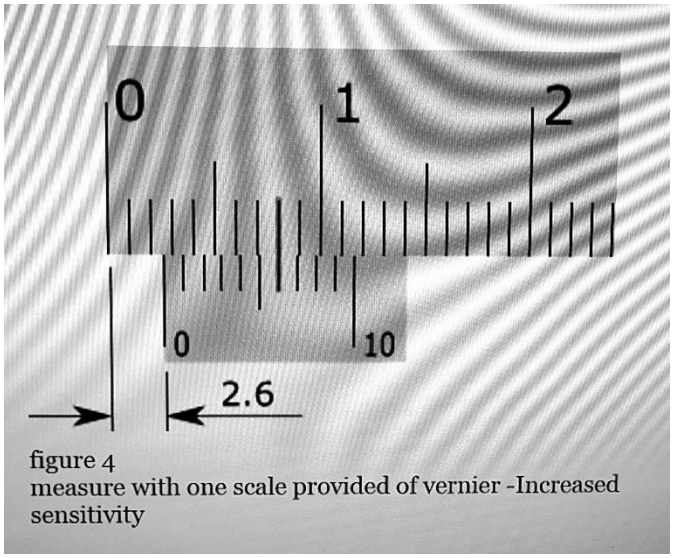


Figure 6.6: Example of a Nonius scale

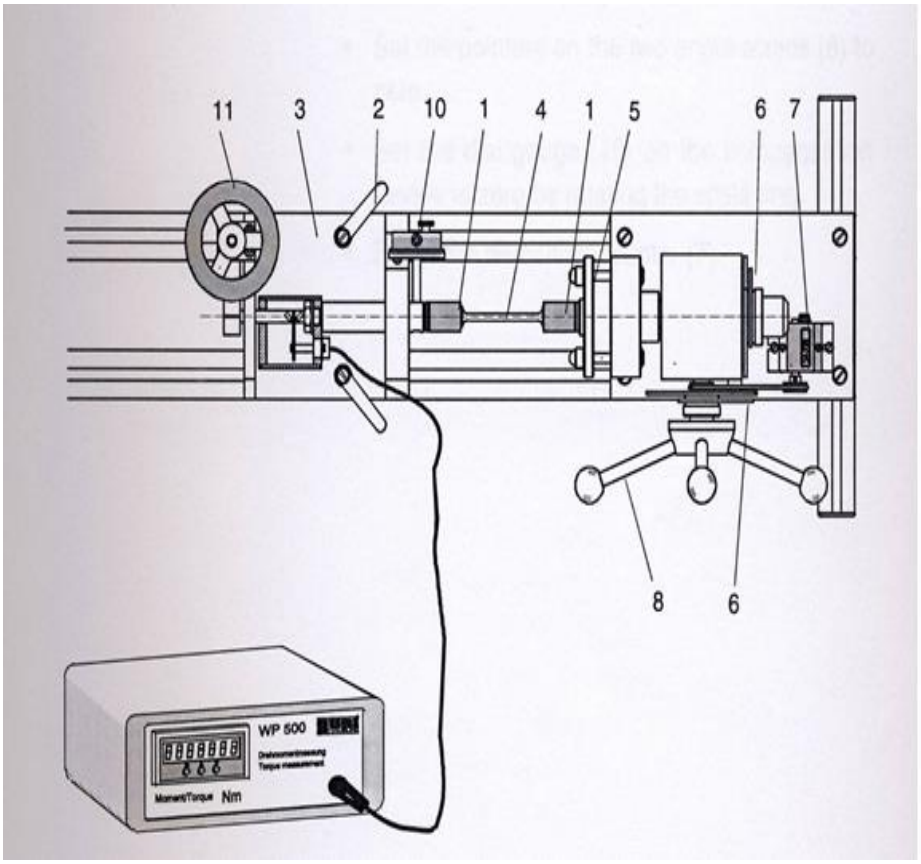


Figure 6.7: WP 500 Torsion Tester

6.5 Requirement

1. Determine the shear modulus G of the testing materials using the linear regression method.
2. Find out the uncertainties of the experimental results and show an uncertainty tree.

6.6 Fatigue and determination of S-N curve

6.6.1 Background Knowledge

Machine parts are frequently subjected to varying stresses and it is important to know the strength of materials under such conditions. It is well known that materials fail under repeated loading and unloading, or under reversal of stress, at stresses smaller than the ultimate strength of the material under static loads. The magnitude of the stress required to produce failure decreases as the number of cycles of stress increases. This phenomenon of the decreased resistance of a material to repeated stresses is called fatigue, and the testing of a material by the application of such stresses is called an endurance test.

If $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum and minimum values of the repeated stress, then the algebraic difference

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} \tag{6.14}$$

is called the range of stress. The cycle is completely defined if the range and the maximum stress are given. The average stress is

$$\sigma_m = \frac{1}{2}(\sigma_{\max} + \sigma_{min}) \tag{6.15}$$

In the particular case of reversed stress $\sigma_{\max} = -\sigma_{\min}$, $\sigma_{\max}\Delta\sigma = 2\sigma_{\max}$ and $\sigma_{\min} = 0$. Any cycle of varying stresses can be obtained by superposing a cycle of reversed stress on a steady average stress. The maximum and minimum values of the varying stress are then given by the following formulas:

$$\sigma_{max} = \sigma_m + \frac{\Delta\sigma}{2}; \sigma_{min} = \sigma_m - \frac{\Delta\sigma}{2} \tag{6.16}$$

There are various methods of applying the load during an endurance test. The specimen can be subjected to direct tension and compression, to bending, to torsion, or to some combination of these. The simplest way is by reversed bending. A common form of a fatigue test bar is a cantilever as shown in Figure. 6.8. The cross section of the specimen is varied along the length in such a manner that the maximum stress occurs at cross-section *mn*. The effect of stress concentrations is eliminated by using a large fillet radius and by increasing the diameter of the bar near the fillet. The load *P* is always downward and the specimen rotates at constant speed. The stress therefore changes sign every half revolution, and the number of cycles of stress is equal to the number of revolutions of the machine. The stress is a completely reversed stress, the average stress being zero and the range of stress being twice the σ_{max}.

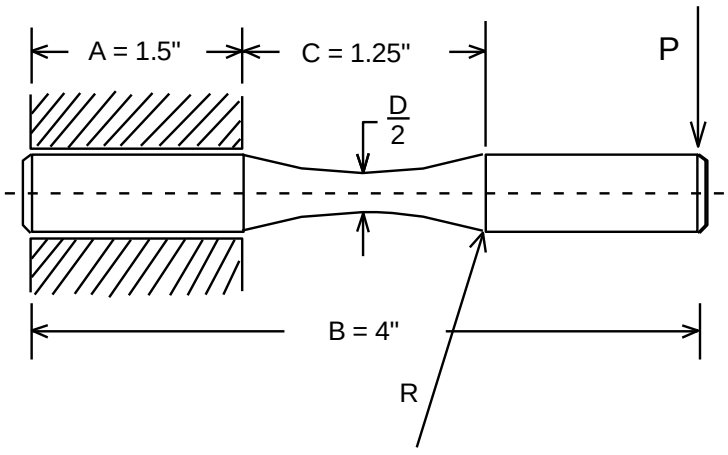


Figure 6.8: Specimen for material fatigue experiment

By taking several specimens and testing them at various loads *P*, a curve such as shown in Figure.6.9 can be obtained.

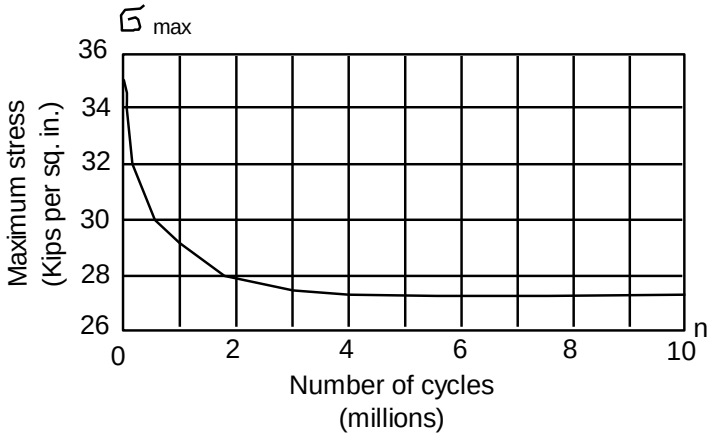


Figure 6.9: Typical material fatigue S-N testing curve

Here, $\sigma_{\max}$ $\sigma_{\max}$ is represented as a function of the number of cycles *n* required to produce a fracture. The curve shown was obtained with mild steel. At the beginning $\sigma_{\max}$ decreases rapidly as *n* increases. After about 4 million cycles, there is no longer any appreciable change in $\sigma_{\max}$, and the curve approaches asymptotically to the horizontal line $\sigma_{\max} = 27000 lb\ per\ sq. - in.$ The stress corresponding to such an asymptote is called the endurance limit of the material. It is now the usual practice in endurance tests to plot $\sigma_{\max}$ against $\log n$. In this manner, the magnitude of the endurance limit is disclosed by a definitive discontinuity in the curve. An example of such a curve is shown in Figure. 6.10.

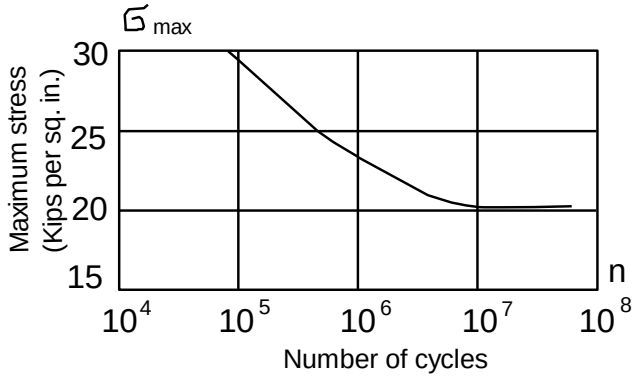


Figure 6.10: A typical S-N curve with different scale

6.7 Experimental Procedure (RBF-200 fatigue testing machine)

The RBF-200 fatigue testing machine, as shown in Figure. 6.11, is a compact, bench mounted machine designed to apply reversed bending loads to unthreaded, straight shank specimen bars. Included is a cycle counter (99,999,900 maximum counts), an adjustable speed spindle (500 to 10,000 cpm), and a calibrated beam and poise system which can apply an infinitely adjustable

moment of up to 200 inch-pounds to the cantilevered end of the specimen bar. Collet size is ½ inch.

- Motor and Spindle

The motor is a ½ HP, 115volt, universal type which is powered by a variable transformer to control the speed from 500 to 10,000 RMP. The motor drives a spindle assembly through a flexible coupling.

CAUTION: The motor must not be operated at a speed over 10,000 RMP!

The spindle assembly consists of the shaft, bearings, and oil filled housing. A sight gage is provided on the back of the machine for maintaining the proper oil level in spindle.

- Bending Moment Loading Beam

The bending moment loading beam is numbered from 0 to 200 inch-pounds at successive 10 inch-pound increments. The interval between each 10 inch-pound increment is marked with successive one inch-pound divisions. A locking screw is provided in the poise weight to secure it at the desired bending moment setting.

- Cutoff Switch

A snap action reset switch is furnished to automatically shut off the machine at specimen failure. It is located under the end of the calibrated beam in such a manner that when the beam drops at specimen failure, the nuts on the screw are adjusted to stop the beam from damaging the switch after actuation. The switch must be reset with the tab at the outside end of the machine before testing can be resumed.

- Cycle Counter

The six-digit resettable counter (99,999,900 maximum count) is actuated by a switch which is directly driven by the spindle through a 100:1 ratio.

6.8 Testing Procedure

1. Loosen the lock screw fixing the poise weight to the calibrated beam and move the weight to the zero position at the extreme left end of the beam (see Figure. 6.11.)
2. Loosen the nuts holding the safety bar at the end of the load arm and swing the bar free of the load arm.
3. Pull the safety guard straight upward free the phenolic block base. The guard is retained only by a friction fit.
4. Swing the load arm up and to the right so that a specimen bar may be inserted into the drive spindle collet. Position the load arm to prevent contact with the free end of the specimen.
5. Before inserting the specimen into the drive spindle collet, wipe the specimen clean and carefully check for any burrs, flats, or ridges. Stone away any discontinuities that might interfere with the even distribution of the collets gripping action. Also wipe clean the specimen bores in both collets.
6. Specimen bars should be pushed into the collets until either the specimen bottoms or the front face of the collets lines up with the end of the tangent on the specimen.
7. Tighten the drive spindle collet onto the specimen. The collet must be tightened sufficiently to prevent any relative movement between the collet and specimen which could cause fretting corrosion.

8. Manually rotate the assembly and check for run-out. The run-out should not exceed .001 inch at the drive spindle collet and .003 inch at the free end of the specimen.
9. If excessive run-out is present, loosen the collet sufficiently. Tighten the collet and recheck the run-out.
10. Insert the free end of the specimen into the load arm collet observing the same procedures and precautions noted above for the drive spindle.
11. In wrenching tight the load arm collet, particular care should be taken to ensure that pure torsional wrenching is used and that no bending forces are imparted to the specimen.
12. Again, rotate the assembly and check the final run-out on the right-hand end of the load arm which should not exceed .006 inch. If excessive run-out is present, repeat the procedure described in 8. It may be necessary to tap the specimen free from the collet. Tighten the collet and recheck the run-out.
13. Set the counter to "zero.
14. Turn the speed control knob counterclockwise to the zero position. Back off the cutoff switch from tripping by the movement of the load arm as it comes up to speed.
15. Push down the cutoff switch reset tab extending through the right-hand end of the machine base.
16. With the fingers of the right hand, grasp the load arm bearing housing to damp out any resonances and slowly rotate the speed control knob clockwise to bring the machine up to the desired speed.
17. Speed may be readily determined from a counter/timer relationship. Two zeroes must be added to the indicated reading of the counter for the actual spindle count.
18. When the spindle speed has been roughly adjusted to its desired rate, slowly move the poise weight along the calibrated beam to the required bending moment setting.
19. While adjusting the position of the poise weight, watch for interference between the cutoff switch adjusting screw and the guard.
20. Fix the weight to the beam by tightening the lock screw and quickly reset the counter to zero without stopping the machine.
21. Machine speed should be rechecked to determine if loading the specimen caused it to slow down.
22. Finally, adjust the cutoff switch actuation by slowly turning the adjusting screw clockwise until the switch actuates and the power is shut off. Immediately, and in the following sequence, back off the adjusting screw ½ turn, and push down the cutoff switch reset tab. This should be done as quickly as possible to minimize the loss of spindle speed.

The intent in this procedure as well as moving the weight to the desired moment setting after the machine has been brought up to speed is to minimize any over load condition on the specimen if the machine passes through a critical (resonant) speed. In addition, it is important to select a non-resonant test speed and to hold the load bearing housing with the fingers during any speed changes to dampen vibration when passing through critical speeds.

The applicable inch-pound moment setting for the poise weight is generally determined on the basis of some desired bending stress level in the specimen. This moment may be determined from the equation:

$$M = 3.1416 \times \frac{\sigma D^3}{32} = 0.0982 \sigma D^3 \quad (6.17)$$

Where
M = Setting for poise weight in inch-pounds
σ = Desired bending stress level in specimen at minimum cross section in pounds per square inch (PSI).
D = Diameter of specimen at minimum cross section in inches

6.9 Requirement

1). Familiarization with the operation of the fatigue testing machine. 2). Familiarization with how to determine the S-N curve of the material.

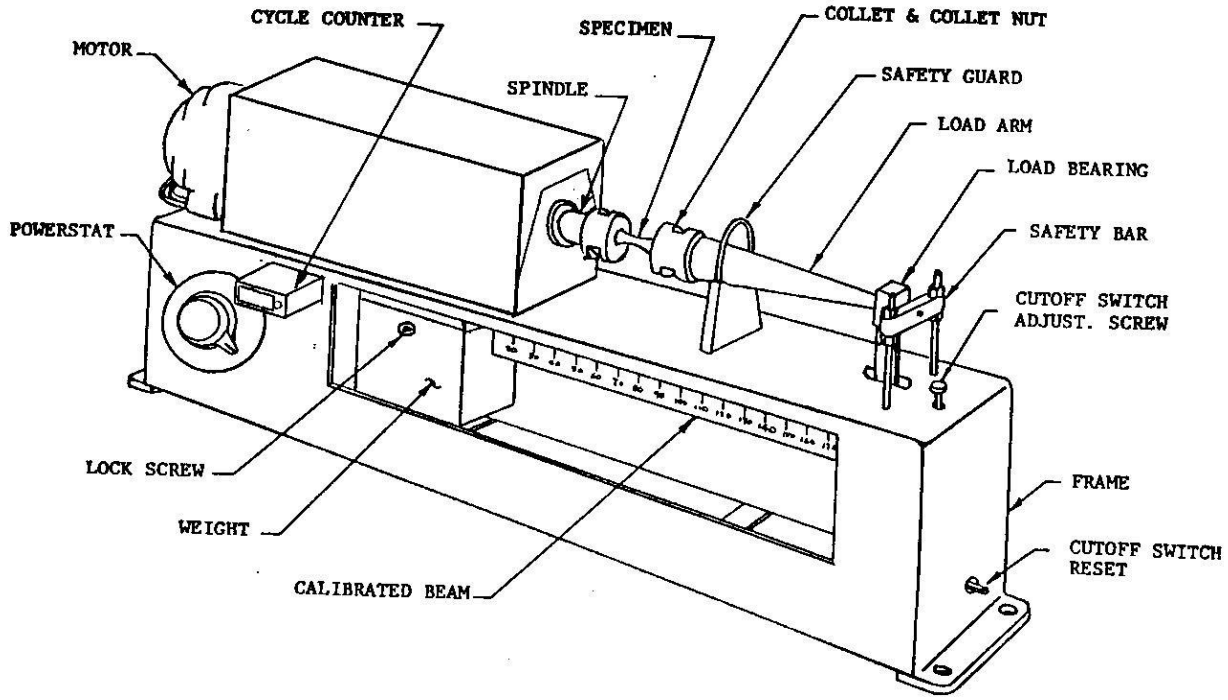


Figure 6.11: Torsional Fatigue Testing Machine

LAB 7: STRUCTURAL INSTABILITY

7.1 Purpose of Experiment

1. Through column buckling test to understand the phenomenon of structural instability.

7.2 Equipment

- Strut apparatus.
- Three steel specimens for buckling test.

7.3 Background knowledge

7.3.1 Determination of load-deflection curves and critical loads for buckling of straight columns with various end conditions.

7.3.1.1 Struts Subject To Axial Load

When the length of a strut is very large in comparison to its sectional dimensions and it is loaded in compression failure will occur not due to the compressive stress rather, failure will occur due to bending since no strut is truly straight, no load truly axial, and no material truly homogeneous. Such bending under axial load is called buckling and the load which produces it is referred to as the Buckling, Crippling or Critical Load. When such failure occurs, the strut remains in equilibrium in the bent position, as shown in Figure. 7.1.

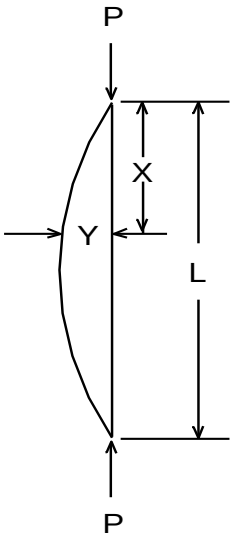


Figure. 7.1: Strut under compressive load

The critical loads can be derived as follows. They are referred to as Euler loads in honor of the Swiss mathematician. Assuming that direct compressive stress is negligible the ends are pin-jointed (i.e. free to change their slope):

$$M = EI\frac{d^2y}{dx^2} = -Py \tag{7.1}$$

or

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0 \tag{7.2}$$

or

$$\frac{d^2y}{dx^2} + k^2y = 0 \tag{7.3}$$

where

$$k^2 = \frac{P}{EI} \tag{7.4}$$

or

$$P = k^2EI \tag{7.5}$$

where *I* is moment of inertia. It can be shown that solution for this differential equation is

$$y = Acos(kx) + Bsin(kx) \tag{7.6}$$

where *A* and *B* are constants. From the boundary condition that *y = 0* when *x = 0*, we obtain *A = 0*. From the boundary condition that *y = 0* when *X = L*, we obtain *sin kL = 0*. By taking the least positive value, we have

$$kL = n\pi, \qquad n = 0, 1, 2, \ldots \tag{7.7}$$

when n is 1, thus:

$$k^2 = \frac{\pi^2}{L^2} \tag{7.8}$$

or

$$P = \frac{\pi^2}{L^2} EI \tag{7.9}$$

From the definition of k^2, *P* is calculated, and it is the critical load for the column under failure by buckling. A safe load shall be no more than this value divided by a suitable safety factor. If one end is fixed so that a change in slope at this end is prevented, and there is no lateral restraint at the other end (Figure. 7.3), bending moment *M* must be introduced to maintain equilibrium. Hence, the strut is now equivalent to half a strut of length *2L* loaded as in Figure. 7.2.,

$$Critical\ Load = \frac{\pi^2}{(2L)^2} EI \tag{7.10}$$

Or

$$P = \frac{1}{4}\frac{\pi^2 EI}{L^2} \tag{7.11}$$

If both ends are built in as shown in Figure. 7.3, there are two points of contraflexure *c*, *c*, at $\frac{L}{4}$ from each end.

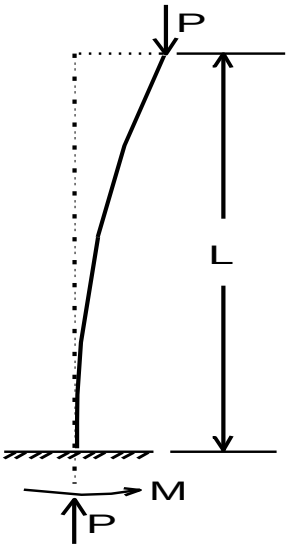


Figure 7.2

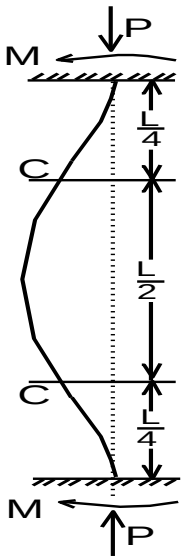


Figure 7.3

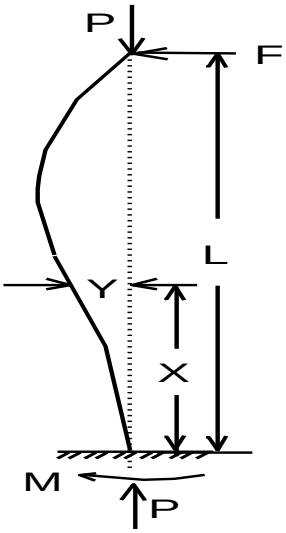


Figure 7.4

The piece of strut between them, of length $\frac{L}{2}$, is similar in shape to the pin-jointed strut of Figure. 7.1 since the bending moment at the Point *C*s is zero.

Hence for this case,

$$Critical\ Load = \frac{\pi^2}{\left(\frac{L}{2}\right)^2} EI \tag{7.12}$$

or

$$P = \frac{4\pi^2 EI}{L^2} \tag{7.13}$$

If the free end of the strut in Figure. 7.2 is prevented from moving laterally by a horizontal force *F* (Figure. 7.4), for any point at x from the fixed end we have

$$M_x = EI\frac{d^2y}{dx^2} = -Py + F(L - x) \tag{7.14}$$

or

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{F}{EI}(L - x) \tag{7.15}$$

or

$$\frac{d^2y}{dx^2} + k^2y = R(L - x) \tag{7.16}$$

where

$$R = \frac{F}{EI} \tag{7.17}$$

and

$$k^2 = \frac{P}{EI} \tag{7.18}$$

It can be shown that the general solution to this differential equation is

$$y = Acos(kx) + Bsin(kx) + \frac{F}{P}(L - x) \tag{7.19}$$

where A and B are constants. From the boundary condition that $y = 0$ when $x = 0$, and $\frac{dy}{dx} = 0$ 2where $x = L$, the constants can be determined as $A = -\frac{FL}{P}$, and $B = \frac{F}{kP}$, respectively. From the boundary condition $y = 0$ when $x = L$ we have

$$BsinkL = -AcoskL \tag{7.20}$$

or

$$\tan kL = -\frac{A}{B} \tag{7.21}$$

or

$$kL = \tan^{-1}\left(-\frac{A}{B}\right) \tag{7.22}$$

or

$$kL = \tan^{-1}(\frac{FL}{P} * \frac{kP}{F}) \tag{7.23}$$

or

$$kL = \tan^{-1}(kL) \tag{7.24}$$

Thus,

$$kL = 4.49\ radians \tag{7.25}$$

and

$$k^2 = \frac{20.2}{L^2} = \frac{P}{EI} \tag{7.26}$$

therefore

$$Critial\ Load = 20.2\frac{EI}{L^2}. \tag{7.27}$$

Since $2\pi^2 = 19.75$, it may be assumed that the critical load is given approximately by

$$P = 2\pi^2\frac{EI}{L^2} \tag{7.28}$$

7.3.2 Buckling test of straight columns.

Elastic instability, or buckling, is a very important part of solid mechanics. This experiment is designed to give students a physical feeling of the nature of buckling by measuring the critical loads for straight column with various end conditions and compare results to those calculated theoretically according to the formula outlined in the previous section.

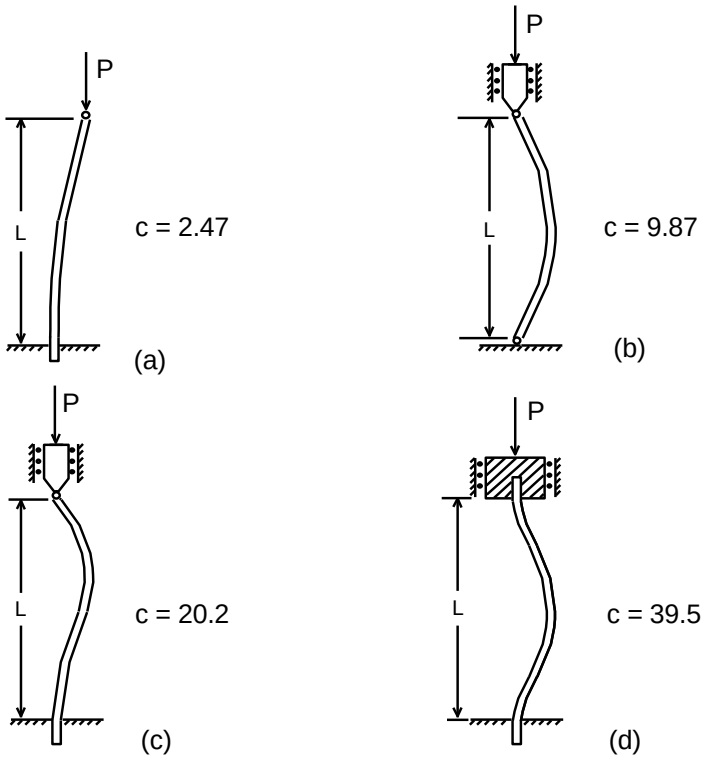


Figure 7.5: Buckling mode of a slender column under compression with different end conditions

Critical loads for:

a) clamped-free columns
b) hinged-hinged columns
c) clamped-hinged columns
d) clamped-clamped columns

In each case the constant C shown in Figure. 7.5 is to be inserted in the formula:

$$P_{crit} = C\frac{EI}{L^2} \tag{7.29}$$

Three sets of conditions are tested:

a) hinged-hinged
b) hinged-clamped
c) clamped-clamped

A set of steel specimens with different length and same width and thickness are provided to fit for different end conditions but the testing length *L* should be the same. While the specimen is being loaded, the load can be read from a load indicator through a load cell. The deflection of the column is measured by a LVDT that connected to a displacement indicator to display the deflections (shown in Figure. 7.6). When the load is applied to the column, the deflection of the column will first increase slowly along with the load. When the load gets to a certain level, the deflection will suddenly increase much faster and the load will remain mostly constant. The maximum load is the critical load. The load is to be applied in small increments, especially near the critical load.

7.4 Experimental Procedure: WP 120 Vertical Buckling Test Device

There are three steel specimens. One is for hinged – hinged end condition. One is for clamped-clamped end condition and one is for hinged – clamped end condition. The Buckling Test Device is shown in Figure. 7.6. The length *L* of the specimens are all the same. The top and bottom specimen holders are shown in Figure. 7.9 and Figure. 7.10 for different end condition specimen. A hydraulic force measuring device for force measurement showed in (Figure. 7.7). Find out the critical load for each end condition.

!Caution!: Never deflect more than max. 6 mm, since there is a risk or plastic deformation and damage to the specimen.

1) Hinged-hinged end condition. Insert thrust piece with V notch into bottom specimen holder. Insert long thrust piece with V notch into top specimen holder (Figure. 7.11). Adjust the load crossbar high and insert specimen with edges in the V notch.
2) Align the measuring gauge to the middle of the specimen. The measuring gauge must be set in the right direction of buckling.

3) Slowly subject the specimen to load using the load nut (Figure. 7.12). Read the deflection from the measuring gauge. Read the load from the force gauge. Read and record the deflection and the load every 0.25 mm. up to 1mm. Above 1 mm deflection, record the load and force every 0.5 mm.
4) The test can be concluded when the force does not change, despite an increasing load.
5) Repeat the test with the opposite buckling direction. To do this, set the buckling direction by initially guiding the specimen by hand.
6) For clamped-hinged end condition (Figure. 7.13) and clamped-clamped end condition (Figure. 7.14) do the same procedures 2-5.

7.5 Requirement

1. Plot graphs of load vs. deflection and extrapolate the curve to obtain the experimental critical load
2. Compare the experimental critical load with those predicted by the Euler equations. Determine the relationship between the experimental critical loads for the various end conditions.
3. A table to show the comparison of the experimental data and the theoretical data is required.
4. Find out the uncertainties of the experimental results and show an uncertainty tree.

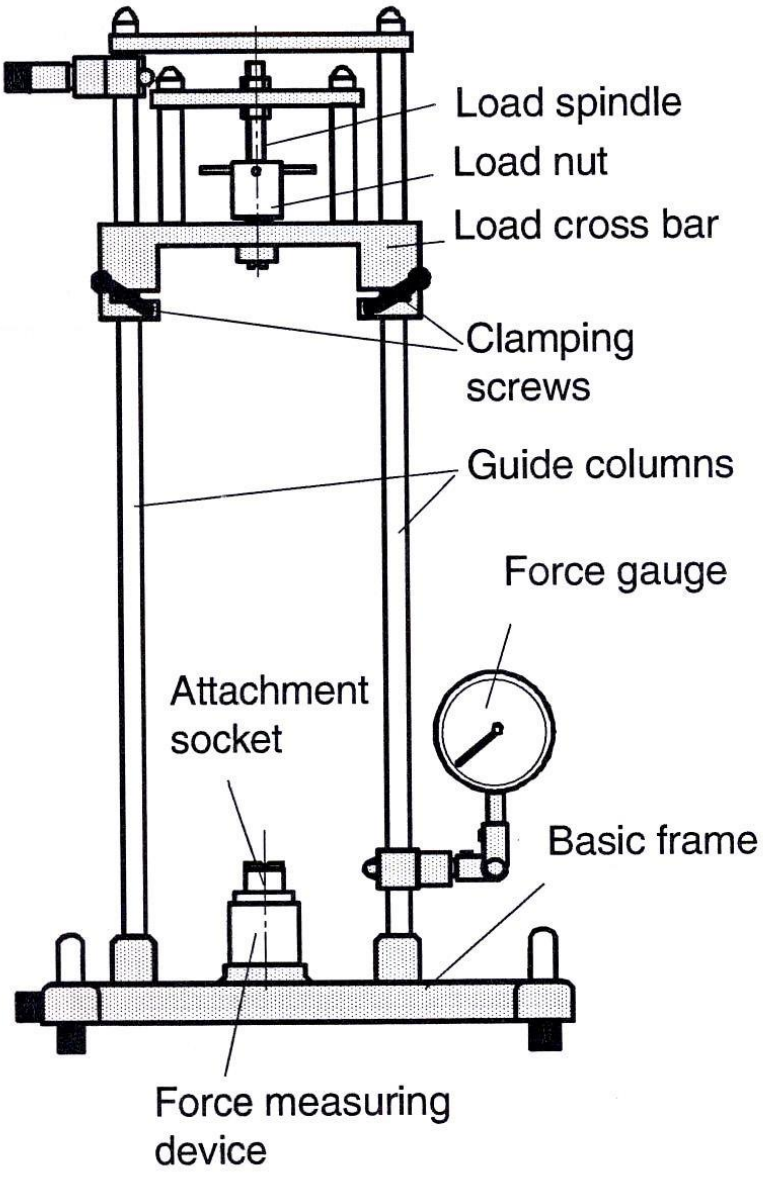


Figure 7.6: WP 120 Buckling Test Device

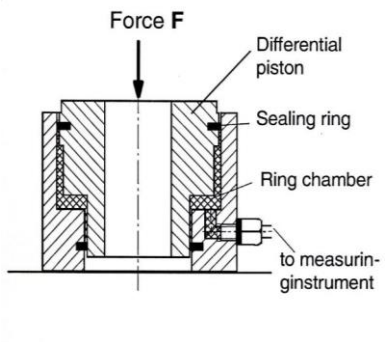


Figure 7.7: Hydraulic Force Measuring Device

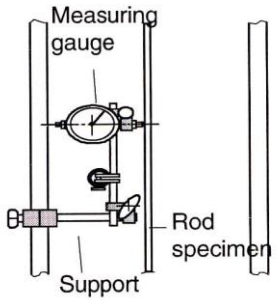


Figure 7.8: Deformation Measuring Device

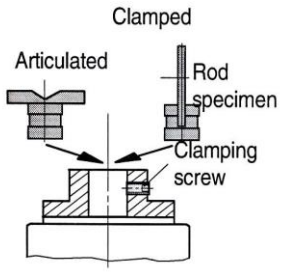


Figure 7.9 Bottom Specimen Holder

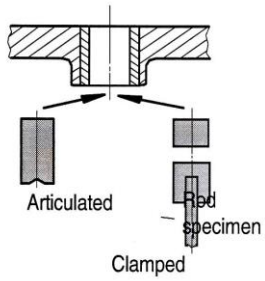


Figure 7.10: Top Specimen Holder

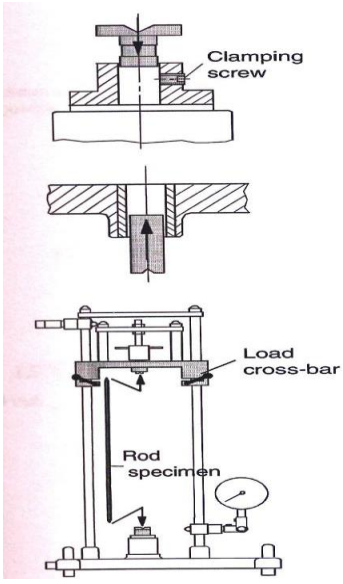


Figure 7.11: Hinged-hinged

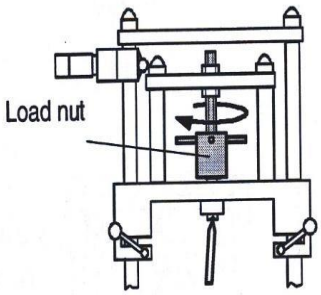


Figure 7.12: Loading parts

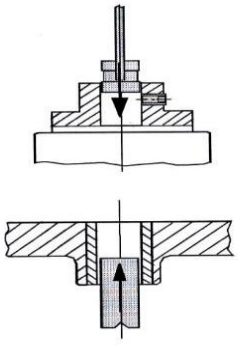


Figure 7.13: Clamped-hinged

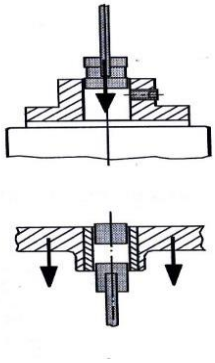


Figure 7.14: Clamped-clamped

LAB 8: STRAIGHTNESS MEASUREMENT OF LINEAR MOTION

8.1 Objectives

The objective of this metrology experiment is to learn how to use a digital indicator, a straightedge, and the principle of straightedge reversal to measure the straightness of a linear slide as well as the shape of the straightedge.

8.2 Equipment

- A digital indicator (Mitsutoyo Digimatic Indicator model 543-142) with a measuring range of 0-.5"/0-12.7 mm, a resolution of .00005"/0.001 mm, and an accuracy of .00015”
- A Rectangular Straightedge
- A linear slide (NS K Monocanier model MCM0803OH-10) with a leadscrew of 10 mm pitch and a differential manual drive (Klinger Model UE2.30.N)
- A full 360^{o} rotary stage (Newport Model RSP- 1)
- An Allen Wrench

8.3 Background Information

Calibration of machine tools is an essential operation in the machine tool industry. It is routinely performed to keep machines in their peak performance and therefore to produce quality products. Some important purposes of calibration can be summarized as follows: 1) error mapping of CNC machines for error compensation, 2) acceptance testing of newly acquired machine tools, 3) periodic calibration for optimized performance of machine tools, 4) troubleshooting, and 5) demonstration of quality to potential customers.

One of the major operations of machine tool calibration is the straightness measurement of machine tool slideways. Conventionally, straightness measurement has been done by using such instruments as dial indicators and straightedges, laser interferometers, etc. The dial indicator and straightedge method is a simple and elegant method, especially when the principle of straightedge reversal is applied. This experiment is designed to let you learn the principle of this method.

8.3.1 Straightness Measurement Method

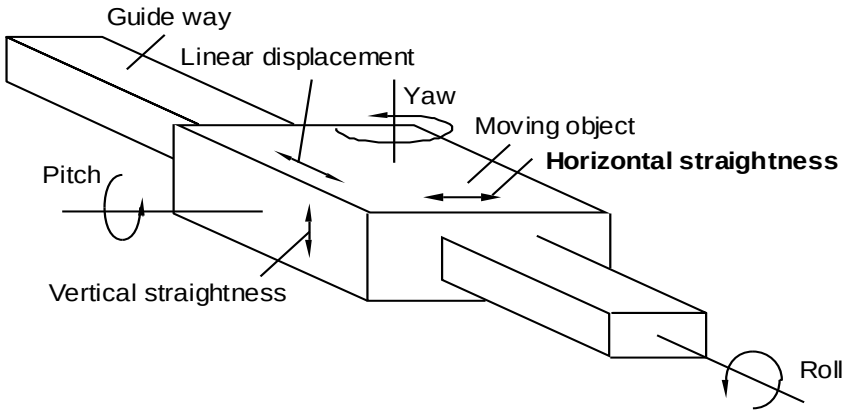


Figure 8.1: Geometric errors of a linear slide.

Figure 8.1 shows all six geometric error components of a linear slide. Among these error components, the horizontal straightness is to be measured in this experiment. One way to measure the straightness of a linear slide is to use a dial indicator and a mechanical straightedge which serves as a reference. Since no real straightedge is perfectly straight, errors in the shape of the reference artifact become mixed with the slide errors one is trying to measure. This problem is readily solved by a simple and elegant technique known as straightedge reversal. Figure 8.2 illustrates the principle.

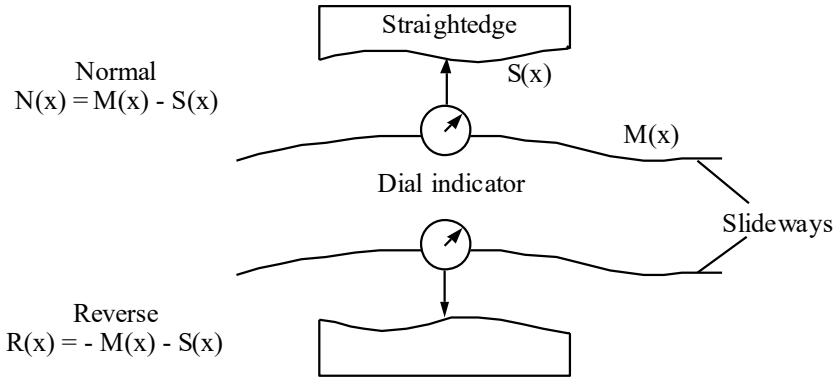


Figure 8.2: Principle of straightedge reversal method.

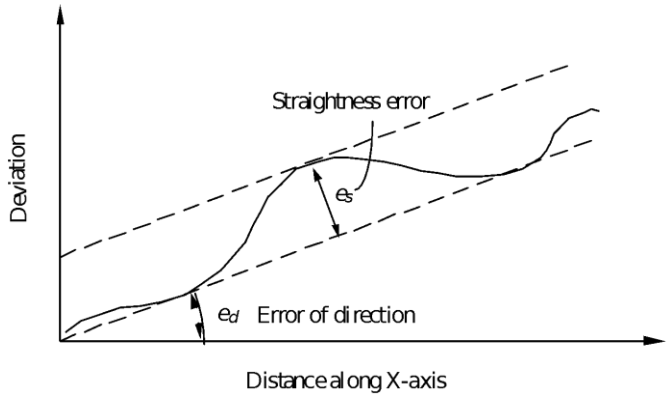


Figure 8.3: Definition of straightness error.

Straightedge reversal consists of two measurement setups. The straightedge is first supported on its side with the gauging surface in a vertical plane, in what is arbitrarily called the "normal" orientation. A series of data is taken at a chosen number of carriage positions, which can be represented as

$$N(x) = M(x) - S(x) \tag{8.1}$$

where M(x) is the error of the machine and S(x) is the shape of the straightedge. In the second setup, the straightedge is rotated 180° about its long axis, and the dial indicator is also rotated so as to sample the reoriented gauging surface. This is called the "reverse" orientation. With this setup, a new set of displacements

$$R(x) = -M(x) - S(x) \tag{8.2}$$

is obtained. From the results of these measurements, both the slide horizontal straightness and the shape of the straightedge can be determined as

$$M(x) = \frac{[N(x) - R(x)]}{2}, \qquad S(x) = -\frac{[N(x) + R(x)]}{2} \tag{8.3}$$

respectively.

Figure 8.3 shows a typical measured result of straightness of motion. It can be considered as consisting of a straightness of motion error e_s (the minimum distance of two parallel lines enclosing the curve) and an error of direction e_d (the slope of these lines). e_d could be caused by misalignment of the straightedge and the motion direction. It cannot be accurately determined from this experiment. Therefore it is eliminated from consideration in this experiment through linear regression analysis. e_s is the error to be measured.

8.4 Experimental Procedure

The experiment can be done following these steps:

1. Make sure that the measuring tip of the digital indicator is roughly aligned with the slide axis, the digital indicator is on the right hand side, and the reading of the counter on the slide drive is 9990.
2. Press the On/Off button to power on the digital indicator.
3. Lightly clamp the straightedge in the fixture as shown in Figure 8.4.
4. Rotate the rotary stage so that the measuring tip of the digital indicator is vertically against the gauging surface of the straightedge and the arrows on the rotary stage are aligned. Lock the rotary stage by tightening the screw on top of the drive of the rotary stage.
5. Press the Reset button twice to reset the digital indicator to zero. If the display is upside-down, rotate the upper part of the indicator until the display is correctly aligned.
6. Sample the gauging surface at 26 slide positions starting from counter number 0 to 25 (the counter number increases by 1 per revolution of the drive, which corresponds to slide position change of 10 mm) and record the readings of the digital indicator as N(x) in your logbook. (Use a table similar to the data sheet shown in Figure 8.6.)
7. Unlock the rotary screw. Rotate the rotary stage to align the digital indicator roughly with the axis of the slide. Move the slide back until the counter number is 9990 again.
8. Rotate the straightedge for 180^{o} about its long axis and lightly clamp it in the fixtures on the other side of the slide (see Figure 8.5) so as to face the same gauging surface of the straightedge to the digital indicator.
9. Repeat steps 4 - 7 and record sampled data as R(x) in the data sheet.
10. Repeat steps 3 - 9 using the opposite gauging surface of the straightedge. Record the sampled data in the corresponding rows of the data sheet.
11. From the above measurements, calculate the horizontal straightness of the slide, $M(x) = [N(x) - R(x)]/2$, and the shape of the straightedge, $S(x) = - [N(x) + R(x)]/2$. Use linear regression analysis to remove the zeroth and first order terms. Plot the straightness of the slide M(x) and the shape of the straightedge S(x) for both gauging surfaces (plot M(x) and S(x) separately). For the straightness of the slide M(x), also calculate the average of the two curves and plot it in the same plot.
12. Analyze and discuss the measured results. State your observations from the results and provide your comments on the measurement technique.

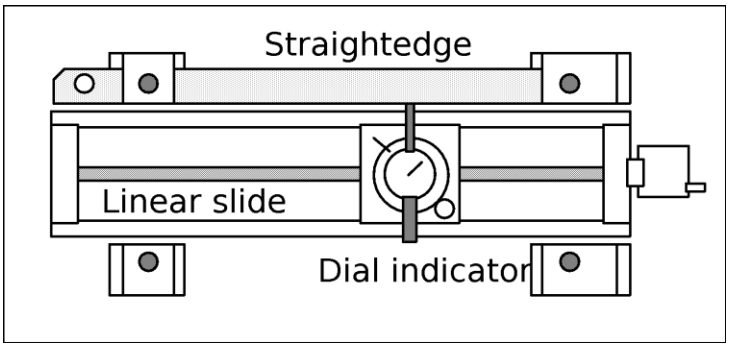


Figure 8.4: First measurement.

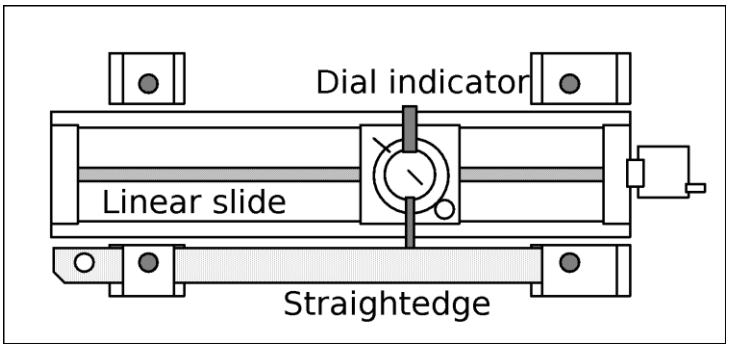


Figure 8.5: Second measurement.

Data Sheet for the Experiment

Distance (cm)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
N(x)																										
R(x)																										
M(x)																										
S(x)																										
N(x)																										
R(x)																										
M(x)																										
S(x)																										

N(x) --- The measured result for the normal orientation.
R(x) --- The measured result for the reverse orientation.
M(x) --- The straightness of the slide which equals [N(x) - R(x)]/2.
S(x) --- The shape of the straightedge which equals -[N(x) + R(x)]/2.

Figure 8.6: Example Date Sheet.

LAB 9: DIGITAL IMAGE CORRELATION/DIGITAL SPECKLE PHOTOGRAPHY TECHNIQUES FOR DEFORMATION ANALYSIS

9.1 Objective

1. Familiarization with white light speckles and their use for strain measurement.
2. Familiarization with the operation of Tinius Olsen H5K-S universal digital testing machine.
3. Familiarization with the capabilities of speckle photography and Digital Image Correlation (DIC).
4. Familiarization with DIC software Ncorr.

9.2 Equipment

- Light source to illuminate the specimen.
- Computer with Matlab and C++ compiler to run Ncorr software.
- Tinius Olsen H5K-S universal digital testing machine.
- Specimen with white light speckles painted on one side.
- DSLR Camera
- Ruler for pixel length scale conversion.
- Projection screen to provide background for photographs.

9.3 Background Knowledge.

9.3.1 Determination of tensile strain ε using the speckle method

Experiments on the extension of bars under tensile load have shown that within certain limits, the elongation of the bar is proportional to the tensile force applied. For many structural materials, this simple linear relationship between the force and the elongation it produces was first formulated by the English scientist Robert Hooke in 1678 and bears his name. Using the notation:

P = the force-producing an extension of the bar
L = length of the bar
A = the cross-sectional area of the bar
δ = total elongation of the bar
E = the elastic constant of the material, called modulus of elasticity or Young's modulus.

Hooke's law may be given by the following equation:

$$\delta = \frac{PL}{AE}. \tag{9.1}$$

In a unit axial tension test, the stress σ in the prismatic bar is the force per unit of cross-sectional area, i.e.:

$$\sigma = \frac{P}{A}. \tag{9.2}$$

Meanwhile, the axial strain is the elongation per unit length, is determined by the equation

$$\varepsilon = \frac{\delta}{L}. \tag{9.3}$$

Using equations (9.1), (9.2) and (9.3), Hooke's law may also be written in the following form:

$$\sigma = E\varepsilon. \tag{9.4}$$

White light speckles can be used as in-plane gages to determine the displacement and strain of the surface to which they are adhered to. The speckle pattern on the specimen will move with the specimen. Using the software Ncorr, a before and after image of the speckle pattern, the displacement can be determined. From the displacement, strain can be derived. The relationship between displacement and strain is shown:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}. \tag{9.5}$$

9.4 Experimental Procedure

9.4.1 Testing Procedure

9.4.1.1 Procedure for painted speckles

1. Take some time to familiarize yourself with the Tinius Olsen digital universal testing machine and the DSLR camera **(Do not make any adjustments to the camera's focus or settings unless explicitly told to by the instructors.)** before running the test.
2. Measure the width and the thickness of the specimen to find out the cross-sectional area of the specimens, additionally measure the initial active length of the specimen.
3. The specimen has been placed inside the universal testing machine for you. Use the slowest tensile loading speed to give the specimen an initial load (about 20 lb.). After the initial loading, the specimen capture an image of the specimen.
4. Set the load and extension readings on the front panel of the digital testing machine to zero.
5. Create a folder on the Desktop of the computer using your own group name for saving your testing files.
6. Load the specimen on the slow speed setting based on its extension, use small extensions (0.0010 – 0.0025in) as your loading steps. At each step wait for the load to settle, then take

readings of the extension, load, and for one of the steps capture an image using the digital camera. Load the specimen 10 steps from the initial point, this will result in 2 total images of the specimen. Take the SD card from the camera and insert it into the computer. Copy the photos from the SD card to the folder for your group.

9.4.1.2 Procedure for naturally-occurring speckles

1. Take some time to familiarize yourself with the Tinius Olsen digital universal testing machine and the DSLR camera **(Do not make any adjustments to the camera's focus or settings unless explicitly told to by the instructors.)** before running the test.
2. Measure the width and the thickness of the specimen to find out the cross-sectional area of the specimens, additionally measure the length of the specimen.
3. The specimen has been placed inside the universal testing machine for you. Use the slowest tensile loading speed to give the specimen an initial load (about 20 lb.). After the initial loading, the specimen capture an image of the specimen.
4. Set the load and extension readings on the front panel of the digital testing machine to zero.
5. Create a folder on the Desktop of the computer using your own group name for saving your testing files.
6. Load the specimen on the slow speed setting based on its extension, use small extensions (0.0010 – 0.0025in) as your loading steps. At each step wait for the load to settle, then take readings of the extension, load, and for one of the steps capture an image using the digital camera. Load the specimen 1 step from the initial point, this will result in 2 total images of the specimen. Take the SD card from the camera and insert it into the computer. Copy the photos from the SD card to the folder for your group.

9.4.2 Analysis procedure

1. Open MATLAB and open the ncorr.m file.
2. Run the ncorr.m file. Click file and then Load Reference Image and select the initial photo that was captured before loading began.
3. Click file then Load Current Image(s) then Load All and select the 3 photos that were captured after the specimen was loaded.
4. Now that all of the images are loaded into Ncorr select Region of Interest(ROI) then Set Reference ROI followed by Draw ROI, then draw a ROI that captures as much of the specimen as possible (It may be best to consult with the instructor if you have a question about your ROI), finally click Finish.
5. Click the Analysis tab, followed by Set DIC Parameters, change Num Threads in the Multithreading Options section to 1, typically the other parameters will not need to be adjusted, consult with the instructor before clicking Finish.
6. Click the Analysis tab, followed by Perform DIC Analysis, then Select Region, and click on the ROI that you previously drew. Once the Set Seeds window opens click set seeds and place the seed at the center of your ROI to the best of your ability, then click Finish.
7. The Seed Preview window will open and show you the placement of the seed on your reference image and all the current image, due to a relatively small extension being used make sure that the seed doesn't move too much from the reference image location. Click Finish and then finally click Finish on the Select Region window. The DIC analysis will now be performed. While the DIC analysis is performed compute the conversion factor for

the number of pixels to inches using the initial image you captured (compute the conversion such that it is units of in/pixel).

8. After the DIC analysis is completed the Displacement will be displayed. The displacements need to be formatted, select Format Displacements in Analysis tab. Using the previously calculated unit conversion convert the data from pixels to inches, all other options in the Format Displacements window can be left as is and click Finish.
9. Click the Analysis tab then Calculate Strains. Typically setting the strain radius to the largest value will produce the best results.
10. Save the displacement plots by selecting Plot then View Displacement Plots then click All. Click file on each of the windows that pops up then Save Image and finally Save Image With Info.
11. Save the strain plots by selecting Plot then View Strain Plots then click All. Click file on each of the windows that pops up then Save Image and finally Save Image With Info.

9.5 Requirements

1. Bring a flash drive or similar storage device to bring the data collected home.
2. Compare the displacement results from Ncorr with the readings from the Tinius Olsen H5K-S's extension output.
3. Use a linear regression compute the Young's Moduli of the specimen using the loads, their corresponding elongation, initial active length, and cross-sectional area of the specimen.
4. Using computed Young's Moduli of the specimen, and the strain plots produced by Ncorr compute only the tensile stress experienced by the specimen.

LAB 10: PHOTOELASITICITY FOR STRESS CONCENTRATION ANALYSIS

10.1 Objectives

1. Familiarization with the operations of a polariscope
2. Understanding of the elementary photoelastic method for stress measurement
3. Observation of stress concentration due to the presence of a hole through the photoelasticity method

10.2 Specimens and Instrumentations

1. Polariscope
2. HD digital camcorder and LCD TV
3. Photoelastic plate with a circular hole at the center

10.3 Background Knowledge

10.3.1 Double Refraction and Stress Optical Law

The method of photoelasticity is based on the principle of double refraction observed in a certain class of transparent materials called photoelastic or birefringent materials. This double refraction is a temporary phenomenon associated with the mechanical stressing of the object. When the loads are removed, the optical property of the material returns to normal, which means it will be optically isotropic. Consider a ray of light *Ri* entering a birefringent medium *i* from free space *o* (see Figure. 10.1). Assuming that the object is free of stresses, one can observe the refraction of light based on Snell's Law. Let *Rr* be the refracted ray and *i* and γ be angles of incidence and refraction measured with respect to the surface normal. The ratio $\frac{\sin i}{\sin \gamma}$ is then the refractive index n_{10}.

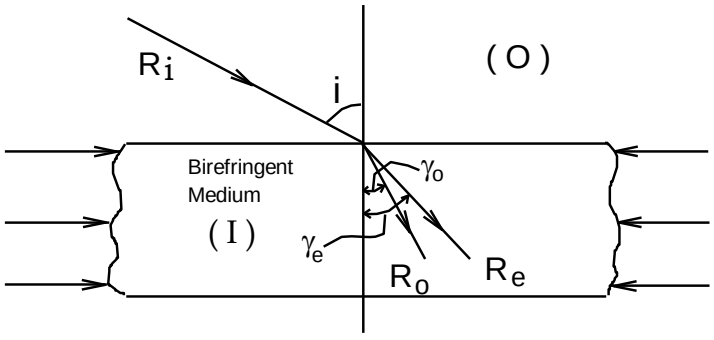


Figure 10.1: Behavior of light in a Birefringent Medium

Now, consider a birefringent medium subjected to external loads. For every incident ray R_i, double refraction gives rise to two refracted rays, R_o and R_e at different angles γ_0 and γ_e respectively with respect to the normal. These two rays propagate inside the medium with different velocities. When they exit from the medium, a phase difference Δ has occurred between the two waves. This phase difference is the result of stress in the medium and their relationship is the stress-optical law given below

$$\sigma_1 - \sigma_2 = \frac{\Delta}{2\pi}\frac{f_\sigma}{D} = \frac{N f_\sigma}{D} \tag{10.1}$$

where N is the fringe order, f_σ is the material stress fringe value and D the thickness of the birefringent material; σ_1, σ_2 are the two principal stresses, and in photoelasticity, it is always assumed that$\sigma_1 \geq \sigma_2$. Thus, once the fringe order is known, the principal stress difference $\sigma_1 - \sigma_2$ at any point can be determined.

10.3.2 Polariscope

The instrument that enables one to determine the stress-induced phase difference is called a polariscope. There are two types of polariscopes. One is called a plane polariscope. Its optical elements consist of one polarizer and one analyzer. A polarizer is an optical element that only allows a light vector to oscillate along a predetermined direction. An analyzer is also a polarizer that is used to analyze the polarization state of the impinging light. As shown in Figure. 10.2, the direction of polarization of polarizer *P* and analyzer *A* are perpendicular to each other.

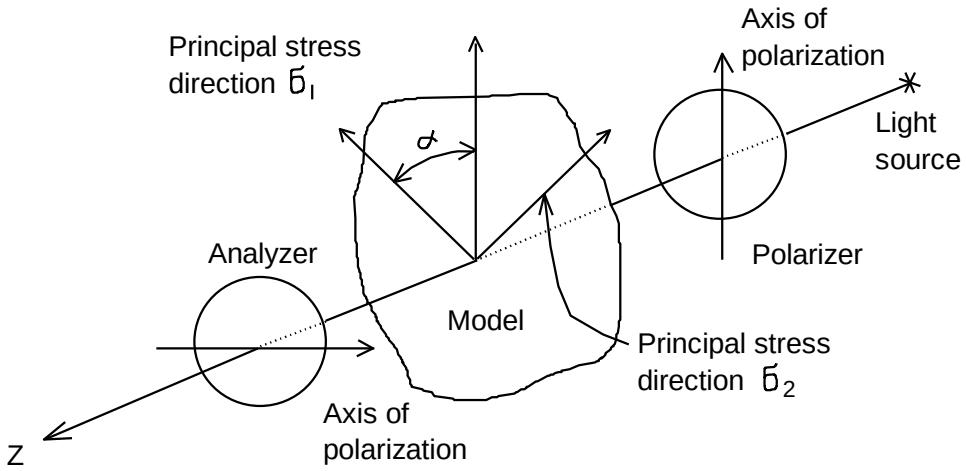


Figure 10.2: Optical arrangement of a plan polariscope

It can be shown that the intensity of light that immerges from the analyzer when a stressed photoelastic model is placed between them is given by the following equation,

$$I = K \sin^2 \alpha \times \sin^2 \frac{\Delta}{2} \tag{10.2}$$

where *K* is a constant, α the angle between σ_1 and the axis of the polarizer and Δ the stress-induced phase difference. Two type of dark fringes are observed. One is the result of

$$I = 0, \text{ when } \sin\alpha = 0,\ \alpha = n\pi;\ n = 0,1,2,3 \ldots$$

These fringes are called isoclinics, or locations of points of equal principal directions. The other type of dark fringes is the result of

$$I = 0, \text{ when } \sin\frac{\Delta}{2} = 0,\frac{\Delta}{2} = n\pi;\ n = 0,1,2,3 \ldots$$

since

$$N = \frac{\Delta}{2\pi} = n\ ;\ n = 0,1,2,3 \ldots$$

it is nothing but the fringe order given in the stress-optical law. These fringes are called isochromatics because they appear as colored in a white light illumination, except the zeroth order fringe which is always dark.

The second type of polariscope is the circular polariscope whose optical arrangement is as shown in Figure. 10.3. It has two more optical elements called quarter wave plates. The first quarter wave plate converts a plane-polarized light emerging from the polarizer with a circularly polarized light. The second quarter wave plate with its fast axis and slow axis orientation reversed cancels the effect of the first quarter wave plate. It can be shown that if a stressed photoelastic model is Iplaced in between the two quarter-wave plates of a circular polariscope the presence of the isoclinics is eliminated. The resulting light intensity emerges from the analyzer is simply,

$$I = K' \sin^2\frac{\Delta}{2}, \tag{10.3}$$

where *K'* is a constant. It can also be shown that if the analyzer is turned $90°$ so that it is parallel to the axis of the polarizer the intensity of the light that emerges is given by

$$I = K' \cos^2\frac{\Delta}{2}. \tag{10.4}$$

Thus,

$$I = 0, \text{ when } \frac{\Delta}{2} = \frac{(2n+1)\pi}{2};\ n = 0,1,2,3 \ldots$$

And the isochromatic fringe order *N* is

$$N = \frac{\Delta}{2\pi} = \frac{2n+1}{2} = n + \frac{1}{2}; n = 0,1,2,3 \ldots$$

These are the half order fringes.

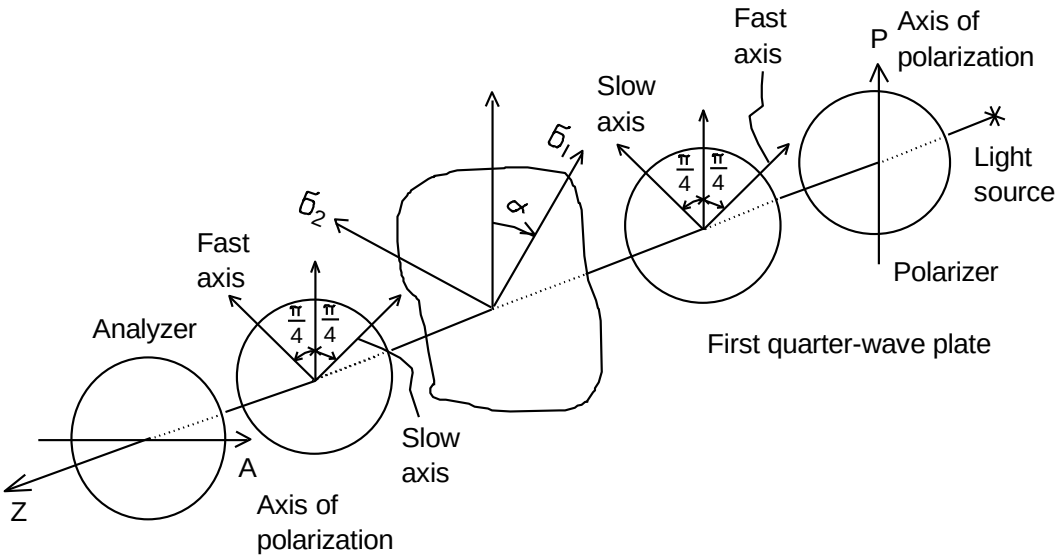


Figure 10.3: Optical arrangement of a circular polariscope

10.4 Stress concentrations and Inglis' solution

In 1913 Charles E. Inglis developed his linear elastic solution for the stress field surrounding an ellipse. This was a major step in the development of Linear Elastic Fracture Mechanics (LEFM) Theory and allows for a simple method of determining stress concentration factor of specimens loaded in tension with elliptical or circular holes. A infinitely large plate with a central ellipse with major axis of width 2a and minor axis of height 2b is subjected to a far-field uniaxial tensile stress as shown in Figure. 10.4.

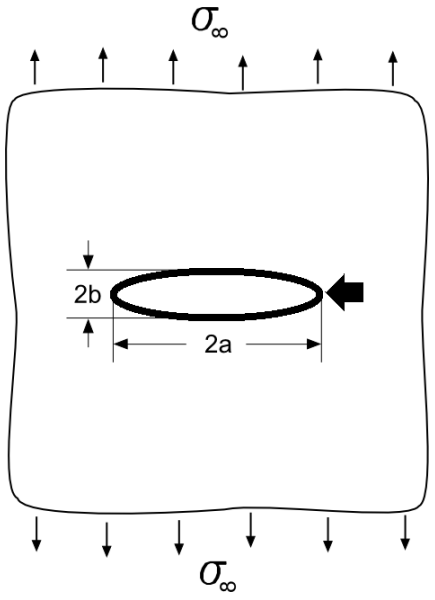


Figure 10.4: Plate with an elliptical hole at the center loaded in uniaxial tension

Inglis' solution allows for a relationship of the stress at the location of the arrow in Figure 10.4 to be dependent on the dimensions of the ellipse and the value of the far-field uniaxial tensile stress. Inglis' solution is as follows:

$$\sigma_C = \left(1 + 2\frac{a}{b}\right)\sigma_\infty, \tag{10.5}$$

10.5 Determination of Stress concentration in a perforated sheet undergoing tensile load.

A strip of photoelastic material of width D with a central hole of diameter 2a is subjected to a far-field uniaxial tensile stress as shown in Figure. 10.5. The stresses at both sides of the edges of the hole along the x-axis will be much higher than the same strip without a hole under the same loading condition. This phenomenon called stress concentration. The stress at the edge of the hole along x-axis compare to the stress in the same strip without a hole under same loading condition is called the Stress Concentration Factor. The stress concentration factor is defined as follows:

$$K_C = \frac{\sigma_C}{\sigma_\infty}, \tag{10.6}$$

where

$$\sigma_\infty = \frac{P}{A} \tag{10.7}$$

and

$$\sigma_c = \sigma_1 - \sigma_2 = \frac{Nf_\sigma}{D} \tag{10.8}$$

Using the Fringe order at x = a, y = 0

D: Thickness of the specimen.

f_σ: material stress fringe value.

P: the load applied to the specimen

A: the section area of the specimen without a hole.

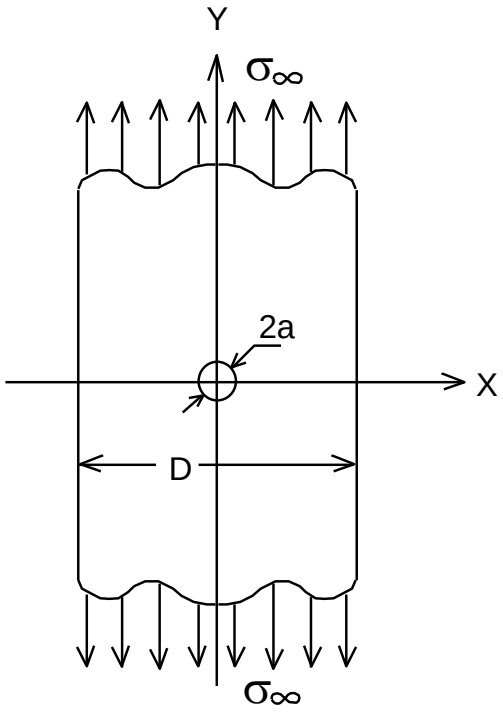


Figure 10.5: Plate with a circular hole at the center loaded in uniaxial tension

10.6 Testing procedure:

1. Change the fixtures and the specimen with a central hole for tensile test.
2. Using white light source with dark background, adjust the height of the moving frame of the loading frame that the image of the central hole and the whole width of the specimen can be observed.
3. Measure the width and the thickness of the specimen.
4. Use the Balance adjustment to set the strain indicator readings of the load to zero before starting to load the specimen.
5. Increase the load slowly (the maximum load does not exceed 400 lb) the color isochromatic fringe pattern can be observed.
6. There are four dark spots along the edge of the hole symmetrically and always in dark even the load is changing. These are four singularities and $\sigma_\theta = 0$ therefore N = 0
7. Start from these singularities, follow the color sequence of the isochromatic fringes to determine the fringe orders.
8. Place the filter in front of the camcorder. The isochromatic fringes change to sharper dark fringes. Start to increase the load (the maximum load does not exceed 400 lb) until the Nth order isochromatic fringe just appears at the edge of the hole along x-axis. Record the load from the strain indicator reading. Set the HD digital camcorder to PHOTO mode and manual exposure mode, the images will save in the memory stick. Take the color and black and white pictures using manual exposure to get the best exposure without and with the filter in front of the HD digital camcorder
9. Run the experiment three times.

10. Repeat for the two specimens with the ellipses, but make sure the maximum load does not exceed 300 lb

10.7 Requirement:

1. Plot the fringe order distribution start from the edge of the hole along x – axis (y = 0).
2. Find out the stress concentration factor K_c for the circular specimen. Using the material stress fringe value of $f_\sigma = 40 \frac{lbf}{in}$, At x = a, y = 0. Note that $\sigma_{yy} = \sigma_1$, and $\sigma_r = \sigma_2 = 0$.
3. Compare your result for the stress concentration factor K_c with the theoretical solution using Inglis' solution.
4. Find out the uncertainties of experimental results and show an uncertainty tree.

Part II:
Report Writing

1 WRITING LAB REPORTS FOR MEC 316

1.1 Introduction

The purpose of technical reporting is to convey information obtained through analysis and/or experiments. The audience can vary widely, depending on the type of report, and can range from colleagues familiar with your field of study to those with very little prior knowledge of the material, *e.g.*, a summary of the experiments for upper management.

A sobering truth of technical reports is that they can easily have a lifetime of decades. It is very common to see papers 10, 20 even 30 years old or more referenced in all types of archival documents (browse through any recent research journal in any field in the library to verify this). Furthermore, it is not uncommon for the experiment and results discussed in the report to represent an investment of tens to hundreds of thousands of dollars.

Finally, the written word is permanent: mistakes, oversights, erroneous conclusions, etc. all remain unabated with time. When a person reads the report, their impression of your work is formed *at that time* and flaws in a written piece of work reflect poorly on the writer.

Fortunately writing is an *acquired skill*. Particularly with regard to technical writing, a reasonable set of guidelines can be established. This document serves to provide a brief overview of such guidelines.

1.2 Lab Report Format

The lab report format for this class consists of the following items, in the order shown:

1. **Title Page** – Include lab title, date, author and fellow group members.
2. **Abstract** – The abstract should be a single, short paragraph that describes the purpose of the experiment, the variables to be measured, and the basic measurement concept. The abstract serves as a summary of the entire work.
3. **Introduction** – An introduction to the topic of the experiment: why it is important, what context the results have to the real world, how the measured values are used or influence engineering design and analysis. Do **not** discuss lab procedures or detailed lab descriptions here.
4. **List of Equipment** – Briefly describe the list of equipment and instrument used to conduct the experiment. Make a sentence to introduce the list and include manufacturer and model number where appropriate.
5. **Experimental Theory** – This section should describe in detail the theoretical basis of the experiment. For example, if the heat flow in a composite cylinder is to be determined from the temperature distribution along the cylinder, describe the relevant governing equations and the relationship between the measured values (the temperatures) and the desired quantities (the heat flow). A schematic diagram of the experimental apparatus may be included and referenced in the discussion.[1] All equations should be numbered.

[1] It is always a good idea to include a schematic diagram of the basic experimental approach. We are not requiring it in this class due simply to time requirements.

6. **Experimental Procedure** – This section describes *in detail* the steps performed during the experiment to obtain the required data. Do not simply copy the steps from the lab manual! This is plagiarism and you will have points taken off. You should understand the experiment well enough to describe it in your own words. Do not write this section as a series of steps or instructions, and do not write in the future tense (we will, etc.). You should be able to describe the experimental procedure in paragraph form, and avoid commanding the reader to wit: "Place the thermometer in the water bath. Record temperature. Record pressure, etc."
7. **Results** – This is the section where your results are actually *presented* with their total uncertainties. Make sentences to describe the calculations involved and the data used. Calculations, figures, and tables should be neatly organized. All figures (i.e., graphs, charts, diagrams, …) and tables must be labeled with a number and caption, and should be included on a single page following the page on which they were referenced. Include units with all physical quantities.
8. **Discussion** – In this section the results that you obtained are *discussed* and *interpreted*. Do the results make sense? Are they what you expected from the theory of the experiment? Is there a trend in the results? The point here is to provide both a quantitative and qualitative analysis of results using deductive reasoning.
9. **Error Analysis and Uncertainty Tree** – This section should detail the calculations associated with the error analysis and uncertainty of the reported results. Make sentences to guide the reader through calculations. Comment on measurement and instrument uncertainties, central tendency and dispersion, statistical samples, confidence interval, and error propagation. What was the largest source of error? How could this error be reduced? An uncertainty tree can be included in this section.
10. **Conclusions** – A *single* paragraph describing the experiment briefly and the results that you obtained. *There should be NO new results in the Conclusions section*! Don't use any symbols or variables in the conclusion section.
11. **References** - If you have any. Be sure to keep the format consistent for all references.
12. **Appendices** - You can place handwritten calculations, spreadsheet lists, and other data here.
13. **Prelab pages** - EACH person from the group must attach his/her prelab pages to the report.

1.3 Grading of the Lab Reports

Each lab report will be graded on a 100-point scale. The breakdown for the grading is:

Abstract	5 points
Introduction	5 points
List of Equipment	5 points
Theory	10 points
Experimental Procedure	10 points
Results	15 points
Discussion	15 points

Error Analysis & Tree	15 Points
Conclusion	5 points
Quality of Writing(Clarity/Style/Format)	15 points
Total	100 points

1.4 Before Beginning Each Lab

The instructor is required to sign off on your prelab sheet before you start the lab. You must have the following items in your sheet before the instructor will sign off on it.

1. Brief objective of experiment (2–3 sentences max).
2. List of equipment: (manufacturer, model and serial numbers). Do this as soon as you arrive at the lab station.
3. Equations for the calculations.
4. Blank tables for all data that is to be collected.

Also, everyone should record the data in their prelab sheets. Even though data are identical, this is useful in the event that some data are not recorded by a person, there is a discrepancy in the data, one person loses their sheet, etc. Every lab member must turn in their prelab sheets with the lab report.

1.5 Deadlines and Late Lab Reports

Lab reports are due at the ***beginning*** of the following lab class. If the lab is not in at this time, it will be considered at least one day late. **LATE LABS WILL HAVE TEN (10) POINTS DEDUCTED FROM THE FINAL SCORE PER DAY.** In addition, every member of the lab group will receive the late penalty, regardless of who was the lead author for the report. Note that this is for your own protection: people who turn lab reports in late have an unfair advantage due to the additional time they have to improve the quality of their lab report. The late penalty compensates for this. There will be *no* exceptions, so don't even bother to ask.

1.6 General Comments on Lab Report Writing

- Lab reports must be typed and **double spaced**[2]. Preferred fonts are Times New Roman or Arial, and Symbol for Greek letters. Use an 11 or 12 pt. font, a 10 pt. font is too difficult to read and a 14 pt. font is too large. Justify the right margin to improve the appearance and readability of your report.
- Be sure to check your spelling. Use the spelling checker on your word processor, but be careful of easily confused words (e.g., *from* vs. *form*) that will not be flagged.
- Avoid single lines at the top of pages and headings without any text at the bottom of pages.
- Don't copy verbatim from the lab manual. Express the concepts in your own words.
- Number all pages.

[2] Quite often, reports and documents—especially drafts or versions to be subsequently edited—are double-spaced. The reason is a practical one: the editors (instructors in this case), will place comments, suggestions, and corrections in the space between lines and the margin. A single-spaced document makes this very difficult to do.

1.7 Figures

Creating clear, well-designed graphs is not difficult, provided several points are kept in mind. Refer to Tables and Figures in your text with capital letters. Also, refer to tables as *Table*, not *Sheet*. Below is an example of a well-designed graph and a list of things to keep in mind when creating graphs. Some points to observe regarding graphs:

1) All figures (graphs, drawings, charts, photographs, …) should have a caption that includes the number of the figure and a brief caption describing the nature of the figure.
2) Axes:
   a) Label **both** axes, using words and symbols.
   b) Include the units on both axes.
   c) Use even, integral values for the tick marks (10, 20, 30… not 9.6, 19.2, 28,8…).
   d) Do not use more significant figures than are required: (9.1, 9.2, 9.3, …, not 9.100 9.200, 9.300, …).
   e) Do not use "e" or "E" to express powers of 10: (1.0×10^4, 1.5×10^4, 2.0×10^4, not 1.0E+04, 1.5E+04, 2.0E+04).
   f) Factor out large multipliers. In the example graph below, the *y*-axis is expressed in mm, rather than 0.004, 0.006, or 0.008 m.
   g) Labels must be outside of graph. Tick marks can be inside or outside the graph. (Both conventions are used in engineering graphs.)
   h) Axes labels should appear at *edge* of *x* and *y*-axes; in the example below, there should be no space to the left of the 0.0, the right of 1.0, above 14, or below 4.
   i) Keep the precision the same in axis labels: (0.0, 0.5, 1.0, 1.5, 2.0, not 0, 0.5, 1, 1.5, 2)
   j) Always place a zero before decimal points: (0.1, 0.2, 0.3 not .1, .2, .3)
3) The plot should span nearly the entire range of the *x* and *y*-axes. Do not leave a lot of dead space in the graph.
4) Use a legend to identify each plot (be sure to use different line types!) OR label each plot explicitly as shown (best for three plots or less).
5) When plotting experimental data, always plot data points as symbols. You may optionally choose to draw a line through the points.
6) Error bars should be used to represent the uncertainty in your experimental values obtained from the error analysis. Note that error bars can also be used in the *x*-direction to represent uncertainty in the value associated with the *x*-axis (not shown in Figure 1).

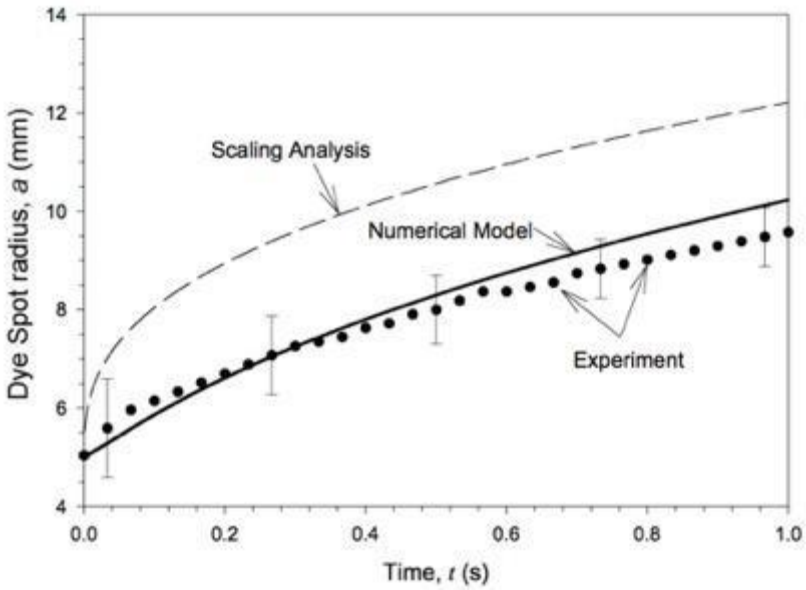


Figure 1. Temporal evolution of the dye spot radius a for ΔT = 1.0 K and initial size a_0 = 5.0 mm. Comparison between numerical model (solid line), scaling analysis (dashed line), and experiment (solid symbols).

2 TIME SAVING TIPS

2.1 Report Writing

Use a good word processor. We recommend using *Microsoft Word* on either Macintosh or PC platforms. Of course, if you are already familiar with other comparable packages, such as *LaTex*, use it. In particular, you should learn to use features such as

- *Styles* for formatting text, headings, lists and tables,
- Frames for placing text,
- The equation editor (note: not installed with Word by default),
- Automatic page and equation numbering, and
- Automated table of contents features, etc.

Use of these features can save a lot of time! Another factor to consider is the word processor your lab partners use. Once you have written the first report, it can serve as the framework for subsequent reports. This is particularly true if you use the automated features discussed above.

You can read *The Elements of Style* by Strunk & White to obtain insights about improving writing style[3]. Other good writing reference books include *The Chicago Manual of Style, 14th ed.* (1993, University of Chicago Press) and *Rules for Writers, 2nd ed*. (1988, Bedford Publishing).

You may write the following by hand:

- Error analysis calculations
- Equations (Note: it is recommended to use Equation Editor in Microsoft Word, rather than handwriting, because changes are easier)
- Drawings (if you want to add them)

2.2 Analysis

There are a variety of packages on the market that can aid analysis. For the level of analysis required in these labs, *MathCAD* is a very useful utility to have. Equations can be entered with functional arguments as they appear in standard math notation. Units and unit conversion are also easily performed. Finally, graphs can be generated that are suitable for the final report. The final calculations can be printed for inclusion in the report. The disadvantages are that it can take some time to get used to the system and the cost.

Other packages such as *Mathematica*, *Maple*, and *Matlab* will probably be more trouble than they are worth for this class. These packages are oriented more towards symbolic manipulations and

[3] The book is not only an excellent and concise summary of the use of English, but it is an entertaining read as well. The book was originally composed by William Strunk, a professor of English at Cornell in the early part of the century. He compiled, over the years, a list of errors and mistakes that students repeatedly made, and provided it to his students as a small booklet. *The Elements of Style* is essentially this same book in form and content. Read it; you will be happy that you did.

numerical analysis. If you are a seasoned veteran with one of these packages, however, by all means use it if it helps.

2.3 Graphs and Figures

Many students use Excel for their graphs. The graphs from Excel are perfectly acceptable, however Excel is a multi-purpose software package, and is oriented towards the businessperson, rather than the scientist and the engineer. Therefore, editing will be required to make Excel graphs appropriate for laboratory reports. For instance, while a title is typically included on top of graphs in Excel, it should be removed as the figure caption of your document is already used to describe the graph content. Axes and labels should also be edited to make graph suitable for laboratory reports. It is a good idea to examine graphs in scientific and engineering textbooks and emulate their style.

In general, software packages dedicated to scientific graphing provide more data analysis power, easier editing and control of graph properties, and a more professional final appearance. Other software include *SigmaPlot*, *DeltaGraph, TechPlot*, *PSIPlot*, *Igor Pro*, etc.

ERROR ANALYSIS

3.1 Introduction

As important as reporting a measured value is the determination of the *uncertainty* of that value. Uncertainty establishes a bound in which the reported value is allowed to fall. A typical expression for a measured variable is expressed as

$$x' = \bar{x} \pm u_x \ (P\%), \tag{1}$$

where x' is the true value of the parameter or system being measured, $\bar{x}$ is the most probable value, or *nominal value* and u_x is the uncertainty, or range within which the measured value may vary. In this manual, Δx will also be used to represent the uncertainty. Here P% is the *confidence interval*, and is a measure of how often the measured value will fall within the reported range. A confidence of 95% is one of the most common values, while values of 68%, 99%, and 50% also find use. Use a confidence interval of 95% for all work in this course. Note that the uncertainty u_x is expressed in the same units as x. Alternatively, the uncertainty can be expressed as a percent of the nominal value:

$$x \pm v\% \ . \tag{2}$$

Reporting a measurement without an uncertainty bound is really an incomplete statement of the measurement value.

Example 1

To simply say that the temperature of an oven is 154 ºC is not necessarily a useful value. The uncertainly of the measurement is also an integral part of the specification of a measurement value. If, for example, the oven temperature is 154 ± 2 ºC, and the process in the oven requires a temperature of 154 ± 0.5 ºC, then the oven temperature is unacceptable. On the other hand, if the required temperature is 154 ± 5 ºC then the temperature is acceptable.

3.2 Accuracy and Precision

Accuracy and *precision* are indicators of the quality of a measurement. Accuracy refers to how close the measured value is to the true value. *Precision* is a measure of how repeatable the measurement is, or, equivalently, how much variation there is from measurement to measurement. Precision is closely related to *repeatability*.

► Accuracy

Accuracy can be expressed several ways.

Absolute error, ε, is simply the difference between the measured value and the true value:

$$\varepsilon = \text{true value} - \text{measured value}. \tag{3}$$

Relative error, ε_r, is more commonly used, and is defined as

$$\varepsilon_r = \frac{\text{true value} - \text{measured value}}{\text{true value}} \ . \tag{4}$$

Note that absolute error has the same units as the measurement value, however relative error has *no units*. Note also that the sign of the error (both relative and absolute) indicates whether the

measured value is greater than, or less than, the true value. For positive values of ε_r, the measured value is *smaller* than the true value, and *vice versa* for negative ε_r.

The relative error is then used to define the *relative accuracy*, *A*, as follows:

$$A = 1 - \varepsilon_r \tag{5}$$

Values of *A* close to unity imply the measurement is accurate, i.e., the measured value is close to the true value.

Example 2

Say a 50.0 g mass is placed on a scale, and the scale reports the measured value as 51.3 g. The *absolute error* is 50 g – 51.3 g = –1.3 g, the *relative error* is (50 g – 51.3 g)/50 g = –0.026, or –2.6%. The *relative accuracy* is then 1 – (–0.026) = 1.026 or 102.6%.

Note that in the above examples, it is assumed that the true value is known. In general, this is not the case, or we wouldn't have to perform the measurement in the first place. Nonetheless, it is important to ascertain the accuracy of a system, and this is done by placing a *known* input into the system, and recording the output. This process, which is part of the process of *calibration*, allows the accuracy to be estimated. A simple example would be to place known weights on a scale and compare the reported measurement from the scale with the known weight that actually is on the scale. The scale would then be adjusted to read the correct weight.

► Precision

The *precision* or repeatability of a measurement system refers to the ability of the system to indicate a particular value upon repeated, but independent, measurements of a constant input value. *Precision error* is a measure of the random variation to be expected during repeatability trials. Note that high repeatability results in a low precision error, but gives no indication of the accuracy of the measurement. This is best illustrated with an example. Consider the following measurements made on a system to measure $\pi = 3.1415926\ldots$ Assume each measurement consists of four readings.

Case 1:	2.1415	2.1414	2.1416	2.1415
Case 2:	3.12	3.17	3.09	3.16
Case 3:	3.1415	3.1413	3.1412	3.1417

Case 1 is *precise*, but not *accurate*, Case 2 is accurate, but not very precise, and Case 3 is both accurate and precise. Note that high accuracy implies high precision.

3.3 Different Types of Measurement Error

There are three basic types of measurement error

1. Systematic, or bias error
2. Precision, or random error
3. Illegitimate error

► Systematic or bias error

These are fixed or constant values of the error in a given set of measurements. In most cases they can be accounted for by *calibrating* the experiment.

Example 3

An everyday example of a bias error would be measuring your height with your shoes on. Every measurement made would have the *same* amount of error, namely the height of your shoes. No amount of statistical averaging would remove this error.

Sources of such bias error include:

1. Errors during calibration

2. Loading errors, i.e., the measurement system alters the value of the original system
3. Unaccounted for effects that remain constant with time, e.g., forgetting to account for the weight of the wax paper when measuring a small amount of chemical on an analytic balance.

► Precision or random error

These are random in nature and can be treated by statistical analysis. They occur due to a variety of causes, including:

1. Varying environmental conditions (e.g., changing room temperature and pressure, or room vibration),

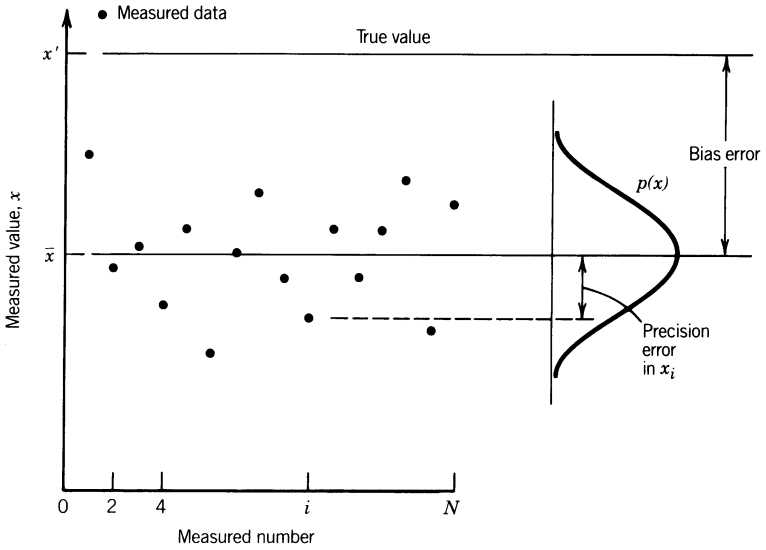


FIGURE. 1. Example of bias and random errors.

2. Insufficient sensitivity of the measuring system, and
3. Drift and fluctuation in the measurement system itself.

The relationship between bias and precision errors is illustrated in Figure 1 above. Note the constant offset produced by the bias error, and the spread of the data about $\bar{x}$ due to precision error.

► Illegitimate errors

These occur mainly due to oversight (or carelessness!) and consists of the following:

1. Blunders or mistakes (e.g., reading mm as cm or inches),
2. Computational errors (e.g., wrong formula or mixed units), or
3. Incorrect system operation.

Note the term *illegitimate*; these errors are <u>not</u> acceptable as reasons for discrepancies between measured and expected values. If illegitimate errors are the cause of the error, the best solution is to repeat the experiment to avoid them. In general, all of the above errors introduce *uncertainty* in the measurement.

3.4 Determining Measurement Uncertainty

<u>Note: This can be very confusing! Please read carefully!!</u>

Uncertainty analysis (or *error analysis*) is the procedure by which uncertainties in measured quantities are ascertained, and the relationship between these measured values and the reported value is established. There are two key steps in performing an uncertainty analysis in a measurement:

1. Establishing the uncertainty of the initial, or *input*, variables measured, and
2. Propagating the error in the measured values to any calculated values.

For example, say you wanted to calculate the density of a piece of wood. You could cut a rectangular piece and measure its length, l, width, w, and height, h, to obtain the volume, V, then measure its mass, m, with a scale, and finally compute the density, ρ. The calculations are as follows:

$$V = lwh \quad \text{and} \quad \rho = m/V.$$

The input variables are the ones actually measured: l, w, h, and m. The variables V and ρ are calculated, or derived from, these initial values. The act of assessing the uncertainty in the density measurement involves (1) assessing the uncertainty in l, w, h, and m, and then (2) propagating that error to the calculated variables V and then ρ.

3.5 Determining the Uncertainty of Input Variables

The first step in determining uncertainty is to determine the uncertainty in the input variables themselves. There are two main sources of uncertainty in measurements: (1) instrument uncertainty and (2) measurement uncertainty.

► Instrument uncertainty

Instrument uncertainty arises from the measurement system itself, and represents errors associated with the equipment responsible for making the measurement. There are two basic kinds of instrument uncertainty:

(i) *Resolution uncertainty*, u_0. Resolution uncertainty is simply the inability of the measurement device to resolve to an infinite number of digits. The resolution uncertainty is generally taken as one-half of the instrument resolution:

$$u_0 = \frac{1}{2} \times \text{instrument resolution.} \tag{6}$$

Consider, for example, a normal thermometer with a scale that reads in 0.1 ºC increments. The instrument resolution is then 0.1 ºC, and the resolution uncertainty is ±0.05 ºC.

(ii) *Manufacturer's uncertainty*, u_c. The manufacturer's uncertainty refers to specific errors in the instrument (which are usually reported by the manufacturer). These include such things as *linearity*, *hysteresis*, *repeatability*, etc. The total manufacturer's uncertainty is combined using the *Root Sum Square* (RSS) method:

$$u_c = \sqrt{u_1^2 + u_2^2 + u_3^2 + \ldots} \tag{7}$$

Here the u_j are the individual instrument uncertainties as reported by the manufacturer. Finally, the total instrument uncertainty is obtained by combing the resolution and manufacturer's uncertainty:

$$u_{inst} = \sqrt{u_0^2 + u_c^2} \tag{8}$$

► Sample measurement uncertainty

Measurement uncertainty also appears in the measured variable itself that must be estimated. Both bias errors and random errors can be present. Bias errors must be ascertained through *calibration*, which is mentioned elsewhere in this document. For random errors, statistics can be used to determine the uncertainty.

Say that N measurements are collected on a system. In general, the measurements will vary from one measurement to the next, and statistics can be used to analyze such variations. The average, or *mean*, $\bar{x}$, and *sample standard deviation*, S_x, are calculated as follows:

$$\bar{x} = \frac{1}{N}\sum_{j=1}^{N} x_j \qquad\qquad S_x = \left[\frac{1}{N-1}\sum_{j=1}^{N}\left(x_j - \bar{x}\right)^2\right]^{1/2}. \tag{9a,b}$$

It is sometimes desirable to know what the range of an additional measurement is expected to be. In other words, after making N measurements, what is the expected range of the next, or N+1, measurement? Statistics can be used to provide this range, which is expressed as follows:

$$x_{N+1} = \bar{x} \pm u_x \quad (P\%) \tag{10}$$

Here x_{N+1} represents the value of the next measurement, $\bar{x}$ is the mean, as defined above, and u_x represents the uncertainty of the next measurement with confidence interval P. For example after collecting a sample and performing the appropriate statistics, the result of a future measurement might be expressed as

$$1.34 \pm 0.23 \ (95\%).$$

This means that, about 95% of the time, the next measurement made will fall between 1.11 and 1.57. About 5% of the time, the measurement will fall outside of this region. The mean of the measurements $\bar{x}$ is 1.34, the uncertainty is ± 0.23, and the confidence interval is 95%.

The calculation of the sample measurement depends on how many measurements are made, with the cutoff being about 60 samples. The following applies:

Small Sample Size ($N \le 60$)

For smaller sample sizes, i.e., $N \le 60$, the uncertainty increases somewhat due to the limited number of samples available for the statistics. To account for this, the so-called *Student-t distribution* is used to determine the uncertainty from the standard deviation, S_x. The format is as follows:

$$u_x = t_{\nu,P} \cdot S_x. \tag{11}$$

Here $t_{\nu,P}$ is the coefficient (coverage factor) that multiplies S_x to obtain the uncertainty, $\nu = N-1$ is number of degrees of freedom[4], and P is the desired confidence interval (in percent). Values of $t_{\nu,P}$ are obtained from tables. A list of $t_{\nu,P}$ for ν ranging from 1 to 60, and for confidence intervals of 50%, 90%, 95%, and 99% is shown in Table 1. Note that as $\nu \to \infty$ the values of $t_{\nu,P}$ approach the values in Eq. (12).

[4] Remember that for a *sample* taken from the *population* the number of degrees of freedom, ν, is one less than the number of measurements, N, i.e., $\nu = N-1$. This is because the sample will have the same central tendency as the population, hence the number of degrees of freedom is reduced by one.

Example 4

Consider the following set of pressure readings made on a compressed air system (in psi):

20.3 21.2 19.8 19.7 17.9 18.2 22.3 21.0 19.8

What is the expected range of a subsequent measurement for a 95% probability?

Solution

Here $N = 9$ so $\nu = 9–1 = 8$, $\overline{x} = 20.02$ and $S_x = 1.40$. For a 95% probability, $t_{8,95\%} = 2.306$ from Table I, therefore $u_P = 2.306 \cdot 1.40 = 3.229$. Thus the next measurement will fall withing 20.0 ± 3.23 psi, i.e., the measured pressure will lie between 16.8 and 23.2 psi 95% of the time, or in 19 out of 20 measurements.

Table I. Values of Student-t distribution for finite sample sizes

ν	t_{50}	t_{90}	t_{95}	t_{99}
1	1.000	6.314	12.706	63.657
2	0.816	2.920	4.303	9.925
3	0.765	2.353	3.182	5.841
4	0.741	2.132	2.770	4.604
5	0.727	2.015	2.571	4.032
6	0.718	1.943	2.447	3.707
7	0.711	1.895	2.365	3.499
8	0.706	1.860	2.306	3.355
9	0.703	1.833	2.262	3.250
10	0.700	1.812	2.228	3.169
11	0.697	1.796	2.201	3.106
12	0.695	1.782	2.179	3.055
13	0.694	1.771	2.160	3.012
14	0.692	1.761	2.145	2.977
15	0.691	1.753	2.131	2.947
16	0.690	1.746	2.120	2.921
17	0.689	1.740	2.110	2.898
18	0.688	1.734	2.101	2.878
19	0.688	1.729	2.093	2.861
20	0.687	1.725	2.086	2.845
21	0.686	1.721	2.080	2.831
30	0.683	1.697	2.042	2.750
40	0.681	1.684	2.021	2.704
50	0.680	1.679	2.010	2.679
60	0.679	1.671	2.000	2.660
∞	0.674	1.645	1.960	2.576

Large Sample Size ($N > 60$)

If the number of measurements is large (typically > 60), then infinite statistics[5] can be used to obtain the uncertainty directly:

$$\begin{aligned} u_x &= S_x && (68\%) \\ u_x &= 1.96S_x && (95\%) \\ u_x &= 2.58S_x && (99\%) \end{aligned} \tag{12}$$

Note the three popular confidence intervals associated with these statistics: 68%, 95%, and 99%.

► Uncertainty of the Mean

It turns out that many times we are not interested in the uncertainty of a *single* future measurement, as the preceding section explains how to do. Rather, we would like to know how close the average, or mean, of a *collection* of measurements is to the true (population) mean.

Let's say you collect a random sample, or make a series of measurements, and perform the appropriate statistics on this new sample. Say you then collected a new random sample. You would find that, in general, each sample will yield a slightly different sample mean and sample standard deviation. This is not surprising, since we are picking different subsets of the entire population each time.

As you might also expect, the uncertainty of the mean of an entire set of measurements from the population mean will be *less* than the uncertainty of the next measurement represented by Eqs. (12) and (11). In other words, the sample mean will be closer to the population mean because the random errors will average out among all the measurements. This is in contrast to Eqs. (12) and (11) in the preceding section that assign the uncertainty of a *single* measurement. The uncertainty of the sample mean takes the form:

$$x' = \bar{x} \pm t_{\nu,P} S_{\bar{x}} \ (P\%) \tag{13}$$

Here $t_{\nu,P}$ is the Student-*t* distribution discussed above, and x' is the true—i.e., the population—mean. Here $S_{\bar{x}}$ is the *standard deviation of the means*, and is defined as[6]

$$S_{\bar{x}} = \frac{S_x}{\sqrt{N}}, \tag{14}$$

where S_x is the sample standard deviation from Eq. 9b, and N is the number of samples. Equation (13) states that the true value of the population x' will like between $\bar{x} - t_{\nu,P} S_{\bar{x}}$ and $\bar{x} + t_{\nu,P} S_{\bar{x}}$ P% of the time.

When expressing uncertainty for a series of measurements, it is the *Standard Deviation of the Mean* (Eq. (14)) that should be used, not Eqs. (12) and (11).

Example 5

Consider the previous example of the pressure readings. Estimate the bounds of the population mean, based on the sample statistics.

[5] *Infinite statistics* assumes there is an infinitely large sample size, i.e., $N \to \infty$. *Finite statistics* deals with finite sample sizes. In real life, of course, every sample is finite in size, but it turns out that for values of $N \geq 60$, infinite statistics is a good approximation. Since infinite statistics are easier to work with, they are often used for large N.

[6] Note the bar over the subscript x in to distinguish it from the sample standard deviation, S_x.

Solution

From the previous example, we know $\bar{x}$ = 20.02, S_x = 1.40, N = 9, and $t_{8,95\%}$ = 2.306. Thus the standard deviation of the means is $S_{\bar{x}} = S_x/\sqrt{N} = 1.40/\sqrt{9} = 0.466$. Thus the true (population) mean is expected to fall within $\bar{x} \pm t_{8,95\%} S_{\bar{x}}$ = 20.02 ± 2.3060×466 = 20.02 ± 1.075 , i.e., there is a 95% chance that the true mean lies between 18.95 and 21.10.

► Total Uncertainty

The total uncertainty is the root of sum of square (RSS) of the measurement and instrument resolution:

$$u_{inst} = \sqrt{u_{inst}^2 + u_{meas}^2}. \tag{15}$$

This value reflects both the intrinsic uncertainty associated with the measurements used as well as the uncertainty during the measurement process.

3.6 Error Propagation

As mentioned earlier, in most measurements, the quantity actually measured, i.e., the *input variable*, differs from the quantity that is being *reported*, i.e., the output variable. Consider, for example, a simple thermometer. When one reads the "temperature", in fact the height of a column of liquid is measured. The scale on the thermometer is then used to relate the liquid height to the temperature itself. The uncertainty of the reported temperature depends on the uncertainty of the measurement of the height of the liquid. The error in the input variables is said to *propagate* to the output.

Having determined the uncertainty in the input variables as described above, we need to determine the error propagation from the input values to the output value(s). To do this, the functional relationship (ideally) must be known between the input and output variables. Consider, for example, the following relationship for an output variable y that depends on a series of input variables x_1, x_2, x_3, etc.

$$y = f(x_1, x_2, x_{3,} \ldots). \tag{16}$$

First, note that the average value of y is obtained from the average values of x_j:

$$\bar{y} = f(\bar{x}_1, \bar{x}_2, \bar{x}_{3,} \ldots). \tag{17}$$

Say also that the uncertainty in the input variables Δx_1, Δx_2, Δx_3, etc. is known. Small changes (i.e., uncertainties) in y, which arise from the uncertainties Δx_j can be determined from a Taylor series:

$$\Delta y_j = \frac{\partial y}{\partial x_j} \Delta x_j \tag{18}$$

Here j = 1, 2, 3, …, and Δy_j refers to the change in y due to the change in x_j. The derivative $\partial y/\partial x_j$ is called the *sensitivity of y to the variable j*.

3.7 Absolute Uncertainty

One way to obtain the total uncertainty in y resulting from all the uncertainties Δx_1, Δx_2, Δx_3, …, would be to add up the absolute value of all of the individual uncertainty contributions[7]:

$$\Delta y = \left|\frac{\partial y}{\partial x_1}\Delta x_1\right| + \left|\frac{\partial y}{\partial x_2}\Delta x_2\right| + \left|\frac{\partial y}{\partial x_3}\Delta x_3\right| + \ldots \tag{19}$$

While this is certainly acceptable, it has the unfortunate result that every possible source of uncertainty will *always* assume its *largest* value. In some situations, for example where human lives are at stake, this might be mandated. Absolute uncertainty is also the easiest uncertainty to calculate.

3.8 Root Sum Square Uncertainty

A more common technique, however, is called the Root-Sum-Square (RSS) method for assessing the output variable uncertainty:

$$\Delta y = \left[\left(\frac{\partial y}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial y}{\partial x_2}\Delta x_2\right)^2 + \left(\frac{\partial y}{\partial x_3}\Delta x_3\right)^2 + \ldots\right]^{\frac{1}{2}}. \tag{20}$$

As the name implies, the square root of the sum of the square of the individual uncertainties is used to compute the total uncertainty in y. Though beyond the scope of this discussion, the above equation assumes that each possible source of error is not necessarily at its maximum value at all times[8]. The RSS technique should be used for <u>all</u> uncertainty analysis done in this course. <u>Also use a 95% confidence interval.</u>

Example 6

Consider an electrical resistor with a voltage $V = 2.38 \pm 0.09$ volt across it. The resistor has a resistance of $503 \pm 2\ \Omega$. Determine the power and the uncertainty of the power dissipated by the resistor.

Solution:

Power dissipated by the resistor is V^2/R, and, using the average values, $\overline{P} = (2.38\text{ volt})^2/(503\ \Omega) = 0.0113$ W.

The uncertainty is obtained as follows:

$$u_p = \left[\left(\frac{\partial P}{\partial V}\Delta V\right)^2 + \left(\frac{\partial P}{\partial R}\Delta R\right)^2\right]^{\frac{1}{2}} = \left[\left(\frac{2V}{R}\Delta V\right)^2 + \left(\frac{-V^2}{R^2}\Delta R\right)^2\right]^{\frac{1}{2}} = \left[\left(\frac{2\times 2.38}{503}\times 0.09\right)^2 + \left(\frac{-2.38^2}{503^2}\times 2\right)^2\right]^{\frac{1}{2}} = 8.53\times 10^{-4}\text{W}$$

Note that the uncertainty in the voltage is the dominant source of uncertainty in the power.

[7] The absolute value is required to avoid cancellations that would otherwise occur when both positive and negative terms are combined in the addition process.

[8] More specifically, the uncertainties are assumed to follow a *Gaussian* distribution, meaning that, in any given situation, only one or two variables will take on their maximum uncertainty, several variables will have some uncertainty, and a handful of variables will have almost no uncertainty at all. Say you were measuring the temperature of the air in your room, and you determined the uncertainty in the temperature to be ± 1ºC. When you make your measurements, however, the temperature you obtain is not necessarily off by as much as 1ºC. In all likelihood, if you made several measurements over time, some would yield errors as much as 1ºC, other would be less, say 0.3 –0.5 ºC, and yet others would be very close to the true value, i.e., the error would be close to zero. The idea of the RSS method is to account for such fluctuations in the uncertainty.

3.9 Practical Matters

It is not always possible to measure quantitatively the uncertainty in a system. Consider, for example, a gauge on an instrument that fluctuates with time, or the wobble of a rotating shaft on an engine as it spins. In such cases, one may not be able to take a meaningful reading from the instrument, and an approximation must be used. Use your engineering judgment to approximate, to the best of you ability, the magnitude of the fluctuations that you observe, and use this estimate directly for u_x.

Similarly, in very complicated systems, the relationship between input and output variable may be very complicated, or may not be known at all. Consider, for example, how the uncertainty in the thickness of an airplane wing will affect its lift. This is a tremendously complicated problem, and such a relationship cannot be determined with a simple expression like the one above. There are a couple of alternative courses of action in such cases.

First, if a numerical simulation of the phenomenon is available, it can be used to approximate the sensitivity to a variable numerically. This is done by evaluating the output variable y at values of $\bar{x}_j + \Delta x_j$ and $\bar{x}_j - \Delta x_j$, and using a central difference to approximate the derivative $\partial y/\partial x_j$:

$$\frac{\partial y}{\partial x_j} \approx \frac{f(\bar{x}_j + \Delta x_j) - f(\bar{x}_j - \Delta x_j)}{2\Delta x_j}. \tag{21}$$

If a numerical model is not available, or otherwise impractical to use, another — albeit more difficult — approach is to determine the sensitivity experimentally. Another wing in the example above could be manufactured with a *known* additional thickness, and the lift measured and compared with the original wing to determine the change in lift with wing thickness. Note that this option is time consuming, expensive, and limited: the results are only valid for the set of parameters that were used for the particular experiment, e.g., air speed, wing length, air density, temperature, etc.

3.10 Some Useful Relationships

Some combinations of uncertainty appear so frequently that they warrant summarizing for future use. In the following, a, b, c, etc. refer to values, and Δa, Δb, Δc, etc. refer to the uncertainty in those values. Note that the sign '±' means either '+' or '–' in the expression, and either one can hold: $a \pm b$ can be $a + b$ or $a - b$. If more than one '±' appears in an expression, then *each* sign can be either '+' or '–': $a \pm b \pm c$ can be $a+b+c$ or $a - b + c$, or $a + b - c$, or $a - b - c$, and so on. Note also that the expression Δa^2 is short for $(\Delta a)^2$, etc.

Another very useful expression can be developed for the special case when the functional relationship between the unknown and known variables takes the form:

$$y = \frac{a^{n_1} b^{n_2} c^{n_3} \cdots}{d^{m_1} e^{m_2} f^{m_3} \cdots}, \tag{22}$$

where $n_1, n_2, \ldots$, and $m_1, m_2, \ldots$, are integers (1,2,3,…) or rational fractions (1/2, 3/4, 1/5, etc.). In this case, the uncertainty in y can be expressed as:

$$\frac{\Delta y}{y} = \left[\left(n_1 \frac{\Delta a}{a} \right)^2 + \left(n_2 \frac{\Delta b}{b} \right)^2 + \cdots + \left(m_1 \frac{\Delta d}{d} \right)^2 + \left(m_2 \frac{\Delta e}{e} \right)^2 + \cdots \right]^{\frac{1}{2}}. \tag{23}$$

Note that this works *only* with variables to simple powers. For example, this will not work with something like $\sin(x)$ or $(a + b^2)^3$.

Example 7

If $y = \rho^3 U^{1/2} L^{2.3} / \mu^4$, find the RSS uncertainty in y, given ρ, U, L, μ and their uncertainies.

Solution:

Since each variable appears to a simple power, the above expression can be used. The uncertainty is obtained as follows:

$$\frac{\Delta y}{y} = \left[\left(3\frac{\Delta\rho}{\rho} \right)^2 + \left(\frac{1}{2}\frac{\Delta U}{U} \right)^2 + \left(2.3\frac{\Delta L}{L} \right)^2 + \left(4\frac{\Delta\mu}{\mu} \right)^2 + \right]^{\frac{1}{2}}$$

The uncertainty in Δy can be obtained by multiplying both sides by y, where y is calcuated using the nominal values of ρ, U, L and μ.

Table II. Some common expressions for uncertainty

Expression	Uncertainty Relationship	Derivation
$c = a \pm b \pm \cdots$	$\Delta c = \left[\Delta a^2 + \Delta b^2 + \cdots \right]^{\frac{1}{2}}$	$\Delta c = \left[\left(\frac{\partial c}{\partial a}\Delta a \right)^2 + \left(\frac{\partial c}{\partial b}\Delta b \right)^2 + \cdots \right]^{\frac{1}{2}}$
$c = a \times b$	$\mathrm{D}c = \left[\left(b\mathrm{D}a \right)^2 + \left(a\mathrm{D}b \right)^2 \right]^{\frac{1}{2}}$	$\mathrm{D}c = \left[\left(\frac{\partial c}{\partial a}\mathrm{D}a \right)^2 + \left(\frac{\partial c}{\partial b}\mathrm{D}b \right)^2 \right]^{\frac{1}{2}}$
$c = a \times b \times c$	$\mathrm{D}c = \left[\left(bc\mathrm{D}a \right)^2 + \left(ac\mathrm{D}b \right)^2 + \left(ab\mathrm{D}c \right)^2 \right]^{\frac{1}{2}}$	(not shown)
$c = \frac{a}{b}$	$\mathrm{D}c = \left[\left(\frac{\mathrm{D}a}{b} \right)^2 + \left(\frac{a}{b^2}\mathrm{D}b \right)^2 \right]^{\frac{1}{2}}$	(not shown)
$c = \frac{a \pm b}{e \pm f}$	$\mathrm{D}c = \left[\left(\frac{\mathrm{D}a}{e \pm f} \right)^2 + \left(\frac{\mathrm{D}b}{e \pm f} \right)^2 + \left(\frac{a \pm b}{(e \pm f)^2}\mathrm{D}e \right)^2 + \left(\frac{a \pm b}{(e \pm f)^2}\mathrm{D}f \right)^2 \right]^{\frac{1}{2}}$	(not shown)

In the following, a, b, c, etc. refer to mean values, and Δa, Δb, Δc, etc. refer to the uncertainty in those values. Note that the sign '±' means either '+' or '–' in the expression, and either one can hold: $a \pm b$ can be $a + b$ or $a - b$. If more than one '±' appears in an expression, then *each* sign can be either '+' or '–': $a \pm b \pm c$ can be $a+b+c$ or $a - b + c$, or $a + b - c$, or $a - b - c$, and so on. Note also that the expression Δa^2 is short for $(\Delta a)^2$, etc.

3.11 Further Reading

1. *Theory award Design of Mechanical Measurements,* R.S. Figliola and D.E. Beasley, John-Wiley, 1991 (used for this write-up).
2. *Measurement Systems, Application and Design, 3rd ed.*, E. O. Dobelin, McGraw-Hill, 1983.
3. *Mechanical Measurements, 5th edition,* T.G. Beckwith, R.D. Marangoni, and J.H. Lienhard, Addison-Wesley, 1993.

4 THE UNCERTAINTY TREE: TOWARDS A MORE ENJOYABLE ERROR ANALYSIS

4.1 Introduction

Error analysis—particularly for students learning the skill—can be a frustrating, error-prone experience. This is especially true if the relationships between the input and output variables are complicated, either from complex individual equations, or if a series of equations relating input and output variables is required. To make matters worse, textbooks on the subject illustrate error propagation concepts with trivial examples that do little to prepare one for real-world error analysis. As a result, errors are made by students and professionals alike, particularly for complicated systems.

This note provides a simple, yet extremely effective tool to properly account for the flow of uncertainty from input to output variables. The tool is called an *uncertainty tree*, and represents a graphical depiction of the variable dependence in an error analysis. The variable whose uncertainty is ultimately desired, i.e., the output variable, appears at the top of the tree, and the variables that it depends upon are listed at sublevels below it. Functional relationships (equations) are used to connect one level to another. If the variables at a sub-level depend, in turn, on yet other variables, then a second sub-layer is created with the appropriate equation relating the two variables. A tree branch is terminated when the uncertainty in a given variable is known, at which point a double underline (<u>____</u>) is used to show the uncertainty in that variable is known.

The idea is best shown by example.

Example 1

Say that the kinetic energy of a spherical object is to be determined by measuring the body's mass and velocity. Assume that the velocity is determined by measuring the time it takes the object to pass a given distance, and that the mass is calculated by measuring the body's diameter, from which the volume and density are used to determine the mass. We will assume the velocity is constant, and that the ball is homogeneous (constant density).

The equations governing this problem are as follows:

$$T_{KE} = \tfrac{1}{2}mv^2 \qquad v = x/t \qquad m = \rho V \qquad V = {}^1\!/_6\,\pi d^3$$

Here:

T_{KE}	is the kinetic energy of the object (J)
d	the object's diameter (m)
m	the object's mass (kg)
v	the velocity (m/s)
x	the distance traveled (m)
t	the time taken for the object to cross the distance x (s)
V	the object's volume (m^3)
ρ	the density (kg/m^3)

Solution

The uncertainty in the kinetic energy, T_{KE}, is desired, which depends on the variables m, v, x, and t. We assume that the uncertainty in the density ρ, diameter d, distance x, and time t, have been determined.

The uncertainty tree thus takes the following form:

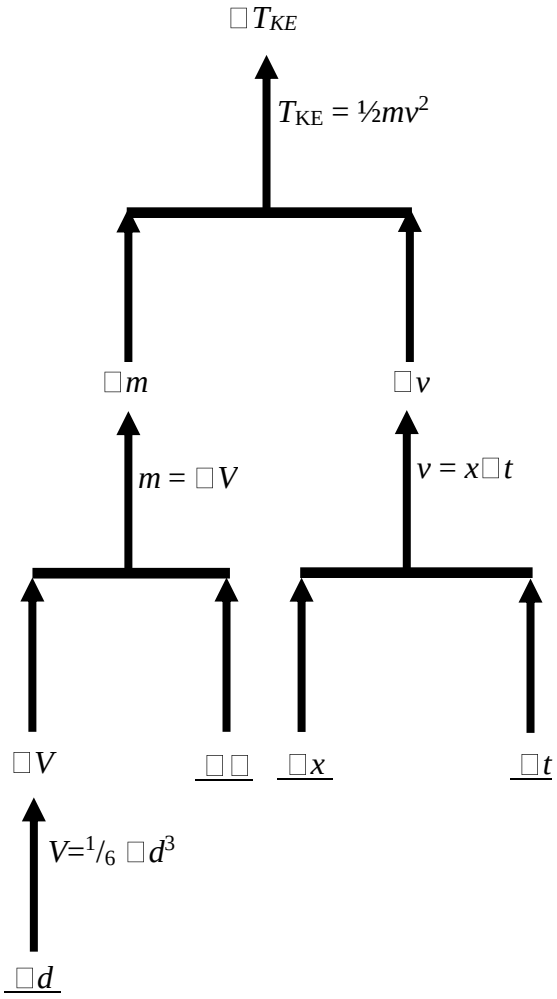


Figure 1 – Uncertainty Tree for Example 1

The uncertainty tree technique is compatible—and indeed, compliments—existing presentations on uncertainty analysis provided in standard textbooks.

4.2 Constructing the Uncertainty Tree

Several guidelines will help in constructing the uncertainty tree:

1. Only one variable whose uncertainty is desired should appear at the top of the tree (ΔT_{KE} in this case).
2. All of the variables that contribute to the desired variable are listed *one level below* the original variable (for example, Δm and ΔV make the second level). The process is repeated until variables with known uncertainty appear at the bottom of the screen.
3. Variables whose uncertainty is known are identified with a double underline (____), and denote a terminus (end) to the tree.
4. Every node of a tree must ultimately be terminated, i.e, have a double underline with it, signifying that the uncertainty for that variable is known.

5. If an explicit analytic expression, i.e., y = $f(x_1, x_2 x_3, \ldots)$, is known for the input and output variable, then the arrow connecting both variables is drawn as a solid. The relationship should then also be written near the top and to the left of the arrow (e.g., $m = \rho V$ in the diagram).

To determine the uncertainty relationships, simply descend the tree one level at a time.
Referring to Figure 1, and using the RSS uncertainty, the uncertainty relationships are (the absolute value uncertainty can just as easily be used with the tree concept):

Level 1

$$\Delta T_{KE} = \left[\left(\frac{\partial T_{KE}}{\partial m} \Delta m \right)^2 + \left(\frac{\partial T_{KE}}{\partial v} \Delta v \right)^2 \right]^{\frac{1}{2}} \tag{1}$$

Level 2

$$\Delta v = \left[\left(\frac{\partial v}{\partial x} \Delta x \right)^2 + \left(\frac{\partial v}{\partial t} \Delta t \right)^2 \right]^{\frac{1}{2}} \tag{2a}$$

$$\Delta m = \left[\left(\frac{\partial m}{\partial \rho} \Delta \rho \right)^2 + \left(\frac{\partial m}{\partial V} \Delta V \right)^2 \right]^{\frac{1}{2}} \tag{2b}$$

Level 3

$$\Delta V = \left[\left(\frac{\partial V}{\partial d} \Delta d \right)^2 \right]^{\frac{1}{2}} \tag{3}$$

Note that, in this example, all uncertainties are ultimately expressed in terms of the uncertainty in the ρ, d, x, and t, which, as the problem states, are known. At this point, simply insert the appropriate expressions into the equations from each level and solve for the desired uncertainty.

APPENDIX: STUDENT MANUAL FOR STRAIN GAGE TECHNOLOGY

M-LINE ACCESSORIES

Student Instruction Bulletin

Strain Gage Installations with M-Bond 200 and AE-10 Adhesive Systems

1.0 INTRODUCTION

Because the strain gage is an extremely sensitive device capable of registering the smallest effects of an imperfect bond, considerable attention to detail must be taken to assure stable, creep-free installations. However, the techniques involved are very simple, and readily mastered.

This manual gives explicit step-by-step instructions for making consistently successful strain gage installations with M-Bond 200 and M-Bond AE-10 Adhesives. These directions should be followed precisely. More detailed information may be found in the Measurements Group VideoTech™ Library and in the following publications:

- Instruction Bulletin B-129, *Surface Preparation for Strain Gage Bonding.*
- Instruction Bulletin B-127, *Strain Gage Installations with M-Bond 200 Adhesive.*
- Instruction Bulletin B-137, *Strain Gage Installations with M-Bond AE-10/15 and M-Bond GA-2 Adhesive Systems.*

All operations described in this manual can be performed with the use of the Student Strain Gage Application Kit. The procedures outlined here are ideally suited to the classroom or teaching laboratory. For most teaching/learning activities involving strain gage technology, the specially priced, first-quality *Student Gages* manufactured by Micro-Measurements Division of the Measurements Group may be used with excellent results.

2.0 STRAIN GAGE ADHESIVES

Because consistently successful installation of strain gages requires the use of an adhesive certified for strain gage use, Micro-Measurements *M-LINE* adhesives undergo extensive laboratory testing to ensure reliability and consistency of those properties required in strain gage bonding. To assure accurate and reliable strain gage measurements, it is strongly recommended that a certified adhesive such as M-Bond 200 methyl-2-cyanoacrylate or M-Bond AE-10 epoxy adhesive be selected for most general laboratory installations.

2.1 M-Bond 200

Micro-Measurements certified M-Bond 200 is an excellent general-purpose laboratory adhesive because of its fast room-temperature cure and ease of application. It is compatible with all Micro-Measurements strain gages and all common structural materials. M-Bond 200 Adhesive can be used for high-elongation tests (+60 000$\mu\epsilon$), for fatigue studies, and for one-cycle proof tests within a normal operating temperature range of –25° to +150°F *(–32° to +65°C).*

The catalyst supplied with M-Bond 200 is specially formulated to control the reactivity rate. For best results, the catalyst should be used sparingly. Since M-Bond 200 bonds are weakened by exposure to high humidity, adequate protective coatings are essential. Because this adhesive will become harder and more brittle with time, M-Bond 200 is not generally recommended for permanent installations over one or two years in duration.

HANDLING PRECAUTIONS

M-Bond 200 is a cyanoacrylate compound. *Immediate bonding of eye, skin, or mouth may result upon contact. Causes irritation.* The user is cautioned to (1) *avoid contact with skin;* (2) *avoid prolonged or repeated breathing of vapors;* and (3) *use with adequate ventilation. For additional health and safety information, consult the material safety data sheet which is available upon request.*

The shelf life of M-Bond 200 is six months when stored under normal laboratory conditions. Life of *unopened* material can be extended by refrigeration [+40°F *(+5°C)*]. Due to possible condensation problems, care should be taken to allow the unopened bottle to return to room temperature before opening. Refrigeration after opening is not recommended.

2.2 M-Bond AE-10

Micro-Measurements certified M-Bond AE-10 is a 100% solids epoxy system for use with strain gages. It offers the advantages of high elongation (10%) and wider operating temperature range [-320° to +200°F *(-195° to +95°C)*]. Because it is highly resistant to moisture and most chemicals, M-Bond AE-10 is recommended for permanent installations over one year in duration.

M-Bond AE-10 Adhesive is supplied in kit form with pre-weighed resin and sufficient curing agent for six separate mixes of adhesive. Allow the materials to attain room temperature before opening the containers. Each of the individual units of resin can be separately activated by filling one of the calibrated droppers with curing agent *exactly* to the number 10 and dispensing the contents into the center of the jar of resin. *Immediately cap the bottle of curing agent to avoid moisture absorption.* Mix the resin and curing agent for five minutes, using one of the plastic stirring rods. The pot life or working time after mixing is 15 to 20 minutes at +75°F *(+24°C)*. The pot life can be somewhat extended by occasionally stirring the mixture, by cooling the jar, or by spreading the adhesive on a chemically clean aluminum plate. Discard the dropper and stirring rod after use.

HANDLING PRECAUTIONS

While M-Bond AE-10 is considered relatively safe to handle, *contact with skin and inhalation of its vapors should be avoided.* Immediately washing with ordinary soap and water is effective in cleansing should skin contact occur. For eye contact, rinse thoroughly with copious amounts of water and consult a physician. For additional health and safety information, consult the material safety data sheet which is available upon request.

The shelf life of unmixed components is one year at room temperature. During storage, crystals may form in the resin. These crystals do not affect adhesive performance, but should be reliquefied prior to mixing by warming the resin jar to +120°F *(+50°C)* for approximately one-half hour. Because excess heat will shorten pot life, allow the resin to return to room temperature before adding the curing agent.

3.0 SURFACE PREPARATION

Strain gages can be bonded satisfactorily to almost any solid material if the material surface is properly prepared. While there are many surface preparation techniques available, the specific procedures and techniques described here are a carefully developed and thoroughly proven system. They are ideal for both M-Bond 200 and M-Bond AE-10 Strain Gage Adhesives.

The purpose of surface preparation is to develop a chemically clean surface having a roughness appropriate to the gage installation requirements, a surface alkalinity of the correct pH, and visible gage layout lines for locating and orienting the strain gage. The Micro-Measurements system of surface preparation will accomplish these objectives for aluminum alloys and steels in five basic operations:

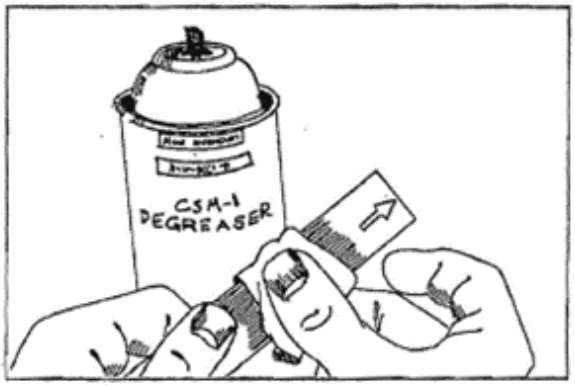
- Solvent degreasing
- Surface abrading
- Application of gage layout lines
- Surface conditioning
- Neutralizing

To ensure maximum cleanliness and best results, the following should be avoided in all steps:

- Touching the surface with the fingers
- Wiping back and forth or reusing swabs or sponges
- Dragging contaminants into the cleaned area from the uncleaned boundary of that area
- Allowing a cleaning solution to evaporate on the surface
- Allowing partially prepared surface to sit between steps in the preparation process or a prepared surface to sit before bonding

Consult Instruction Bulletin B-129 for other test materials and for special precautions and considerations for surface preparation.

3.1 Solvent Degreasing

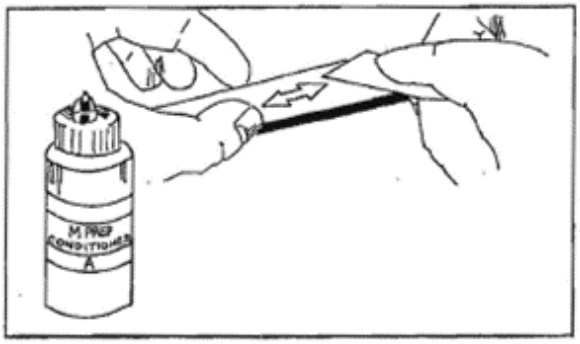


Degreasing is performed to remove oils, greases, organic contaminants, and soluble chemical residues. Degreasing should *always* be the first operation.

Degreasing can be accomplished using a solvent such as CSM-1 Degreaser. Spray applicators are preferred to avoid back-contamination of the parent solvent. Use a clean gauze sponge to clean the entire specimen, if possible, or an area covering 4 to 6 in *(100 to 150 mm)* on all sides of the gage location.

3.2 Surface Abrading

The surface is abraded to remove any loosely bonded adherents (scale, rust, paint, coatings, oxides, etc.), and to develop a surface texture suitable for bonding. For rough or coarse surfaces it may be necessary to start with a grinder, disc sander, or file; but, for most specimens a suitable surface can be produced with only silicon-carbide paper of the appropriate grit.



Place a liberal amount of M-Prep Conditioner A in the gaging area and wet-lap with clean 320-grit silicon-carbide paper for aluminum, or 220-grit for steel. Add Conditioner A as necessary to keep the surface wet during the lapping process.

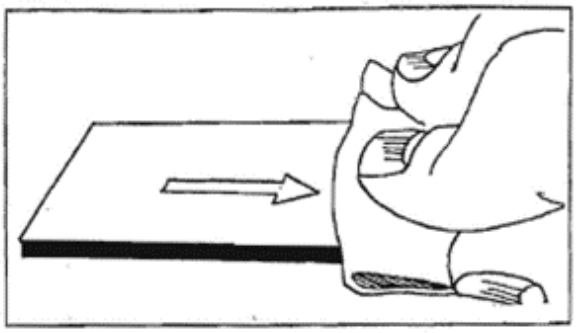
When a bright surface is produced, wipe the surface dry with a clean gauze sponge. A clean surface of the gauze should be used with each wiping stroke. A sufficiently large area should be cleaned to ensure that contaminants will not be dragged back into the gaging area during the steps to follow.

Repeat the above step, using 400-grit silicon-carbide paper for aluminum, or 320-grit for steel.

3.3 Layout Lines

The desired location and orientation of the strain gage on the test surface should be marked with a pair of crossed, perpendicular reference lines. The reference or layout lines should be *burnished*, rather than scored or scribed, on the surface. For aluminum, a medium-hard drafting pencil is satisfactory. For most steels, a ball-point pen or a tapered brass rod may be used. All residue from the burnishing operations should be removed in the following step.

3.4 Surface Conditioning



After the layout lines are marked, Conditioner A should be applied repeatedly, and the surface scrubbed with cotton-tipped applicators until a clean tip is no longer discolored by scrubbing. The surface should be kept constantly wet with Conditioner A until the cleaning is completed. When clean, the surface should be dried by wiping through the cleaned area with a *single* slow stroke of a gauze sponge. The stroke should begin inside the cleaned area to avoid dragging contaminants in from the surrounding area. Throw the used gauze away and, with a fresh gauze, make a *single* slow stroke in the opposite direction. Throw the second gauze away.

3.5 Neutralizing

To provide optimum alkalinity for Micro-Measurements strain gage adhesives, the cleaned surfaces must be neutralized. This can be done by applying M-Prep Neutralizer 5A liberally to the cleaned surface, and scrubbing the surface with a clean cotton-tipped applicator. The cleaned surface should be kept completely wet with Neutralizer 5A throughout this operation. When neutralized, the surface should be dried by wiping through the cleaned area with a *single* slow stroke of a clean gauze sponge. Throw the gauze away and with another fresh gauze sponge, make a *single* stroke in the opposite direction. Always begin within the cleaned area to avoid recontamination from the uncleaned boundary.

If the foregoing instructions have been followed precisely, the surface is now properly prepared for gage bonding. The gages should be installed within 30 minutes on aluminum or 45 minutes on steel.

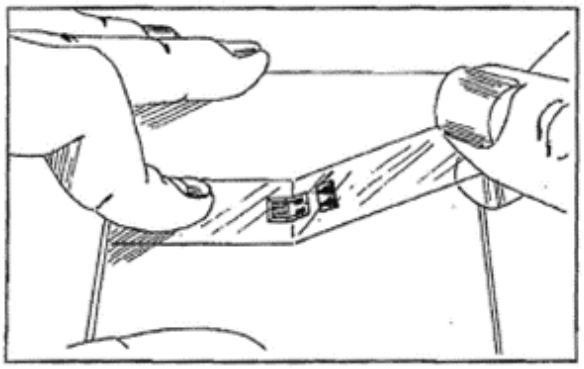
4.0 STRAIN GAGE BONDING

The electrical resistance strain gage is capable of making accurate and sensitive indications of strains on the surface of the test part. Its performance is absolutely dependent on the bond between itself and the test part. The procedures outlined below will help ensure satisfactory bonds when using M-Bond 200 or AE-10 Adhesives. While the steps may appear unduly elaborate, these techniques have been used repeatedly in strain gage installations which have yielded consistent and accurate results. The steps shown assume that a terminal strip will be used. When CEA-Series gages are used, no strip is required.

4.1 Handling and Preparation

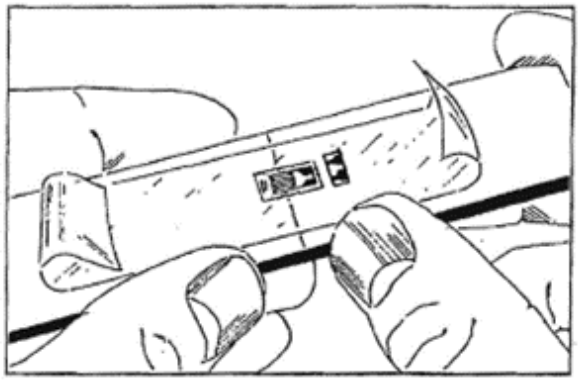
Micro-Measurements strain gages are specially treated for optimum bond formation with all appropriate gage adhesives. No further cleaning is necessary if contamination of the prepared bonding surface is avoided during handling. (Should contamination occur, clean with a cotton swab moistened with a low residue solvent such as *M-LINE* Neutralizer 5A or GC-6 Isopropyl Alcohol. Allow the gage to dry for several minutes before bonding.) Gages should never be touched with the hands.

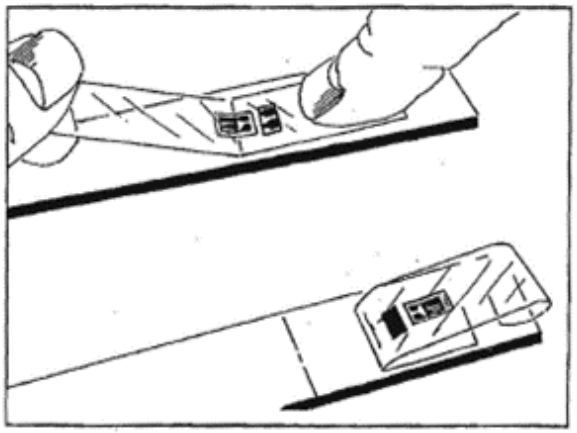
Remove the strain gage from its acetate envelope by grasping the edge of the gage backing with tweezers, and place on a chemically clean glass plate (or empty gage box) with the bonding side of the gage down. Place the appropriate terminals (if any) next to the strain gage solder tabs, leaving a space of approximately 1/16 in *(1.5 mm)* between the gage backing and terminal.



Using a 4-to-6-in *(100-to-150-mm)* length of *M-LINE* PCT-2A cellophane tape, anchor one end of the tape to the glass plate behind the gage and terminal. Wipe the tape firmly down over the gage and terminals. Pick the gage and terminals up by carefully lifting the tape at a shallow angle (30 to 45 degrees) until the tape comes free with the gage and terminal attached. (The shallow angle is important to avoid over-stressing the gage and causing permanent resistance changes.) **Caution: Some tapes may contaminate the bonding surface or react with the bonding adhesive. Use only tapes certified for strain gage installations.**

The strain gage is now prepared for positioning on the test specimen. Position the gage/tape assembly so the triangle alignment marks on the gage are over the layout lines on the specimen. Holding the tape at a shallow angle, wipe the assembly onto the specimen surface. If the assembly is misaligned, lift the tape again at a shallow angle until the assembly is free of the specimen. Reposition and wipe the assembly again with a shallow angle.

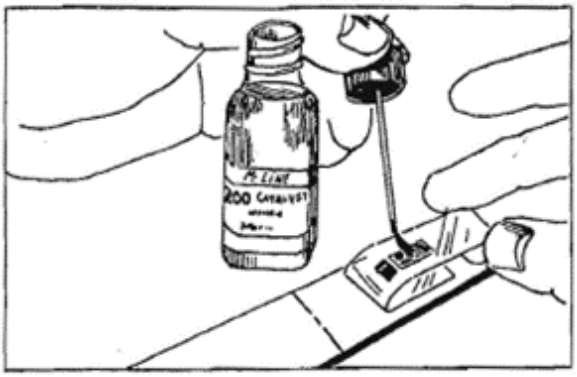




In preparation for applying the adhesive, lift the end of the tape opposite the solder tabs at a shallow angle until the gage and terminal are free of the specimen. Tack the loose end of the tape under and press to the surface so the gage lies flat with the bonding side exposed.

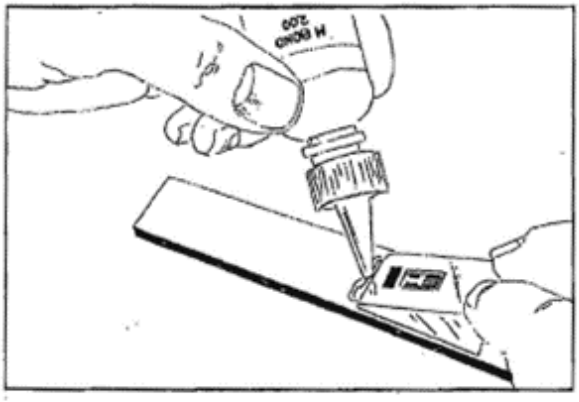
The appropriate adhesive may now be applied. The procedures for M-Bond 200 and M-Bond AE-10 are described in the two sections which follow.

4.2 Bonding with M-Bond 200



M-Bond 200 Catalyst should be applied sparingly in a thin uniform coat. Wipe the brush against the lip of the bottle approximately ten times to remove most of the catalyst. Set the brush down on the gage and swab the gage backing by sliding — not brushing in the painting style — the brush over the entire gage surface. Move the brush to an adjacent tape area prior to lifting from the surface. Allow the catalyst to dry at least one minute under normal ambient laboratory conditions.

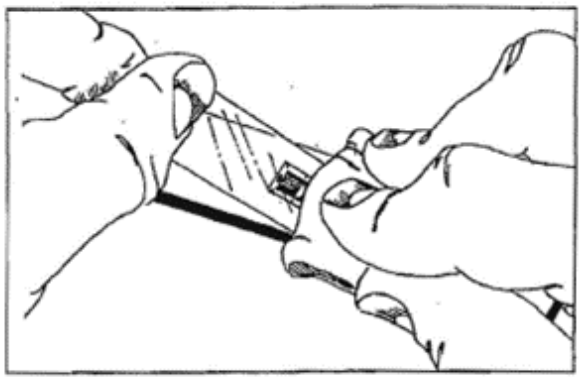
The next three steps must be completed in sequence within three to five seconds. Read these steps before proceeding.



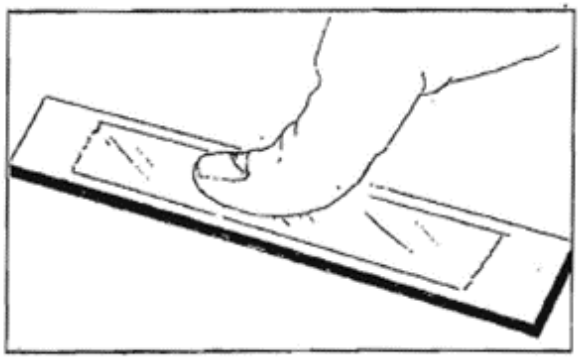
Lift the tucked-under tape. Holding the gage/tape assembly in a fixed position, apply one or two drops of M-Bond 200 Adhesive at the junction of the tape and specimen surface, about 1/2 in *(13 mm)* outside the actual gage installation area.

Immediately rotate the tape to approximately a 30-degree angle so that the gage is bridged over the installation area.

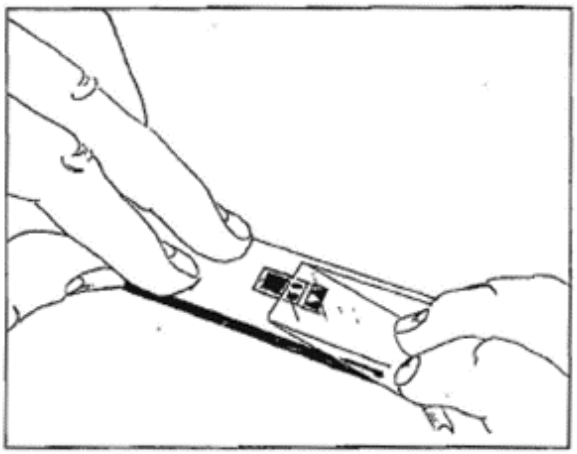
Holding the tape slightly taut and beginning from the tab end of the gage, slowly and *firmly* make a single wiping stroke over the gage/tape assembly with a clean gauze sponge to bring the gage back down over the alignment marks on the specimen. Release the tape.



Immediately upon completion of the above step, *discard the gauze* and apply firm thumb pressure to the gage and terminal area. This pressure should be held for at least one minute. Wait two minutes before the next step (tape removal).

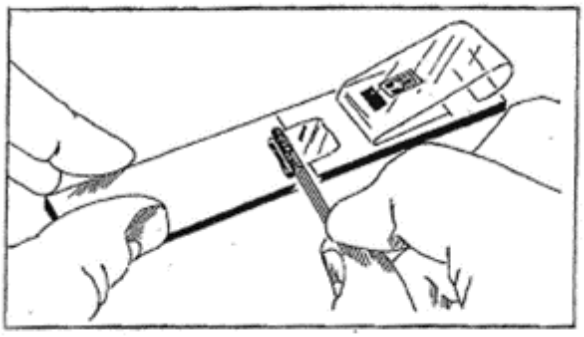


The gage and terminals should now be bonded to the specimen. To remove the tape, pull it back directly over itself, *peeling* it slowly and steadily off the surface.

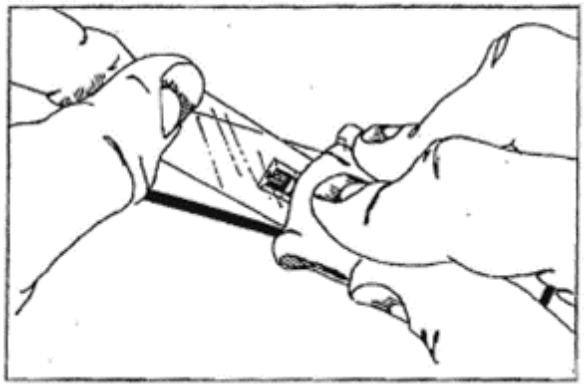


4.3 Bonding with M-Bond AE-10

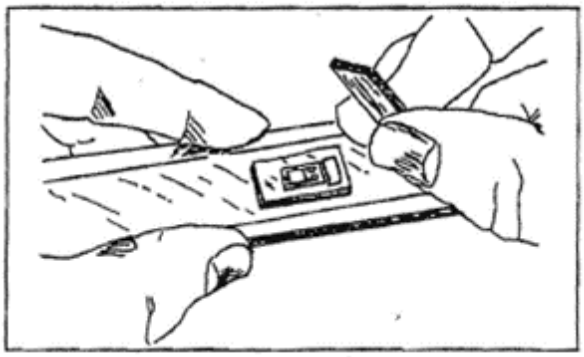
(This section follows 4.1 when using M-Bond AE-10 Adhesive.) Mix the Resin AE with Curing Agent Type 10 per the instructions in Instruction Bulletin B-137 supplied with the adhesive.



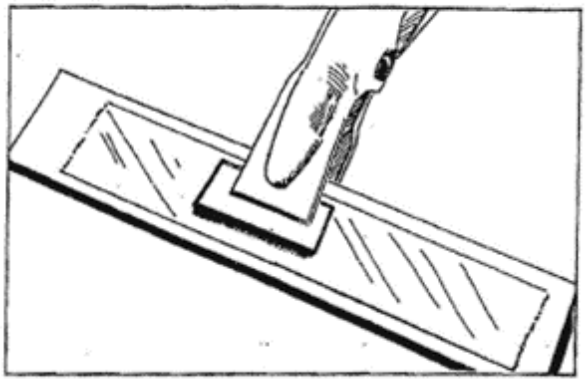
Coat the specimen and back of the gage and terminal with the prepared M-Bond AE-10 Adhesive. The mixing rod may be used to apply a thin layer of adhesive over both surfaces. *Be careful not to pick up any unmixed components of the adhesive.* To ensure this, wipe the mixing rod clean and then pick up a very small amount of adhesive from the central area of the adhesive jar. After applying the adhesive, proceed immediately to the next step.



Lift the tucked-over end of the tape and bridge over the specimen installation area at approximately a 30-degree angle. Beginning from the tab end of the gage and using a clean gauze sponge, slowly and firmly make a single wiping stroke over the gage/tape assembly to bring the gage back down over the alignment marks on the specimen.

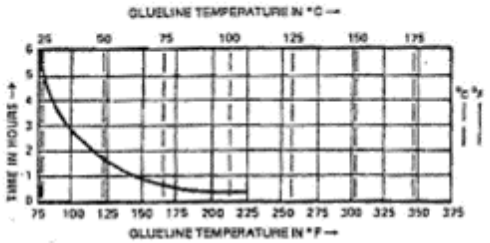


Place a silicone rubber pad and a back-up plate over the gage installation. Apply force by dead weight or spring clamp until a pressure of 5 to 20 psi *(35 to 135 kN/M²)* is attained. Take care to ensure the pressure is equal over the entire gage surface.



The M-Bond AE-10 Adhesive will develop adequate bonding strength in six hours at room temperature [+75°F *(+24°C)*]. The time may be reduced by increasing the temperature of the glueline per the schedule below. **Warning: For curing temperature above +150°F *(+66°C)* a special mylar tape must be used for gage handling, and a Teflon® strip should be placed between the gage and the silicone rubber pad.**

RECOMMENDED CURE SCHEDULE



After the adhesive is cured, remove the clamps or weights, the silicone pads and Teflon strip (if used). To remove the tape, pull it back directly over itself, *peeling* it slowly and steadily off the surface.

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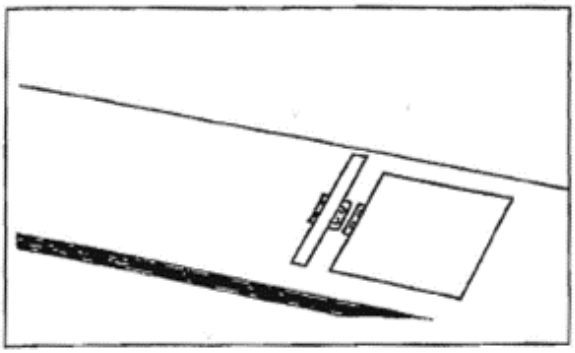
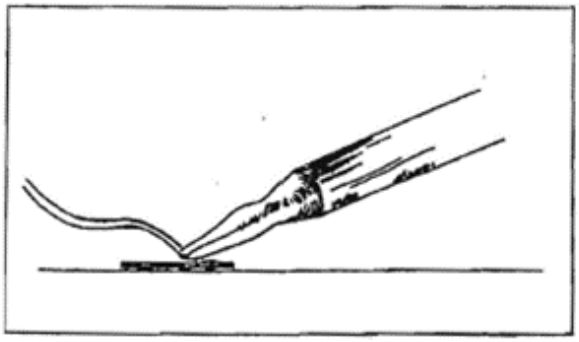
5.0 SOLDERING TECHNIQUES



If the strain gage is without encapsulation or preattached lead ribbons, mask the gage grid area with drafting tape, leaving only the tabs exposed.

After the soldering iron has reached operating temperature, clean the tip with a gauze sponge and tin it with fresh solder. Tin the gage tabs and terminal tabs (if used). Melt a small amount of solder on the tip of the soldering iron, lay the rosin-core solder wire across the gage tab or copper terminal. Firmly apply the iron tip for one second, then *simultaneously* lift both solder and tip. A bright, shiny, even mound of solder should have been deposited on the tab. If not, repeat the process. If spikes are formed rather than smooth beads, it is a sign of inadequate flux, dwelling too long with the iron, and/or an improper iron temperature. Feeding the cored solder into the tab area during heat application will increase the amount of flux available.

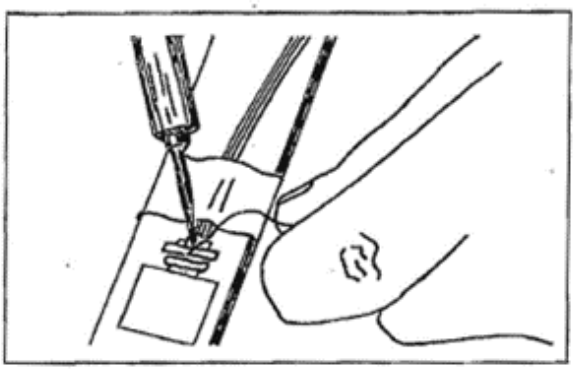


For a three-conductor lead-in wire, separate the individual leads for 3/4 in *(20 mm)*. Strip away 1/2 in *(13 mm)* of insulation by using the soldering tip to melt the insulation on both sides of each end of the wire 1/2 in *(13 mm)* from the ends and quickly pulling off the insulation. **Warning: Do not use a knife or other blade to cut the insulation.** When the main leadwire is stranded and terminal strips are used, it is often convenient to cut all strands but one to fit the size of the copper pad. The long strand can then be used as the jumper wire. Soldering is made considerably easier by this method. This is unnecessary when the leadwires are bonded directly to the solder tabs on CEA-Series strain gages.

Holding the tip of a finger on the tip of the tinned wire for safety, cut each wire with diagonal wire cutters leaving 1/8 in *(3 mm)* of exposed, tinned wire.

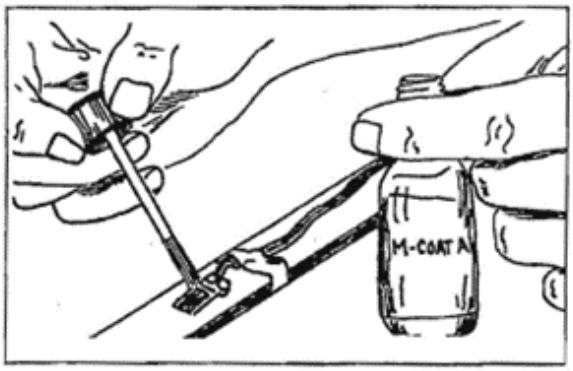
Tack the lead-in wires to the specimen with drafting tape so the tinned end of the wire is spring-loaded in contact with the solder bead. Complete the solder connection as before by applying solder and iron tip for one second and removing simultaneously.

Apply rosin solvent liberally to the solder joints. Drafting tape may be removed by loosening the mastic with rosin solvent. Remove all solvent with a gauze sponge, using a dabbing action. Repeat.



Tape or otherwise secure the lead-in wires to the specimen to prevent the wires from being accidentally pulled from the tabs. A stress relief "loop" should be placed between the tape and the solder connections.

Apply a protective coating over the entire gage and terminal area. For most laboratory uses, M-Coat A will provide adequate long-term protection. The coating should be continuous up to and over at least the first 1/8 in *(3 mm)* of leadwire insulation.



The properly installed strain gage will have a resistance to ground of at least 10 000 to 20 000 megohms. Checking leakage resistance with the Model 1300 Gage Installation Tester is highly recommended.