Simultaneous elections in a polarized society make single-party sweeps more likely

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Abstract

In a country with multiple elections, it may prove economically expedient to hold some or all of them on a common polling date. We show that in a polarized society, where each voter has a preferred party, such a decision will bring about a *systemic* change at the political level: an increase in the simultaneity of polling will increase the likelihood of a *single-party sweep*, namely, that a single party wins all the elections.

Our result holds under fairly general conditions and we discuss its applicability to the two most common real world electoral systems, namely *first-past-the-post* (most voters) and *party list proportional representation* (most countries). In the course of our analysis, we obtain a generalization of the well-known Harris correlation inequality.

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1 Introduction

Most major democracies around the world conduct multiple elections, allowing citizens to engage in the democratic process at regular intervals across various levels of government. Americans participate in elections every two years to choose a member of the House of Representatives, every four years to elect a President, and twice within a six-year period to vote for two Senators; additionally, various local and municipal elections occur throughout these cycles. In India, the national parliament and state assemblies are legally mandated to have five-year terms; however, as in many parliamentary democracies, the fall of a government can lead to elections being held earlier than the scheduled term.

Some elections are tradionally held together with others, so that a voter on a given poll date might cast votes in multiple elections. For example, the three major American election cycles are synchronized so that every other House election coincides with a Presidential election, and, for each voter, two out of three House elections also involve a senatorial candidate on the ballot. In India, the national and state elections were held simultaneously until 1967 but the premature fall of various governments have caused them to become unsynchronized over the years.

Simultaneous elections reduce cost, time, and effort, and consequently there is a strong economic argument to be made in their favor. The ruling party in India, has proposed a constitutional amendent—"One Nation One Election"—which seeks to mandate the simultaneity of national and state elections. This has been opposed by other parties out of concern that the current Prime Minister's popularity might increase the state-level dominance of the ruling party. Indeed, the tendency of a popular leader to attract votes for other candidates of the same party is a well-known and much-studied phenomenon in the American context, where it is referred to as a "coattail" or "down-ballot" effect.

A separate development, observed and studied in both the American and Indian contexts, is that of increasing "political polarization"[1]. This refers to a ten-

[1] See, e.g., [7], [21], [40] and the references therein.

dency of voters to identify closely with a political party, with a corresponding reduction in the number of "uncommitted" or "issue-based" voters. At the ballot level this results in more "party-line" votes, with relatively few "split" or "cross-over" votes. While the outcome of an election depends on who shows up to vote, and how they choose to vote; in a polarized society the second factor is much less important. The political parties have a clear idea of who their core supporters are, and much of the election effort is focused on a get-out-the-vote campaign, targeted towards getting these supporters show up on election day.

In this paper we study the likelihood of a "single-party sweep"; namely, given some set of elections, what is the probability that *a single party wins all of them?* In order for this to be a well-defined question, one must clarify what it means to win an election. In some real-world situations, such as a presidential contest or a two-party parliamentary election, there is an unambiguous notion of what constitutes a "win"; in other settings, there might be several natural candidates.

For example, in the case of Indian state elections, one might be interested in analyzing either the total number of assembly seats won, or the total number of state governments formed by a party. Alternatively one might choose to focus on a single state, examining whether a single party wins majority in that state as well as the national level. The ruling party in India refers to this as "double engine governance", and portrays it as a favorable outcome that accelerates developmental initiatives within the state[2]. For maximal applicability of our analysis, we choose to work with a somewhat abstract notion of a *win*, subject only to two mild requirements, which fits all these scenarios and many more besides.

Our main finding is that in a country with a polarized electorate, a more simultaneous polling schedule increases the likelihood of a single-party sweep.

We make this statement precise below in three scenarios of increasing generality. Our most general scenario allows for several parties and staggered polling, which is often the case in Indian elections. Although our two motivating exam-

[2]The Indian Railways sometimes use trains with two engines for increased speed and relaibility, especially when transporting heavier loads.

ples, the US and India, both involve first-past-the-post voting, *this is not assumed in our model*, and our results are equally applicable to several other major electoral systems, including party list proportional representation.

Finally, it is worth noting that we actually prove a stronger result below; namely, that probability of a sweep goes up for every party. Thus the effect we describe is systemic in nature and does not depend on the coattails of a popular leader. It stems solely from statistical aggregates of individual voters' actions, and is best regarded as a macroscopic manifestion of political polarization. Our model brings to light a novel cause for sweeps which is not at odds with other causes discussed in the literature, but rather independent of them.

1.1 Main results

Country A has two political parties and two national elections, Presidential and Parliamentary; in each election the candidate with the most votes wins, with ties decided by a coin toss. The election commission is considering whether to hold the elections simultaneously or on separate polling dates. Each voter h has a fixed probability p_h of voting on any poll date[3], and the population is *polarized* in the sense that each voter prefers the same party for both elections. A *sweep* is an outcome where one party wins both the presidency and a parliamentary majority.

Theorem A *A simultaneous election makes a sweep more* [4] *likely.*

Country B has several political parties, and several elections with different notions of what constitutes a "win". However the following two conditions hold.

(a) Each election can be "won" by *at most* one party.

(b) For any turnout in any election, an extra vote for some party cannot decrease its win probability[5], nor can it increase the win probability of a rival party.

[3] This assumption is in the spirit of Banzhaf [5] and Penrose [33] (see section 3.1).

[4] Throughout, "more" is to be understood in the weak sense of "greater than or equal to".

[5] We speak of "win probabilities" since, for example, tied vote counts might lead to coin tosses.

The election commission is exploring various polling schedules. We will say that schedule π is *coarser* than schedule π' if any two elections that are simultaneous (have the same poll date) in π' are also simultaneous in π. As before, each voter h has a fixed probability p_h of voting on any poll date and always votes for the same party. A *sweep* is an outcome in which one party wins all the elections.

Theorem B *A coarser polling schedule makes a sweep more likely.*

Country C has a setup similar to country B. However the election commission is considering *staggered* schedules in which each election might be spread out over several days, with different poll dates for different regions. We will say that staggered schedule Π is coarser than Π' if any two elections that are simultaneous for a voter h in Π' are also simultaneous for h in Π.

Theorem C *A coarser staggered schedule makes a sweep more likely.*

In a country with many political parties, it often happens that several parties come together in an alliance in order to form a government. Here one can distinguish two kinds of alliances – a pre-poll alliance which involves seat sharing, and a post-poll alliance in which a group of parties comes together after having fought the elections separately, perhaps competing against each other for more seats. In this case, one may ask whether Theorem C continues to hold if we treat each alliance as single party. In section 3.3 we show that is indeed the case for many real-world electoral systems.

1.2 Structure of the paper

In section 2 we formulate and prove a mathematical result, Theorem D, which readily implies Theorem C and thus Theorems A and B. Theorem D also implies the Harris inequality, Theorem E, which is a well-known correlation inequality with many mathematical applications [2, 19]. As our proof shows, Theorem D may be regarded as a generalization of the Harris inequality to n functions.[6]

Section 3.1 compares our model of random voter turnout to other models of voter behavior. In Section 3.2 we discuss the applicability of our results to various electoral systems that are in use around the world, and have been studied extensively in the theory of Social Choice [3, 16]. Section 3.3 is on alliances. Section 3.4 contrasts our anaysis of sweeps to the earlier literature. Finally, Sections 3.5, 3.6 focus on the implications of our results to elections in the USA, India.

2 Proofs

We will deduce Theorem C from Theorem D, which we now formulate and prove.

Let $L = \{1,\dots,n\}$ be a finite set, let π be a partition of L, and let $\mathscr{L}$ be the power set of L. Given $0 \le p \le 1$, construct a random subset $S \in \mathscr{L}$ as follows: for each M in π, toss a coin with probability p of heads; if heads then include M in S.

Definition 1 *We write $\mu_{\pi,p}$ for the probability measure on $\mathscr{L}$ such that $\mu_{\pi,p}(S)$ is the probability of obtaining the subset S by the above random procedure.*

Let $H = \{1,\dots,m\}$ be another finite set, and let $\mathscr{H}$ and $\mathscr{T}$ be the power sets of H and $H \times L$. Then we have bijections $\lambda : \mathscr{L}^m \to \mathscr{T}$, $\theta : \mathscr{H}^n \to \mathscr{T}$, given by

$$\lambda(L_1,\dots,L_m) = \bigcup_h \{(h,l) : l \in L_h\}, \quad \theta(H_1,\dots,H_n) = \bigcup_l \{(h,l) : h \in H_l\}.$$

Definition 2 *We say that a function f on $\mathscr{H}$ is* increasing *(resp.* decreasing*) at $h \in H$, if for all $S \subset H$ one has $f(S \cup \{h\}) \ge f(S)$ $\big($resp. $f(S \cup \{h\}) \le f(S)\big)$.*

We say that a tuple $F = (f_1,\dots,f_n)$ of functions on $\mathscr{H}$ is aligned *if for each $h \in H$, either all f_l are increasing at h or all f_l decreasing at h.*

[6] The Harris inequality is a special case of the still more general FKG inequality [17]. One of the authors of the present paper has proposed a different generalization of the FKG inequality to n functions and obtained some partial results in this direction. However the general problem remains open, and seems much harder than the generalization considered here; see [25, 36, 37, 38, 39].

Let $\Pi = (\pi_1, \ldots, \pi_m)$, $\mathbf{p} = (p_1, \ldots, p_m)$, $F = (f_1, \ldots, f_n)$ be tuples such that each π_h is a partition of L, each p_h is in $[0,1]$, and each f_l is a function on $\mathscr{H}$. We write $\mu_{\Pi,\mathbf{p}}$ and $\bar{F}$ for the product measure and function on $\mathscr{T}$ induced by λ and θ:

$$\mu_{\Pi,\mathbf{p}}(L_1, \ldots, L_m) := \mu_{\pi_1,p_1}(L_1) \cdots \mu_{\pi_m,p_m}(L_m),\ \bar{F}(H_1, \ldots, H_n) := f_1(H_1) \cdots f_n(H_n).$$

Definition 3 *For* Π, $\mathbf{p}$, F *as above, we define* $E(\Pi, \mathbf{p}, F) := \sum_{T \in \mathscr{T}} \bar{F}(T) \mu_{\Pi,\mathbf{p}}(T)$.

Theorem D *Suppose* $F = (f_1, \ldots, f_n)$ *is a tuple of aligned non-negative functions on* $\mathscr{H}$. *If* Π' *is coarser than* Π *then we have* $E(\Pi', \mathbf{p}, F) \geq E(\Pi, \mathbf{p}, F)$ *for all* $\mathbf{p}$.

Proof. It suffices to prove $E(\Pi', \mathbf{p}, F) \geq E(\Pi, \mathbf{p}, F)$ for the case where

1) $\Pi' = \{\pi'_h\}$ and $\Pi = \{\pi_h\}$ differ only for a single h, say $h = 1$;

2) π'_1 is obtained by combining two parts of π_1, say the first two, as follows:

$$\pi_1 = \{M_1, M_2, M_3, \ldots, M_k\}, \quad \pi'_1 = \{M_1 \cup M_2, M_3, \ldots, M_k\}.$$

Indeed, by iterating 2) we obtain the inequality for any π'_1 coarser than π_1. Now by iterating 1) we obtain the inequality for any Π' coarser that Π.

Thus we may assume that Π and Π' are as in 1) and 2). Now the measures $\mu_{\Pi,\mathbf{p}}$ and $\mu_{\Pi',\mathbf{p}}$ involve the same coin tosses for $\pi_2, \ldots, \pi_m$ and for $M_3, \ldots, M_k$ in π_1. It suffices to show that for each subset T' of $L \times H$ resulting from these tosses, the conditional expectation of $F(T) = f_1(H_1) \cdots f_n(H_n)$ is higher if we toss a single coin for $M_1 \cup M_2$ rather than separate coins for M_1 and M_2.

To study this we write $\theta(T') = (H_1, \ldots, H_n)$, and we set

$$a_i = \prod_{l \in M_i} f_l(H_l \cup \{1\}),\ b_i = \prod_{l \in M_i} f_l(H_l),\ \text{for } i = 1,2; \quad c = \prod_{l \notin M_1 \cup M_2} f_l(H_l).$$

Then the conditional expectations for the single and double toss are, respectively,

$$A = [p a_1 a_2 + (1-p) b_1 b_2] c,\ B = [p^2 a_1 a_2 + p(1-p)(a_1 b_2 + b_1 a_2) + (1-p)^2 b_1 b_2] c,$$

where $p = p_1$. Now by an easy calculation we get

$$A - B = p(1-p)(a_1 - b_1)(a_2 - b_2)c.$$

Since $0 \leq p \leq 1$, we have $p(1-p) \geq 0$, and since $f_i \geq 0$, we have $c \geq 0$. Further, since the f_i are aligned, either they are all increasing or all decreasing, at $h = 1$. In the first case we have $a_i \geq b_i$ and in the second case $a_i \leq b_i$, for $i = 1, 2$. In either case we get $(a_1 - b_1)(a_2 - b_2) \geq 0$, which implies $A - B \geq 0$. ■

Theorem C follows immediately from Theorem D.

Proof of Theorem C. Let $H = \{1, \ldots, m\}$ be the set of voters and let $L = \{1, \ldots, n\}$ be the set of elections. A voter *turnout*, $T \subset H \times L$, is the set of pairs (h, l) such that voter h has voted in election l. The staggered schedules Π and the voter probabilities $\mathbf{p} = (p_h)$ induce a probability measure on voter turnouts, which is seen to be precisely $\mu_{\Pi, \mathbf{p}}$. We will prove that if we replace Π by a coarser partition then the probability of a sweep goes up for *every* party.

Fix a party s and define $F = (f_1, \ldots, f_n)$ where $f_i(H')$ is the probability that party s wins election i when H' is the set of votes cast. Then the f_i are non-negative and aligned by our assumptions. Now the probability of a sweep by party s is precisely $E(\Pi, \mathbf{p}, F)$, and so the result follows by Theorem D. ■

Theorem D can also be regarded as an n-function generalization of the Harris correlation inequality for two functions, which we recall. For $\mathbf{p} = (p_1, \ldots, p_m)$ as before, we define a probabilty measure $\mu = \mu_{\mathbf{p}}$ and expectation $E(f, \mu)$ on $\mathscr{H}$ by

$$\mu(S) = \prod_{h \in S}(p_h) \prod_{h \notin S}(1 - p_h), \quad E(f, \mu) = \sum_{S \in \mathscr{H}} f(S)\mu(S).$$

Theorem E *If f_1, f_2 are increasing functions on $\mathscr{H}$ then we have*

$$E(f_1 f_2, \mu) - E(f_1, \mu)E(f_2, \mu) \geq 0. \tag{1}$$

Proof. This is due to Harris [19], but it is also an immediate corollary of the case $n = 2$ of Theorem D as we now explain. First, if we add a constant to f_1

or f_2 then the left side of (1) is unchanged, thus we may assume without loss of generality that f_1 and f_2 are non-negative. Evidently then the pair $F = (f_1, f_2)$ is aligned and non-negative. Now let $\Pi = (\pi_1, \ldots, \pi_m)$ and $\Pi' = (\pi'_1, \ldots, \pi'_m)$ where $\pi_h = \{\{1\}, \{2\}\}$ and $\pi'_h = \{\{1, 2\}\}$ for all h, then we have

$$E(\Pi', \mathbf{p}, F) = E(f_1 f_2, \mu), \quad E(\Pi, \mathbf{p}, F) = E(f_1, \mu) E(f_2, \mu).$$

Since Π' is coarser than Π, Theorem D implies (1) as desired. ■

3 Discussion

3.1 Random Voter Turnout

A key hypothesis of our model is that the probabilites of turning out to vote are *independent* across individuals, and also across different polling dates for any given individual[7]. This is very much in the spirit of Penrose [33], and later Banzhaf [5]), both of whom considered the special case where these probabilities were identically $1/2$.

The recent "instrumental theory of turnout"[8] provides theoretical underpinnings for the independence assumption. Some of the analyses in the field are focused on individual rational choice (in an exogenously specified environment), modeled in terms of adaptive learning (e.g., [22]), or utility maximization (e.g., [43]) or minmax regret (e.g.,[14]). Others discuss full-blown strategic interaction among the voters (and sometimes also candidates) and derive Nash equilibria of "participation games" (e.g., [24], [29], [30]). These analyses differ considerably from one another on how individual turnout probabilities are formed, but they all

[7]Furthermore, while these probabilities may vary arbitrarily across individuals, they remain constant (across different polling dates) for each individual.

[8]The theory posits individuals for whom the act of voting is instrumental in maximizing their own "payoffs", though these payoffs may be very broadly defined, incorporating not only the joy of seeing their party win but also the joy of participating in the election (alongside the cost of participating).

imply the independence of those probabilities (see the surveys in [10] and [18]). Some recent models (e.g., [34]), while maintaining the instrumentality assumption, introduce the possibility of communication — broadly defined — between candidates, media and voters. This gives rise to correlated equilibria where the turnout probabilities of voters are no longer independent. (Also, there are several behavioral models which openly depart from the "instrumental theory" and directly incorporate correlated turnout among the voters, such as voting in "teams" (see [13], [41] and the references therein).

The independence hypothesis is central to our analysis[9]. However there is a "special kind" of correlation that can be admitted in our model (which, while mathematically obvious, may be useful both conceptually and for applications). As in Harsanyi ([20]), define the *type* of voter $h \in H$ to consist of two components: the party $t \in T$ that h will vote for; and the probability p_h with which h will turn out to vote on *any* polling date. Before the announcement of the polling schedule, assume that nature picks the vector of voter-types according to some *a priori* probability distribution on a given set.[10] Thus nature's move is *ex ante*, and every voter's type stays fixed throughout the elections. Now, if schedule Π is coarser than Π', then the probability of a sweep is higher under Π than under Π' contingent on *every* move of nature (by Theorem C), therefore obviously also in expectation across all the moves of nature. (However, if nature were to move *ex post* independently between different poll dates, the argument given above breaks down.)

[9]We take it to be a *behavioral* hypothesis which governs voters' turnout. In other words, no matter how the behavior might once have originated from rational (optimal) choice, it has got entrenched as a "rule of thumb" for a voter (see [4]), which he follows without bothering to recompute the exact optimal choice, each time the election scenario changes on account of alterations in the polling schedule.

[10]A "move" of nature could represent public news that affects voters' types; or — alternatively — independendent (idiosyncratic) perturbations in every voter's p_h.

3.2 Electoral systems

There is a wide variety of election methods in use across the world. For an extensive survey, covering all the methods we cite below, see [1].

Our analysis applies to first-past-the-post elections that are held in several countries including India, USA, Canada and UK (the first two of which we shall discuss below in some detail).

It also applies to elections based on proportional representation, where each person votes for a single party of his choice, and the percentage of parliamentary seats accorded to any party equals (as nearly as possible) the percentage of total votes it received. Of course, for our analysis to be valid, the method for "rounding" the seats (into integers) should be "monotonic" [11] in the obvious sense, so that condition (b) is not violated; and, furthermore, condition (a) must also hold (e.g., the party with the most seats is deemed to win; or, alternatively, if no party gets a strict majority of the seats, it is a non-win for every party). Such elections are in use — with minor modifications — in many countries and also in the European Parliament (see, again, [1] for details).

There is, however, a category of elections that lies outside the ambit of our model, and it includes the Two Round System (used in the French Presidential Election), Single Transferable Vote, Alternative Vote, Supplementary Vote, Borda Count and so on (described in detail in [1]). In all these cases, a vote is tantamount to a ranking of the electoral candidates. Such elections have received the lion's share of attention in the theoretical literature on "social choice" (SC) ever since Arrow's pioneering monograph [3]. (For a succinct survey, see, e.g., [16].) The concerns of the SC literature are quite different from ours, but even from a "high level", there are salient differences. First, our voter only specifies his top choice instead of ranking the candidates. More importantly, in SC, the entire voter population is assumed to always be present. In sharp contrast, it is the *variable, stochastic turnout* of voters on which our analysis turns. It might be interesting to

[11] Monotonicity holds for most of the rounding methods used in practice, e.g., the Highest Average or the Largest Remainder Methods [1]

examine, along the lines of this paper, how the probability distribution of electoral outcomes depends on the stochastic turnout, in the context of the more sophisticated elections considered in SC (e.g., Borda Count, see [26]), but that would be a topic for another paper.

3.3 Party Alliances

Consider a partition of parties into sets, each of which constitutes an electoral alliance. Define the *type* of an alliance to be *pre-poll* if the parties in the alliance jointly field one candidate in every election, and do not contend against one another; and to be *post-poll* if they contend every election on their own, but come together afterwards by pooling their elected candidates. Our analysis will remain intact if conditions (a) and (b) hold, viewing each alliance as a single contender in all the elections. This is easily seen to be so in elections based on first-past-the post criterion or on proportional representation, provided the type of each alliance is fixed across all polling schedules (i.e., either it is pre-poll for all schedules, or post-poll for all). To check (b) in the presence of post-poll alliances (the only scenario where it is not immediately obvious), note that if a vote is cast in favor of an alliance it does not benefit any of the *rival* alliances, and therefore cannot hurt the alliance either, since the total number of elected candidates is fixed in every election.

However, if the set of parties that form an alliance, or the type of an alliance, varies across elections, or across the polling schedules, or with the electoral turnout, then our analysis breaks down.

3.4 Single-Party Sweeps

A single-party sweep (also known as "one-party monopoly" or "one-party dominance") is a widely prevalent phenomenon, both in local and national elections ([31], [32], [42]) Its causes have been discussed from different standpoints, including: the strategic timing of elections ([23]), economic intervention ([23]), manipulation of electoral rules ([27]), and fragmented opposition ([44]). There does not seem to be much in the literature connecting polling schedules to single-party sweeps, which is the focus of this paper. Two notable exceptions are the "coattail effect" in the context of US elections (see, e.g., [8, 9, 28]). and a recent study ([6]) of the Indian elections. Both analyses are based on the postulate that a voter's behavior *changes* when elections are held simultaneously, on account of either the popularity of the Presidential candidate or the "salience" of one of the political parties (see the next two sections). In sharp contrast, voters' behavior is *fixed* in our model, regardless of the polling schedule or the characteristics of the candidates standing for election. Nevertheless the voter turnout varies with the schedule and drives our result.

3.5 US Elections and The Coattail Effect

The data from USA shows that a single party is more likely to win multiple elections when they are held simultaneously. Indeed, during Congressional elections in USA that coincide with the presidential race, the party of a popular president tends to win more seats in Congress compared to elections held in the midterm of the Presidency (the latest instance of this being the US elections in November 2024). This phenomenon is referred to as the "coattail" effect. The rationale is that a popular presidential candidate fosters additional wins for his party in Congress, with many members of Congress voted into office "on the coattails" of the president.

Our analysis brings to light a completely different mechanism for a single-party sweep, which is not at odds with the coattail effect, but independent of it. Two behavioral causes are commonly postulated for the coattail effect: a popular presidential candidate can mobilize each member in his party base to turn out to vote with higher probability; and, moreover, he can sway undecided voters to his party's fold.[12] These causes are ruled out in our model since the party that h will

[12]A cause of a different nature is the "anti-incumbency factor" so often alluded to by political analysts. It can come into play during midterm elections, especially if there is widespread percep-

vote for, as well the probability p_h that he will vote on any poll date, are both exogenously fixed for every $h \in H$ and not susceptible to change. It is purely "the statistics of voting" that enhances the single-party sweep for simultaneous elections.[13]

The probability of a single-party sweep can, of course, be very small if there are many parties or many elections. It becomes significant when, as is often the case, there are two parties contending two (or three) elections. To the extent that "Duverger's Law" operates and a representative democracy settles into a two-party system, the sweep can be quite a robust phenomenon [12, 15, 35]

3.6 Indian Elections: "ONOE" & "Double-Engine" Governance

Elections in India are held as follows. Political parties contend for seats in the Lok Sabha at the national level and also in 28 Vidhan Sabhas, one for each state. The country (resp., state) is partitioned into electoral districts for the Lok (resp., Vidhan) Sabha election, the candidate with the most votes is elected from each district, and the party with the most candidates is deemed to have won the election.[14]

The possibility of simultaneous elections is no longer theoretical but it looms large in India and is the subject of a fierce ongoing debate, set into motion by the BJP and its allies (currently in power in the Lok Sabha) seeking a Constitutional Amendment to implement their "One Nation, One Election" (ONOE) proposal

tion that the ruling party has been unable to fulfil its electoral promises. For then the rival party makes gains in the midterm elections, further accentuating the benefit of simultaneous elections for the ruling party.

[13] We could incorporate the "coattail" in our model as an increase in the probabilities of turnout of the adherents of the President's party in (at least) all other contemporaneous Congressional elections (while the turnout for its rivals stay fixed or go down). This will obviously increase the probability of win for the party in every election. In this sense, our model is accomodative of the coattail effect.

[14] W.l.o.g. we may assume that the entire electorate of India participates in each Vidhan Sabha election, viewing voters outside the state as "strategic dummies" whose votes do not count (since condition (b) still holds for such dummies.)

(which would make all Vidhan Sabha elections simultaneous with the Lok Sabha election). At the same time, the BJP has been strongly campaigning for "double-engine" governance, especially during elections in the states. Theorem C shows that the likelihood of one party winning the Lok Sabha and the Vidhan Sabha of any given state is higher when both elections are simultaneous compared to when they are not, i.e., under ONOE, the probability of "double-engine" governance goes up for every state. This happens by way of a *systemic change*, no matter which party wins the Lok Sabha. The magnitude of the change can, in general, be quite significant[15]. Indeed consider two parties, BJP and its allies versus the "INDIA" coalition of all their rivals (as in the 2024 elections) contending the Lok Sabha at the Center and the Vidhan Sabha in Bihar. For ease of computation assume that each contender is favored by half the population of India, and of Bihar; and that the turnout probability is $1/2$ for every voter. Then, by the law of large numbers, the probability of a sweep (i.e., of double-engine governance) rises from about 50% to about 100% when the polling schedule of the two elections is made simultaneous (and rises from 25% to 50% for each contender).

The study in [6] refers to the pronounced correlation in the Indian data between simultaneous elections and single-party sweeps (over those elections). To explain it, the authors posit a model of behaviorally-constrained voters for whom information acquisition is costly. If a party is salient in the Lok Sabha elections, then the electorate has a higher probability to also vote for it in any contemporaneous Vidhan Sabha election, rather than acquire costly information about its rivals which could alter that decision. Such is not the case if the Vidhan Sabha election is held at a very different time, for then the "salience factor" is missing and information acquisition becomes less costly. By fixing the type of a player during elections, we have ruled out this kind of varition in voter behavior. Our model highlights an alternative, purely stastistical cause of the correlation.

Finally recall that the notion of "win" is very general in our model and can vary across elections, subject only to conditions (a) and (b). As mentioned in the

[15]We thank Vijay Vazirani for asking this question.

introduction, this enables us to consider Indian elections from diverse viewpoints, including the following two starkly different ones: $1+28=29$ "literal" elections, or 2 "figurative" elections (with the 28 Vidhan Sabha elections viewed as one conglomorate). The literal elections clearly fit into our model, but here the likelihood of a sweep is so miniscule that it is unlikely for either party to be focused on it. Sweeps occur with higher ptobability in the 2-election framework, where "win" in Vidhan Sabhas can be defined in diverse ways, e.g., (i) winning a specific set of states[16]; (ii) winning most states overall; (iii) winning most seats overall (relevant for the future Rajya Sabha); and so on. All these scenarios are covered by Theorem C, which allows for staggered polling schedules (i.e., personal partitions of the elections in L).

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[16]In particular, the set could consist of a single state, in which case a sweep reduces to the "double-engine governance" discussed earlier. More generally, in theory if not in practice, the contending parties may have different sets of states in their sights but, so long as the intersection of their sets is non-empty, condition (a) holds and our analysis remains intact.

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