Inflation Targeting with Sovereign Default Risk

Working Paper 2018-14

Cristina Arellano, Yan Bai and Gabriel Mihalache

December, 2018

Stony Brook University

Inflation Targeting with Sovereign Default Risk*

Cristina Arellano
Federal Reserve Bank of Minneapolis,
University of Minnesota, and NBER

Yan Bai
University of Rochester
and NBER

Gabriel Mihalache
Stony Brook University

December 2018
Preliminary

Abstract

Since the early 2000s, many emerging markets have adopted inflation targeting as their monetary policy, against a background of recurring sovereign debt crises. We develop a framework that integrates inflation targeting monetary policy with sovereign default risk and identify important interactions. Monetary policy alters incentives for international borrowing and sovereign default risk leads to more volatile nominal interest rates, needed to target inflation. We show that this framework replicates the positive co-movements of sovereign interest rate spreads with domestic nominal rates and inflation, a salient feature of emerging markets data. Our framework rationalizes the experience of Brazil during the 2015 downturn, which featured high inflation, high nominal rates, and high sovereign spreads. Our counterfactual experiment suggests that by raising the domestic rate the Brazilian central bank not only reduced inflation but also alleviated the debt crisis.

*We thank our discussants Giancarlo Corsetti and Vivian Yue for insightful comments and suggestions. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. Contact: arellano.cristina@gmail.com; yanbai06@gmail.com; mihalache@gmail.com

1 Introduction

Since the early 2000s, following the steps of advanced economies, many central banks in emerging markets have achieved independence from the central government and have adopted an inflation targeting framework for monetary policy. These emerging markets have conquered their historical inflationary episodes, recently experiencing low and stable inflation. Emerging markets have historically also faced recurrent foreign debt crises, with high and volatile interest rate spreads as well as outright default. The theoretical literature studying inflation targets and sovereign debt crisis, however, has largely studied each of these topics in isolation.[1] In this paper, we develop a framework that integrates inflation targeting monetary policy with sovereign default risk and identify important interactions. Monetary policy alters incentives for international borrowing and sovereign default risk leads to more volatile nominal interest rates, needed to target inflation.

We provide a framework that combines the workhorse New Keynesian monetary model of Gali and Monacelli (2005) with the canonical sovereign default model in Arellano (2008). In our framework, recessions due to low productivity are associated with high inflation and high spreads, due to rising default risk. The monetary authority increases the domestic nominal rate to fight inflation and such tightening reduces the government's incentives to borrow and ameliorates the debt crisis. A reduction in borrowing is desirable for the government because it induces a depreciation, which boosts exports and production. This framework delivers a positive co-movement between sovereign interest rate spreads with domestic nominal interest rates and inflation, which we document as a salient feature for inflation targeting emerging markets.

We apply our framework to the recent Brazilian downturn of 2015, which featured increases in sovereign spreads together with high inflation and tight monetary policy. The central bank increased nominal interest rates in response to high inflation relative to its target. We show that our framework can rationalize these dynamics and then use it to evaluate a counterfactual looser monetary policy. In this alternative scenario, we find that if Brazil had kept nominal interest rates low throughout, inflation would had been higher and the debt crisis would have been exacerbated. Hence we conclude that by adhering to its inflation target, Brazil's central bank was able not only to bring inflation down but also to alleviate the debt crisis.

The small open economy model we consider consists of households, firms, a mone-

[1] For example, the influential paper by Gali and Monacelli (2005) analyzes monetary policy in the context of perfect financial markets and the work of Aguiar and Gopinath (2006) and Arellano (2008) study sovereign default in real models.

tary authority, and a fiscal government that borrows internationally. Households value consumption of domestic goods and imported foreign goods. They work in intermediate goods firms that produce domestic varieties. The intermediate goods firms face frictions in setting their prices and are subject to productivity shocks. Firms have to pay an adjustment cost whenever they change their prices, in the tradition of Rotemberg (1982). Final goods firms are competitive and use intermediate goods varieties to produce domestic output, which is both consumed by domestic households and exported to the rest of the world. The monetary authority sets policy by following a nominal interest rate rule, that depends on the gap between inflation and its target. Monetary policy is subject to shocks, so that the interest rate can deviate from the prescriptions of the rule.

The government borrows from the rest of the world by issuing discount bonds denominated in foreign currency. It is benevolent and uses international borrowing for transfers to households to smooth fluctuations. The government, however, lacks commitment over repaying the debt and can choose to default. The risk of it doing so is reflected in the borrowing rate it faces on world markets.

We consider a Markov problem for the fiscal government. The government internalizes that its borrowing and default decisions induce an allocation and prices for the private economy and monetary authority. It understands that increasing borrowing not only increases consumption of foreign goods but also tends to appreciate the terms of trade, leading to lower exports and production. The government also understands the impact of its policies on future allocations, taking as given the decisions for borrowing and default of future governments. Increasing borrowing depresses future consumption and leads to higher inflation. Today's government can choose to manipulate these future allocations, through its choice of borrowing, to relax constraints it's facing today. Within this structure, debt choices interact with the nominal side of the model and with monetary policy.

We derive an optimal foreign borrowing condition that reflects the government's trade-offs. The condition is an Euler equation for foreign consumption, distorted by *monetary* and *default* wedges, and where the foreign interest rate reflects default risk. The interactions between monetary policy and the government in our model can be analyzed by combining the Euler equations for domestic and foreign consumptions. Absent wedges, this combination yields a standard Uncovered Interest Rate Parity (UIP) condition, which relates interest rate differential to the expected exchange rate depreciation: a higher domestic nominal interest rate, relative to the foreign rate, leads to appreciation on impact and expected depreciation. In our model, with monetary and default wedges, this standard UIP logic breaks down. High domestic nominal rates do not necessarily lead to appreciations, since

the government can respond by adjusting its borrowing and default policies, in a way that alters both the foreign rate, through default risk, but also the terms of trade. We find that in response to a domestic nominal rate increase the government reduces borrowing, inducing a decline in the foreign rate and a depreciation.

We parameterize our model to Brazilian data and perform a quantitative evaluation of our model. We present impulse response functions to monetary and productivity shocks and compare them to a reference model without default risk. This reference model is a variant of the Gali and Monacelli (2005) model with uncontingent debt but otherwise frictionless financial markets. In response to a contractionary monetary shock, inflation, output, domestic consumption, and borrowing fall on impact. Nominal rates in our model rise by more than in the reference model because inflation fall by less. Government spreads fall in our model in response to contractionary shock and remain low for several periods while debt is below its initial level. The reference model exhibits no spreads.

These impulse responses show that with default risk, contractionary monetary policy is less powerful in bringing down inflation as inflation falls less in the benchmark than in the reference despite nominal interest rates increasing by more in the benchmark. Tight monetary policy, nevertheless, also brings down government spreads because it reduces the incentives to borrow.

In response to a low productivity shock, consumption and output fall, while spreads, inflation, and nominal interest rates rise. The increases in inflation and nominal rates are higher in the benchmark than in the reference model. This amplification arises in the benchmark because the bond price schedule is tighter with low productivity leading to an increase in spreads. The tight borrowing conditions discourage imports and stimulate exports, to pay off the debt. Such dynamics lead to an appreciation of the exchange rate which dampens the decline of domestic output. Inflation then increases by more because of a higher unit cost of production. In the reference model without default the economy can smooth the shock by taking on additional debt without reducing imports or increasing exports; the abundant borrowing also leads to a depreciation.

The impulse responses to a productivity shock show that with default risk, nominal interest rates and inflation are more volatile. Pursuing an inflation target requires more aggressive movements in nominal rates in environments with default risk.

We compare our model implications with Brazilian data in terms of second moments, correlations, and dynamics in the 2014-2016 downturn. In terms of second moments we show that our model delivers a volatility of inflation and nominal interest rates close to the data. In terms of correlations, our model delivers strong positive comovement of spreads

with inflation and nominal interest rates.

Finally, we perform an event analysis and evaluate a counterfactual monetary policy scenario. We focus on the period from 2014 to 2016. During the event output fell in Brazil about 6% below trend, inflation and nominal interest rates increased about 4%, and spreads increased about 3%. We apply our model to this event by feeding the model a sequence of productivity shocks such that it reproduces the dynamics of output. We then compare the model implications for inflation, nominal rates, and spreads to the data. Our model reproduces sizable increases in inflation, nominal rates and spreads. Nominal interest rates increase as the inflation target rule calls for such tightening.

We then perform the counterfactual experiment. We feed in the same sequence of productivity shocks but impose that the monetary authority does not increase nominal rates. We then compare the paths of inflation, spreads, and output in the counterfactual scenario to the benchmark. We implement this experiment by feeding in expansionary monetary shocks large enough such that nominal rates remain at the 2015 level throughout. In the counterfactual, inflation increases more, output decreases less, and spread increase substantially more. We conclude that the increase in nominal rates in Brazil, during the event, not only controlled inflation but also moderated the debt crisis.

Related Literature Our project builds on insights from two distinct and hitherto unconnected literatures on emerging market business cycle: the work on New Keynesian monetary policy in small open economies, following Gali and Monacelli (2005), and the literature on fundamental sovereign default risk, following Eaton and Gersovitz (1981).

We follow the quantitative approach to default of Aguiar and Gopinath (2006) and Arellano (2008), but abstract from the many extensions and applications developed in the recent literature, including long-term debt and maturity choice (Chatterjee and Eyigungor (2012), Arellano and Ramanarayanan (2012), Hatchondo et al. (2016)), taxation and government spending (Cuadra et al. (2010)), debt restructuring and renegotiation (Yue (2010), Pitchford and Wright (2012)), or the work on contagion and transmission of country risk (Arellano et al. (2017), Bocola (2016)). Similarly, our domestic monetary environment is close to the reference model of Gali and Monacelli (2005) and abstracts from the many extensions considered in the (medium-scale) open economy DSGE literature, e.g. Christiano et al. (2011). One methodological difference from such projects is that we use global methods rather than local approximations around the steady state. Furthermore, we focus on a simple interest rate rule as a model of inflation targeting, and do not address optimal monetary policy, along the lines of Schmitt-Grohé and Uribe (2007) or Corsetti et al. (2010).

The literature on sovereign default recently turned to questions raised by nominal rigidities and the currency denomination of debt. Na et al. (2014) emphasize downward rigidity of nominal wages and the incentives it creates for exchange rate management together with the monetary authority's inability to pursue such policies in the presence of an exchange rate peg. In their setting inflation is desirable in that, in the presence of a rigid nominal wage, adjustments in the price level can return the real wage to its efficient level. Our projects differs in two main ways: first, we consider price-setting frictions as opposed to nominal wage rigidity and, second, we model an inflation targeting monetary authority, via an interest rate rule that calls for monetary tightening in the face of rising inflation. Bianchi et al. (2018) study an environment similar to Na et al. (2014) and focus on the public spending dimension of fiscal policy. There, additional spending stimulates aggregate demand and lowers unemployment, but at the cost of worsened terms of borrowing. We also address the fiscal authority's role in determining demand, not directly through public spending but rather via international borrowing, in particular as a response to domestic monetary policy.

Kondo et al. (2016) and Sunder-Plassmann (2018) study the interaction of inflation with defaultable debt. The former considers exogenous inflation, for given covariance structures with fundamentals, while the latter builds on a cash-and-credit model with a constant money supply. Nuno and Thomas (2018) build a continuous time model with local currency debt and a discretionary choice of inflation. In contrast with these papers, we emphasize the joint dynamics of endogenous inflation and country risk, under rule-based monetary policy, and focus on the structure of domestic economy as constraints facing fiscal policy-making.

A large literature, following Calvo (1988), studies whether monetary policy can alleviate self-fulfilling debt crises. Aguiar et al. (2013) study trade-offs raised by monetary policy credibility in a tractable, continuous time model of self-fulfilling default. Concerning the multiplicity of equilibria and the role inflation can play in selecting among them, Corsetti and Dedola (2016) focus on unconventional monetary policy while Bacchetta et al. (2018) embed New Keynesian features in the "slow moving debt crisis" model of Lorenzoni and Werning (2013) to quantify the role of inflation for crisis prevention.

Finally, our model's implications for the terms of trade, nominal and real exchange rate, and centralized borrowing raise a natural comparison with the work on capital controls and exchange rates in small open economies, such as Farhi and Werning (2012), Devereux et al. (2018) and more recently Fanelli (2017).

2 Model

We consider a small open economy which is composed of households, final good producers, intermediate goods firms, a fiscal authority, and a monetary authority. There are three types of goods: imported, intermediate, and final. The final good is produced using all varieties of differentiated, intermediate goods. Each variety is produced with labor. The final good is demanded by both domestic and foreign consumers.

Foreign demand for domestic goods (export demand) is given by

$$X_t = \left(\frac{P_t^d}{\varepsilon_t P_t^*} \right)^{-\rho} \xi,$$

where P_t^* is the price of foreign goods in foreign currency, ξ is the level of overall foreign demand and ρ is the elasticity of demand. P_t^d is the price of domestic goods in local currency and ε_t is the nominal exchange rate, with an increase in ε_t being a depreciation of the home currency. Because the law of one price holds we can write the price of the foreign good in local currency as

$$P_t^f = \varepsilon_t P_t^*.$$

The terms of trade are defined by

$$e_t = \frac{P_t^f}{P_t^d} = \frac{\varepsilon_t P_t^*}{P_t^d}.$$

The foreign demand for domestic goods is a function of the terms of trade and the foreign shock

$$X_t = e_t^\rho \xi. \tag{1}$$

We normalize the foreign price P_t^* and ξ to one.

2.1 Households

Households preferences are defined over consumption of domestic C_t and foreign goods C_t^f and labor N_t. Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t,\ C_t^f, N_t),$$

where the per-period utility function is given by

$$u(C_t,\ C_t^f, N_t) = \theta \log(C_t) + (1-\theta)\log(C_t^f) - \frac{N_t^{1+1/\zeta}}{1+1/\zeta}.$$

Taking aggregate prices as given, the households choose consumption, labor supply, and domestic bonds B_t^d holdings, denominated in local currency. Households own intermediate goods firms and receive profits Φ_t from them. They also receive labor income and government transfers T_t. The budget constraint is given by

$$P_t^d C_t + (1+\tau_f) P_t^f C_t^f + q_t^d B_{t+1}^d \leq W_t N_t + B_t^d + \Psi_t + T_t.$$

where q_t^d is the nominal discount prices of domestic bonds, and τ_f is a constant consumption tax that households pay on imported consumption. It is convenient to write the budget constraint in real terms, deflating by the domestic price index P_t^d

$$C_t + (1+\tau_f) e_t C_t^f + q_t^d b_{t+1}^d \leq w_t N_t + \frac{b_t^d}{\pi_t} + \psi_t + t_t. \tag{2}$$

where real domestic bonds are $b_{t+1} = B_{t+1}^d / P_t^d$, the real wage is $w_t = W_t / P_t^d$, real profits and transfers are $\psi_t = \Psi_t / P_t^d$, $t_t = T_t / P_t^d$, and the gross domestic inflation is $\pi_t = P_t^d / P_{t-1}^d$. We can characterize the representative consumer's problem with the following optimality conditions

$$-\frac{u_{N,t}}{u_{c,t}} = \frac{W_t}{P_t^d} = w_t, \tag{3}$$

$$\frac{u_{c^f,t}}{u_{c,t}} = (1+\tau_f) e_t, \tag{4}$$

$$q_t^d = \beta \mathbb{E}_t \left[\frac{u_{c,t+1}}{u_{c,t}} \frac{1}{\pi_{t+1}} \right]. \tag{5}$$

The nominal interest rate is defined as the yield of the discount bond price

$$i_t \equiv \frac{1}{q_t^d}.$$

2.2 Final goods producers

The final good is produced from a variety of differentiated intermediates y_{it}, $i \in [0,1]$ under perfect competition,

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}. \tag{6}$$

where η is the elasticity of substitution between intermediate goods. Let the prices of intermediate goods be $\{p_t(i)\}$. The profit maximization problem of the final good producer is

$$\max P_t^d \left[\int_0^1 y_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} - \int_0^1 p_t(i) y_t(i) di.$$

inducing a demand function and a price aggregator

$$y_t(i) = \left(\frac{p_t(i)}{P_t^d} \right)^{-\eta} Y_t, \tag{7}$$

$$P_t^d = \left[\int_0^1 p_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \tag{8}$$

2.3 Intermediate goods producers

Each differentiated intermediate good is produced with labor n_{it}, using a constant returns to scale production function, subject to productivity shocks z_t

$$y_{it} = z_t n_{it}. \tag{9}$$

Intermediate goods firms are monopolistically competitive and set the prices for their products taking as given the demand system (7). These firms, however, face price setting frictions in that they have to pay a quadratic adjustment cost when they change their price relative to the target inflation $\overline{\pi}$, as in Rotemberg (1982). Taking as given the wage rate W_t and the final good price P_t^d, an intermediate firm i chooses labor and its price to maximize the present discounted value of their profits

$$\max_{\{p_{it}, n_{it},\}} \mathbb{E}_0 \sum_t Q_{t,0} \left\{ p_{it} y_{it} - (1-\tau) W_t n_{it} - \frac{\varphi}{2} \left(\frac{p_{it}}{p_{it-1}} - \overline{\pi} \right)^2 P_t^d Y_t \right\}$$

subject to the production function, where $Q_{t,0}$ is the stochastic discount factor of households, denominated in units of domestic goods, and τ is a labor subsidy. (This subsidy is assumed

constant, a fiscal policy designed to alleviate inefficiencies induced by market power, standard in the New Keynesian literature.)

Using the households' stochastic discount factor and the production function this problem is

$$\max_{\{p_{it}\}} \mathbb{E}_0 \sum_t \beta^t \frac{u_{c,t}}{u_{c,0}} \frac{P_0^d}{P_t^d} \left\{ p_{it} y_{it} - \Omega_t y_{it} - \frac{\varphi}{2} \left(\frac{p_{i,t}(j)}{p_{i,t-1}(j)} - \overline{\pi} \right)^2 P_t^d Y_t \right\}$$

where $\Omega_t = \frac{(1-\tau)W_t}{z_t}$ which we denote as the unit cost. The first order condition for each firm, after imposing symmetry across all firms $p_{it} = P_t^d$, results in

$$\frac{\Omega_t}{P_t^d} = \frac{\eta - 1}{\eta} + \frac{1}{\eta} \varphi \left(\pi_t - \overline{\pi} \right) \pi_t - \frac{1}{\eta} \mathbb{E}_t \left[\beta \frac{u_{c,t+1}}{u_{c,t}} \frac{Y_{t+1}}{Y_t} \varphi \left(\pi_{t+1} - \overline{\pi} \right) \pi_{t+1} \right]. \tag{10}$$

This equation is a standard New Keynesian Philips Curve (NKPC) that relates inflation to a measure of contemporaneous unit cost and expected inflation.

2.4 Monetary Authority

The monetary authority conducts policy using nominal interest rates rules. Nominal rates i_t are set based on the level of inflation relative to the target $\pi_t / \overline{\pi}$, and also respond to a monetary shock m_t

$$i_t = R \left[\left(\frac{\pi_t}{\overline{\pi}} \right)^{\alpha_P} \right] m_t. \tag{11}$$

In equilibrium, the average value of the domestic nominal rate R must satisfy the usual steady state condition,

$$R = \frac{\overline{\pi}}{\beta}. \tag{12}$$

In the quantitative section, we implement interest rate smoothing, common in the New Keynesian literature, to prevent excess volatility of domestic nominal rates. We delay the introduction of this extension to economize on notation in the exposition of the model.

2.5 Fiscal Government and External Debt

The fiscal government is benevolent and maximizes the utility of households. It borrows short-term B_{t+1} at price q_t in foreign currency from international lenders. As in standard New Keynesian models we also let the fiscal government subsidize employment at a time-invariant rate τ, as to undo the markup in goods markets. The government transfers to

households the net receipts from its operations. Letting B_t denote the outstanding foreign currency debt of the government, the budget constraint in local currency is

$$T_t + \tau W_t N_t = P_t^f [q_t B_{t+1} - B_t] + \tau_f P_t^f C_t^f \tag{13}$$

where the net capital inflow from debt operations is multiplied by P_t^f to convert it to domestic currency. The government budget constraint in terms of domestic goods is

$$t_t + \tau w_t N_t = e_t [q_t B_{t+1} - B_t] + \tau_f e_t C_t^f \tag{14}$$

Every period the government chooses B_{t+1} and decides whether to default on its outstanding debt. Default induces costs both in terms of productivity and utility. If the government defaults, the economy experiences a reduction in productivity to $z_t^d \leq z_t$, and a utility cost ν_t. The utility cost ν_t is an i.i.d. shock generated by the process $\nu_t \sim \mathcal{N}(\overline{\nu}, \sigma_\nu)$. Default has the benefit that it reduces the level of debt to $\underline{B}$, an exogenous recovery level. For simplicity, we assume that the government continues to borrow immediately after default.[2]

The government's objective is to maximize the present discounted value of the utility derived from consumption by the representative household, net of any default costs,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(C_t, C_t^f, N_t) - D_t \nu_t),$$

where $D_t = 1$ when the government defaults and zero otherwise. The fiscal government borrows from competitive, international lenders that discount the future at a foreign currency rate r^*. The break-even condition for lenders requires that discount bond prices compensate lenders for the expected loss from the default such that

$$q_t B_{t+1} = \frac{1}{1 + r^*} \mathbb{E}_t \left[(1 - D_{t+1}) B_{t+1} + D_{t+1} \underline{B} \right].$$

3 Equilibrium

We now describe the equilibrium of this economy. We consider a Markov equilibrium where the government takes into account that its borrowing and default policies effect

[2] We introduce the iid default cost shock ν_t to ease computation. It smooths out the bond price function and facilitates uniform convergence. In the robustness appendix we document its role in the quantitative results.

the allocations of the private equilibrium and the monetary authority's response. In the beginning of the period, the aggregate state of this economy include the productivity, monetary, and enforcement shocks $s = \{z, m, \nu\}$ and the government debt B. The fiscal government chooses its policy to default and borrowing D, B'. The private equilibrium and monetary authority's response depend on both the state $\{s, B\}$ and on the government choices because they affect government transfers $t(S)$. Let $S = \{s, B, D, B'\}$ be end of the period state relevant for the private equilibrium.

Definition 1. *Private and Monetary Equilibrium. Given state $\{S\}$, the government policy functions for borrowing $B''(s', B') = H_B(s', B')$, default $D'(s', B') = H_D(s', B')$, and transfer function $t(S)$ consistent with the government budget constraint, the symmetric private and monetary equilibrium consists of*

- *Households policies for domestic goods consumption $C(S)$, foreign goods consumption $C^f(S)$, labor $N(S)$, and domestic debt $B^d(S)$,*
- *Intermediate and final goods firms policies for labor $n(S)$, prices $p^d(S)$ and final domestic goods output $Y(S)$ and exports $X(S)$,*
- *The wage rate $w(S)$, domestic nominal interest rate $i(S)$, aggregate domestic price $P^d(S)$, domestic inflation $\pi^d(S)$, and the terms of trade $e(S)$*

such that: (i) *the policies for households satisfy their budget constraint (2) and optimality conditions (3), (4), (5);* (ii) *the policies of intermediate and final goods firms satisfy their optimization problem (6), (7), (9), and (10);* (iii) *export demand (1) is satisfied* (iv) *labor, domestic goods, and domestic bond markets clear, and balance-of-payment constraint is satisfied* (v) *nominal interest rate satisfies the monetary authority interest rate rule (11).*

The labor market clears, so that labor demanded by firms equal labor supplied by households $n = N$. Domestic bonds are in zero net supply in the economy, reflected in the market clearing condition $B^d = 0$. The resource constraint for domestic goods requires that domestic final good output equal domestic consumption and exports net of the adjustment costs

$$C(S) + X(S) + \frac{\phi}{2}(\pi - \overline{\pi})^2 Y(S) = Y(S) \tag{15}$$

where aggregate output $Y(S) = z\, N(S)$.

The balance-of-payments constraint requires that net export equals the net capital outflow which here equals to the government transfer, net of the labor subsidy

$$X(S) - (1 + \tau_f) e(S) C^f(S) = t(S) - \tau w(S) L(S). \tag{16}$$

3.1 Government Recursive Formulation

We now describe the recursive problem of the government, which borrows in international financial markets and can default. The government is benevolent and picks its policies internalizing that its choices affect the private and monetary equilibrium.

The government trades one-period foreign denominated discount bonds with international lenders and can default on its debt. The government starts with debt B and decides on default D and new borrowing B' that carries price $q(s, B')$. The bond price is an endogenous function that depends on the amount of borrowing B' and the shocks s, in a way that compensates lenders for default risk. These risk-neutral lenders discount the future at the international interest rate r^*. The break-even condition for them imply that the bond price schedule satisfies

$$q(s, B') = \frac{1}{1+r^*}\mathbb{E}_{s'|s}\left[(1 - H_D(s', B')) + H_D(s', B')\underline{B}/B'\right]. \tag{17}$$

where $\underline{B}$ is the recovery level, if the government defaults, and $H_D(s', B')$ is the default policy function of the government.

The government internalizes that its choice of borrowing and default effect the private equilibrium. As is standard in New Keynesian models we set that labor subsidy $(1-\tau) = \frac{\eta-1}{\eta}$ to offset the market power of firms in the steady state. and set $(1+\tau_f) = \frac{\rho}{\rho-1}$ to be equal to the static optimal tariff in steady state. [3]

By combining the equilibrium conditions and the government budget constraint, the private and monetary allocations can be summarized with the decision rules for domestic and foreign consumption $\{C(S), C^f(S)\}$, labor $N(S)$, inflation $\pi(S)$, nominal interest rate $i(S)$, and terms of trade $e(S)$, which satisfy the following system of dynamic equations

$$C(S) + e(S)^\rho \xi = \left[1 - \frac{\varphi}{2}\left(\pi(S) - \overline{\pi}\right)^2\right] zN(S) \tag{18}$$

$$e(S)^\rho \xi = e(S)[C^f(S) + B - q(s, B')B'] \tag{19}$$

$$\frac{u_{c^f}(S)}{u_c(S)} = \frac{\rho}{\rho-1}e(S) \tag{20}$$

$$u_c(S) = \beta\, i(S) M(s, B') \tag{21}$$

$$i(S) = R\left[\frac{\pi(S)}{\overline{\pi}}\right]^{\alpha_p} m \quad \text{with} \quad R = \frac{\overline{\pi}}{\beta} \tag{22}$$

[3] By setting this tariff, we neutralize the incentive of the government to use debt to exert market power with respect to the downward-sloping demand for its exports, in steady state.

$$\frac{1}{z}\frac{u_n(S)}{u_c(S)} = 1 + \frac{1}{\eta - 1}\varphi\left(\pi(S) - \overline{\pi}\right)\pi(S) - \frac{1}{u_c(S)zN(S)}F(s, B') \tag{23}$$

where $q(s, B')$ satisfies (17) and the functions $M(s, B')$ and $F(s, B')$ are the expectations in the households' Euler condition and the firms' pricing condition (the NKPC) given by

$$M(s, B') = \mathbb{E}_{s'|s}\frac{u_c(s', B')}{\pi(s', B')} \tag{24}$$

$$F(s, B') = \frac{\beta}{\eta - 1}\mathbb{E}_{s'|s}\left[z'N(s', B')u_c(s', B')\varphi\left(\pi(s', B') - \overline{\pi}\right)\pi(s', B')\right]. \tag{25}$$

These equilibrium conditions are analogous to those arising from the standard New Keynesian small open economy, e.g. Gali and Monacelli (2005). The difference in our model is that the government understands that its choice of borrowing B' and default D, which are elements in S, affect the equilibrium. The equilibrium depends on government choices because current *and* future allocations and prices, as characterized by the system of equations (18) to (23), depend on B' and D. Consider, for example, the impact of increasing government borrowing B'. Through the balance of payments condition (19) increasing borrowing tends to decrease e, an appreciation in the terms of trade. Such appreciation reduces exports X and, through the relative demand condition (20), it increases demand for domestic consumption C relative to foreign consumption C^f. The increase in domestic consumption leads to a rise in wages and hence labor.

The government's choices also determine future state variables, which means that future allocations and prices also depend on current government's choices. These future effects are encoded in the functions $q(s, B')$, $M(s, B')$, and $F(s, B')$, which are the bond price function, the households' expected marginal utility function, and the firms' expected inflation function, respectively. These functions are the marginal changes associated with a future change in state B' taking as given future governments policies $H_D(s', B')$ and $H_B(s', B')$.

We now set up the the recursive problem of the government which follows the quantitative sovereign default literature. The government can choose to default in any period. Let $V(s, B)$ be the value with the option to default. After default, the debt B is reduced to $\underline{B}$, productivity is reduced to z^d, and the government suffers the default cost ν. The value of the option to default is then

$$V(s, B) = \max_{D=\{0,1\}}\left\{(1 - D)W(s, B) + D\left[W(z^d, m, \underline{B}) - \nu\right]\right\} \tag{26}$$

where $D = 1$ in default and 0 otherwise, and $W(s, B)$ is the value of repaying debt

$$W(s, B) = \max_{B'} \left\{ u(C, C^f, N) + \beta \mathbb{E}_{s'|s} V(s', B') \right\} \tag{27}$$

subject to the private and monetary equilibrium which is characterized by conditions (18) through (23), and the break even condition for the bond price schedule (17).

It is convenient to write the default decision of the government as a cutoff rule based on the default cost ν. Given that default costs are i.i.d., the default decision $D(s, B)$ can be characterized by a cutoff default cost $\nu^*(z, m, B)$ at which the repayment value is equal to the default value such that

$$\nu^*(z, m, B) = W(z, m, B) - W(z^d, m, \underline{B}) \tag{28}$$

and $D(s, B) = 1$ if $\nu \leq \nu^*(z, m, B)$ and zero otherwise. Using this cutoff we can then write the repayment value function as

$$W(s, B) = \max_{B'} \left\{ u(C, C^f, N) + \beta \mathbb{E}_{s'|s} \left[W(s', B') - \int^{\nu^*(z', m', B')} \nu d\Phi(\nu)) \right] \right\} \tag{29}$$

We now define the recursive equilibrium for the economy.

Definition 2. *Equilibrium. Given the aggregate state $\{s, B\}$ a recursive equilibrium consists of government policies for default $D(s, B)$ and borrowing $B'(s, B)$, and government value functions $V(s, B)$ and $W(s, B)$ such that*

- *Taking as given future policy and value functions $H_D(s', B')$, $H_B(s', B')$, $V(s', B')$, and $W(s', B')$, government policies for default and borrowing $D(s, B)$ and $B'(s, B)$ solve the government's optimization problem.*
- *Government policies and values are consistent with the future policies and values.*

3.2 Borrowing with Default Risk and Monetary Policy

In this section we characterize the optimal borrowing decision for the government. As described in the recursive equilibrium, the government chooses its borrowing taking into account the effect that borrowing has on the private equilibrium, both contemporaneously and in the future. We manipulate the government's problem and derive its optimality condition for borrowing to illustrate the forces at play. In this derivation we have assumed

that the functions in the government problem are differentiable.[4] Optimal borrowing satisfies the following Euler equation

$$q\left[u_{C_f}\right]\underbrace{(1-\tau_0)}_{\text{monetary}}-\underbrace{\tau_1}_{\substack{\text{monetary}\\ \text{future}}}-\underbrace{\tau_2}_{\text{default}}=\beta\mathbb{E}\left[u'_{C_f}(1-\tau'_0)\right] \tag{30}$$

where the the wedges τ_0, τ_1, and τ_2 are given by following conditions

$$\tau_0=-\theta\frac{\lambda}{u_C} \tag{31}$$

$$\tau_1=\zeta\frac{1}{M(s,B')}\frac{\partial M(s,B')}{\partial B'}+\gamma\frac{1}{u_C Y}\frac{\partial F(s,B')}{\partial B'} \tag{32}$$

$$\tau_2=-u_{Cf}\left(1-\tau_0\right)\frac{\partial q(s,B')}{\partial B'}B'-\beta\mathbb{E}_{s'}\left[\Phi(\nu^*(s',B'))\frac{\partial\nu^*}{\partial B'}\right]. \tag{33}$$

λ is the multiplier on the relative consumption demand of domestic to imported goods (20), ζ is the multiplier on the domestic Euler condition (21), and γ is the multiplier on the New Keynesian Phillips Curve (23).

We label the wedges τ_0 and τ_1 as *monetary* wedges, because one can show that they are nonzero only because of the pricing frictions ($\phi>0$). We label the wedge τ_2 as *default* wedge because it is nonzero even absent any price frictions, due to default risk.

We now discuss the forces that give rise to the monetary wedges. Price frictions in our model are costly because they distort allocations and generate resource costs. In this open economy model with elastic demand for exports, the government understands that its borrowing influences the terms of trade which impact exports and hence domestic production and allocations. The monetary wedge τ_0 in (31) reflects these contemporaneous effects from borrowing.

This result can be seen more sharply if we consider the limit case when price frictions are infinity and inflation is constant. The private and monetary equilibrium in this case is characterized by conditions (18) to (22) and we can show that the monetary wedge τ_0 is equal to

$$\tau_0=\frac{\rho e^{\rho}}{\rho e^{\rho}u_C+1-\theta}\left(u_C-\frac{u_N}{z}\right).$$

This monetary wedge is positive, $\tau_0>0$, when consumption is too low relative to

[4]We do not require this assumption for the computation of the model, nor do we employ the Euler equation derived in this section.

leisure. As is standard in New Keynesian models, price frictions lead to labor wedges that impede the efficient production condition that equates that marginal product of labor to the marginal disutility of consumption relative to labor namely, $u_C = \frac{u_N}{z}$. In our model, this wedge matters for government borrowing because external borrowing affects the terms of trade. In particular, a positive $\tau_0 > 0$ decreases the marginal benefit from borrowing B' because high borrowing tends to appreciate the exchange rate, reduce exports, and lower the demand for domestic goods.

The second monetary wedge τ_1 contains the effect that borrowing has on expected marginal utility $M(z, B')$ and inflation $F(z, B')$ for households and firms in equations (24) and (25) times the multipliers on those conditions, ζ and γ. These terms capture the time-inconsistency problem of the government, arising from wanting to manipulate at time t both expected marginal utility and expected inflation at time $t+1$ with its choice of borrowing B', the resulting level of debt next period. We show below that these functions are increasing in B'. Marginal utility is increasing in the level of debt because with high debt both foreign and domestic consumption are low. Inflation also increases with debt because high debt depreciates the terms of trade boosting exports and production. The monetary wedge is positive, $\tau_1 > 0$, reducing the marginal benefit of borrowing, when the multipliers on the domestic Euler equation and the firms' pricing condition are positive. The domestic Euler equation multiplier ζ is positive when current consumption is too high relative to tomorrow's consumption. This situation gives the government the incentives to reduce borrowing and boost consumption tomorrow. The multiplier on the firms' pricing condition is binding when inflation is too low relative to the marginal cost of production. Such an effect also gives incentives for the government to reduce borrowing and boost exports to increase inflation. This is beneficial also because it smooths out inflation costs over time.

The default wedge τ_2 contains two components. The first component is negative and arises because the government understands that the price for its debt falls with borrowing, $\frac{\partial q(z,B')}{\partial B'} < 0$, as default incentives rise with debt. Such elasticity decreases the marginal benefit from borrowing. The monetary wedge also changes the valuation of such a force. The second component is positive and reflects the discount for the government from not repaying back the debt in case of default.

It is useful to compare our model's Euler equation (30) with an undistorted Euler equation that arises in the standard Gali and Monacelli (2005) model, under market incompleteness.

$$q\, u_{Cf} = \beta \mathbb{E}\left[u'_{Cf}\right]. \tag{34}$$

Our Euler condition (30) collapses to this equation when all wedges are zero. Recall that, given a level of borrowing B' and its price q, the allocations and prices in our model are exactly the same as the allocations in this reference model as both models satisfy the system of equations (18) through (23).

We now derive the Uncovered Interest Parity (UIP) conditions. It is useful to define a few relations between terms of trade, exchange rate, consumer and producer price indices. We can derive the consumer price index as the price of bundle of domestic and imported consumption,

$$P^{\text{CPI}} = \frac{(P^d)^\theta e^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}.$$

The CPI inflation is then

$$\pi^{\text{CPI}} = \pi^\theta \left(\frac{e}{e_-}\right)^{1-\theta}.$$

The real exchange rate e^{CPI} equals the nominal exchange rate ε times the ratio of the world price P^* (set to 1) and P^{CPI},

$$e^{\text{CPI}} = \frac{\varepsilon\, P^*}{P^{\text{CPI}}}.$$

Using these relations, the real exchange rate and nominal devaluation depend on terms of trade and domestic prices,

$$e^{\text{CPI}} = \theta^\theta (1-\theta)^{1-\theta} (P^d)^{1-\theta} e^\theta,$$

$$\frac{\varepsilon'}{\varepsilon} = \frac{e'}{e}\pi.$$

In the reference model, with the undistorted Euler condition for foreign consumption, the standard UIP condition holds. By combining the Euler equation for domestic bond (21) and the one for foreign bonds (34), we can derive the log-linearized UIP as follows

$$\hat{i}_t = \hat{i}_t^f + \mathbb{E}_t\left[\hat{\pi}_{t+1} + \hat{e}_{t+1} - \hat{e}_t\right] \tag{35}$$

where $i^f = 1/q$, the expected nominal devaluation equals $\mathbb{E}_t[\hat{\pi}_{t+1} + \hat{e}_{t+1} - \hat{e}_t]$, and $\hat{x}$ denotes the percentage deviation of variable x from its steady state.

Consider the effects from an increase in the domestic nominal interest rate i_t in the reference model. This UIP condition calls for an expected nominal depreciation. With pricing frictions, this change leads to an expected terms of trade depreciation which means that on impact the terms of trade appreciate. Exports decline due to the appreciation. Production in this economy is depressed, because of the low domestic consumption arising

from the increase in interest rates and also because of the low exports.

The UIP condition in our model is different because of the wedges in the optimal foreign borrowing condition. By combining equations (21) and (30), we can derive an uncovered interest rate parity condition with sovereign default risk, a log-linearized version, as

$$\hat{i}_t = \hat{i}_t^f + \mathbb{E}_t \left[\hat{\pi}_{t+1} + \hat{e}_{t+1} - \hat{e}_t \right] + \mathbb{E}_t \hat{\tau}_{t+1}^{\text{UIP}}, \tag{36}$$

where $\hat{\tau}^{\text{UIP}}$ is the percentage deviation of the composite wedge τ^{UIP} defined as

$$\tau^{\text{UIP}} = \frac{q u_{cf} \left(1 - \tau_0'\right)}{q u_{cf} \left(1 - \tau_0\right) - \tau_1 - \tau_2}.$$

The UIP wedge τ^{UIP} includes all the monetary and default wedges in condition (30).[5]

We now consider the effects from an increase in the domestic nominal interest rate i_t in our model. The wedges in the UIP condition allow for the dynamics of the terms of trade to be different than in the reference model. In particular, the amount of borrowing that the government undertakes impacts the terms of trade and hence alter its dynamics. Less borrowing than in the reference model will lead to an smaller appreciation of the exchange rate which boosts exports. As we see below, this is precisely how the government responds. Facing a high monetary shock, the government in our model borrows less than in the reference.

Moreover, in our model the foreign rate responds to domestic conditions as this rate includes sovereign default risk. In response to the tight monetary shock, the borrowing response will determine the foreign interest rate. If government borrowing falls in response to high nominal rates, then the foreign rate decreases which widens the interest rate differential which alters the dynamics of the allocation. On the other hand, if government borrowing increases, then the foreign rate increases which dampens the interest rate differential and associated responses.

4 Quantitative Analysis

We now describe the parameterization of the model, discuss policy rules, impulse responses, and compare the model implications to the data.

[5] We relegate the detailed derivation to Appendix A.

4.1 Inflation, Nominal Rates, and Spreads for Inflation Targeters

Starting in the late 1990s, many emerging markets have worked towards central bank independence and have adopted an inflation target for monetary policy.[6] Inflation rates in these countries have been in the single digits since the 2000s, which is a dramatic change from the 1980s when many of these countries experienced episodes of very high inflation. At the same time, emerging countries continue to face challenges with their public debt, although debt crises have been dampened as well for many countries.

	Brazil	Mexico	Colombia
Means (%)			
Inflation	6.6	4.3	5.2
Nominal Rates	14.3	6.2	7.9
Spreads	4.5	2.3	3.2
Standard Deviations (%)			
Output	1.6	1.7	1.3
Inflation	2.6	2.9	4.0
Nominal Rates	4.6	2.6	3.6
Spreads	3.5	0.8	1.9
Correlations with Spreads			
Output	-0.16	-0.39	-0.17
Inflation	0.52	0.22	0.34
Nominal Rates	0.79	0.27	0.76

Table 1: Emerging Markets in Inflation Target Era

In this section we present the data of three emerging markets that are inflation targeters: Brazil, Mexico, and Colombia. We focus on data since 2000. We are interested in the connection between of inflation, interest rates, and sovereign spreads. Data are quarterly series. Inflation is computed as a annual change of the CPI. Nominal interest rates are the 3 month rate on government bonds denominated in local currency. Spreads are the EMBI+ spread which are measures of the difference in yields for government bonds denominated in U.S. dollars relative to a U.S. treasury bond. Output is real GDP filtered with an HP filter.

[6]See Roger (2009) for a description of this process and for a list of Emerging Markets that adopted Inflation Targeting.

Table 1 presents means and standard deviations for inflation, nominal interest rates, and spreads as well as correlations for these variables with spreads. Inflation in these three countries have been low with a mean close to 5%. Nominal interest rates and spreads have also been fairly low ranging from 6% to 14% and 2% to 5% respectively. The volatility for inflation, nominal rates, and spreads have also been quite modest. In terms of correlations, inflation and nominal rates are positively correlated with spreads. The co-movement between nominal rates and spreads is strong, specially for Brazil and Colombia and equal close to 0.8. As is typical in emerging market, output is negatively correlated with spreads, see Neumeyer and Perri (2004), although the magnitude of these correlations are weaker than the correlation of spreads with nominal rates.

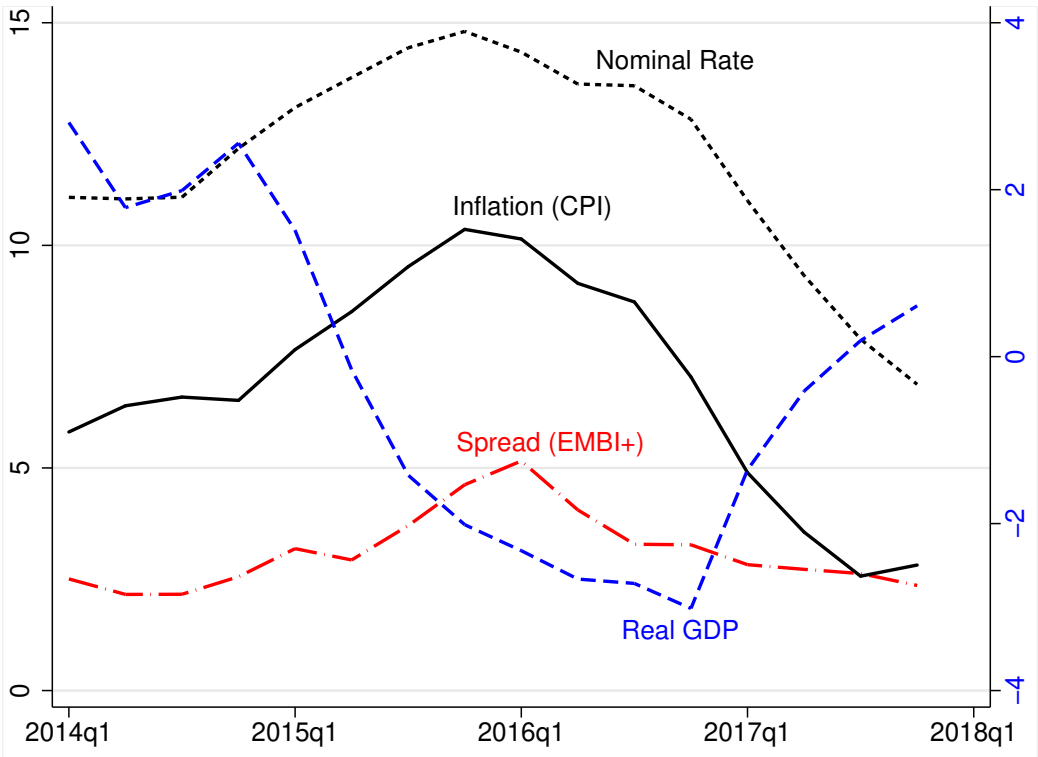


Figure 1: The 2015 Recession in Brazil

We focus our quantitative analysis on Brazil because this is the country that has experienced the largest recession. During 2016, Brazil experienced a severe recession that originated with the decline in commodity prices. As output fell, inflation rose and the central bank tightened policy by raising nominal rates following their inflation target goal. In Figure 1 we plot the dynamics of output, nominal rates, inflation, and spreads from 2014 to 2018. From the beginning of 2015 to mid 2016 output fell about 5%. Inflation increased

from about 6% to about 10% and nominal rates increase from about 11% to about 15% in response to the high inflation. Spreads also increased during this period, from about 2% to 5%. At the end of 2016, the economy recovered with output increasing and inflation, nominal rates, and spreads falling.

The dynamics surrounding the Brazilian event illustrate the correlations observed in the longer time series for Brazil, Mexico, and Colombia. Emerging markets that are inflation targeters raise nominal rates in response to high inflation, just as developed countries do. In emerging markets, however, government spreads rise reflecting elevated default risk during these events.

4.2 Interest Rate Smoothing

For our quantitative results, we extend the model by incorporating interest rate smoothing by replacing the interest rate rule (11) with

$$i_t = \left[R\left(\frac{\pi_t}{\overline{\pi}}\right)^{\alpha_P}\right]^{1-\kappa}[i_{t-1}]^{\kappa}\, m_t. \tag{37}$$

Such a specification is standard in the New Keynesian literature and is aimed at preventing excess interest rate volatility (cf. Clarida et al. (2000)). This extension requires us to include the lag of the domestic nominal rate (i_-) as an additional state variables in S but otherwise leaves all other model equations unaltered.

4.3 Parameterization

The model includes three shocks: productivity z, monetary m, and utility cost ν. We assume that productivity shocks follow an AR(1) process

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_t$$

with ε_t following standard normal distribution. Monetary shocks are log-normal distributed with mean zero and standard deviation σ_m. Utility cost of default are normally distributed with mean $\bar{\nu}$ and standard deviation σ_ν. Following Chatterjee and Eyigungor (2012), we assume that the productivity after default takes the following form

$$z^d(z) = z - \max\left\{0, \lambda_0 z + \lambda_1 z^2\right\}.$$

We calibrate our model to quarterly Brazilian data from 2004 to 2017. Table 2 presents

all the parameters values with the source or targeted moments. There are two sets of parameters. For the first set, we assign their values directly, by relying on one-to-one mappings with the data or reference values in the literature. The second set is unique to Brazil and we choose them jointly to match a set of data moments. The first set of parameters include the Frisch elasticity ζ, the share of domestic consumption θ, export demand elasticity ρ, international interest rate r^*, the goods elasticity η, and the persistence of TFP shock ρ_z. For the Frisch elasticity, we choose a value of 0.72, within the broad range of values used in the IRBC/Open Macro literature, and document robustness in Appendix C. The import share of Brazil is 15 percent, we therefore set θ as 0.85. Export demand elasticity ρ is chosen to be 5 following Devereux et al. (2018). This number is also within the plausible range of estimates in the trade elasticity literature. The international risk-free rate is 2%, consistent with US Treasury yields. The elasticity of substitution between goods varieties η is chosen to be 6, as is standard in the literature, inducing a 20% markup. Given that we are considering a short horizon of the data and that the employment data of Brazil has many missing values, it is difficult to estimate precisely the TFP process. Instead, we set ρ_z as 0.95 and estimate the volatility of the standard error, σ_z, to match the volatility of Brazilian output.

The second group includes the discount factor β, inflation target $\overline{\pi}$, Taylor rule parameter α_p, the Rotemberg adjustment cost φ, and the shock parameters. We choose them jointly so that the model generates the observed moments. Roughly speaking, the volatility of TFP shock is closely related to the output volatility. Under the Taylor rule and the Rotemberg adjustment cost, it is costly to deviate from the inflation target $\overline{\pi}$, and thus the average inflation rate helps pin down $\overline{\pi}$. Both α_p and the Rotemberg adjustment cost affects the inflation volatility, but α_p matters more for the relative volatility of nominal rate to inflation. The standard deviation of utility cost σ_ν and the productivity loss parameters λ_0 and λ_1 all matter for the mean spread and volatility of spread. We therefore target the mean and volatility of EMBI in the data. Financial frictions affect risk sharing and we therefore target the comovement of consumption and export with output. Domestic interest rate target is chosen to be 1.05 to match the average nominal rate of 14.3% in the data.

4.4 Policy Rules

Before describing the model time series, we illustrate the model mechanisms by describing policy rules and the equilibrium functions.

Assigned Parameters		
Share of domestic goods	$\theta = 0.85$	Import share of Brazil
Export demand elasticity	$\rho = 5$	Literature: Devereux et al. (2018)
International rate	$r^* = 2\%$	US Treasury yields
Goods elasticity	$\eta = 6$	Literature: 20% markup
Frisch elasticity	$\zeta = 0.72^{-1}$	Literature, Appendix C
Persistence of TFP shock	$\rho_z = 0.95$	Literature, Appendix C
Export demand level	$\xi = 1$	Normalization
Interest rate smoothing	$\kappa = 0.6$	Literature
Natural rate	$R = \overline{\pi}/\beta$	Equilibrium condition

Parameters from Moment Matching		*Brazilian Data, 2004–2017*
Discount factor	$\beta = 0.99$	Debt service to GDP = 6%
Inflation target	$\overline{\pi} = 1.013$	Mean inflation = 5.9%
Inflation weight in rule	$\alpha = 1.125$	Inflation volatility = 1.8%
Rotemberg adjustment	$\varphi = 12$	Mean nominal rate = 11.2%
Std of TFP shock	$\sigma_z = 0.55\%$	GDP volatility = 1.9%
Std. of monetary shock	$\sigma_m = 0.31\%$	Nominal rate volatility = 2.2%
Productivity in default	$\lambda_0 = -0.275$	Mean govt spread= 2.6%
	$\lambda_1 = 0.3$	Govt spread volatility = 1%
Std. of default cost	$\sigma_\nu = 0.675\%$	corr(GDP, spread) = −0.40
Recovery level	$\underline{B} = 0.2$	Brazil recovery rate = 86%

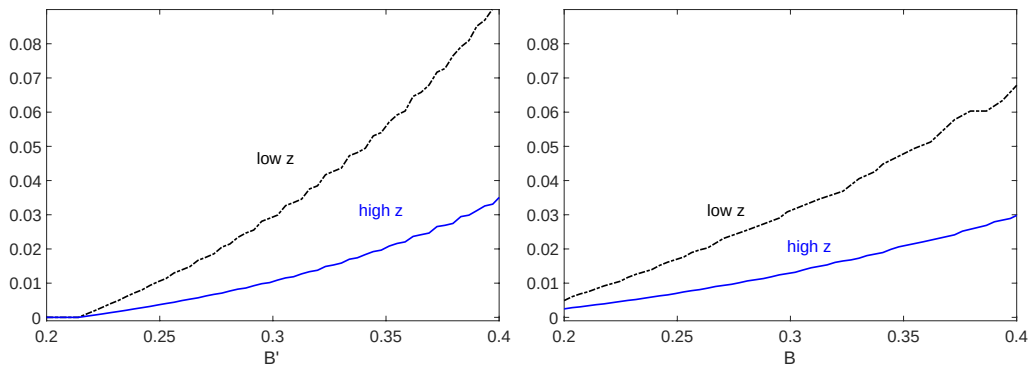
Table 2: Parameter Values

We start by describing the spread function which is an important force driving allocations. We define government spreads as the inverse of the bond price relative to the risk free rate

$$\text{spread}(s, B') = \frac{1}{q(s, B')} - (1 + r^*). \tag{38}$$

In our model, and in the standard sovereign default models, the government's default incentives increase with the level of debt B and with low productivity. The spread function reflects default risk and hence increases with borrowing B' and decreases with productivity. Panel (a) in Figure (2) plots the spread function, as a function of B' relative to average output, for two z levels, a high level z_H and a low level z_L. These functions are independent of the monetary shock m and default cost ν because these shocks are i.i.d. Panel (a) shows that spreads increase with borrowing. When productivity is low, spreads are higher and

increase faster with borrowing. Panel (b) plots the equilibrium government spread as a function of the state of debt B. The equilibrium spread is the spread taking into account the optimal borrowing rule spread$(s, B'(s, B))$. As the plot shows, equilibrium spreads are increasing in the level of debt but the optimal borrowing dampens the dependency of spreads on debt.

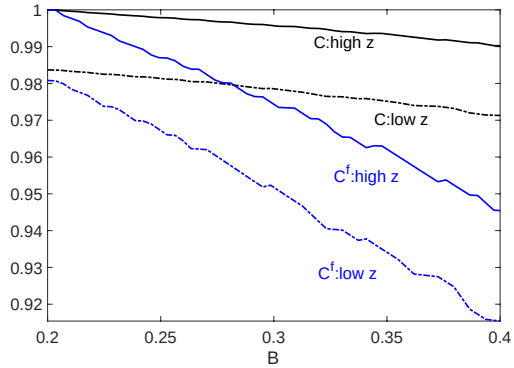


(a) Spread Function (b) Equilibrium Spread

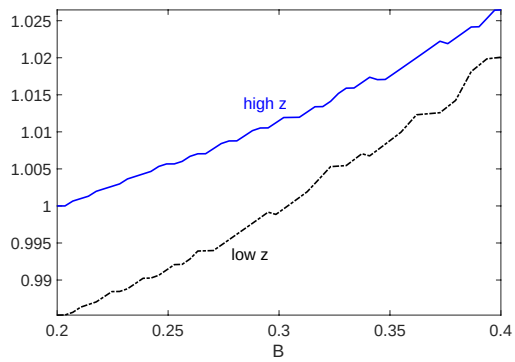
Figure 2: Spread Function an Equilibrium Spread

We now present the policy rules for the other allocations and prices in our model. Recall that the allocations in our model depend on in equilibrium on the government's state $s = \{z, m, \nu, B\}$ which contain the shocks to productivity, monetary, and default costs, as well as the endogenous level of debt. In Figure 3 we plot the policy rules for consumption, output, inflation, nominal rates, and terms of trade as a function of the debt level (relative to average output) for two levels of the productivity shock, a high level z_H and a low level z_L holding constant the monetary and enforcement shock at their median levels.

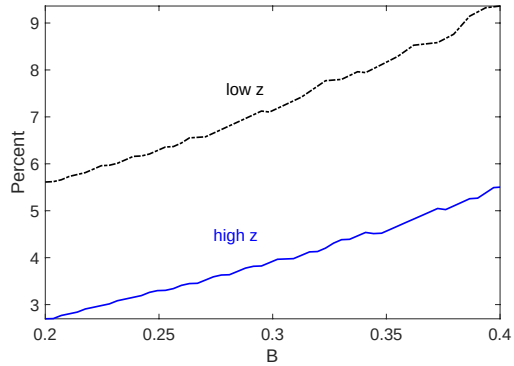
Panel (a) plots domestic consumption and imported consumption as a function of debt and shows that consumption falls with debt. In our economy with financial restrictions arising from default risk high levels of debt are associated with larger net debt repayment $B - qB'$, which requires larger net exports to pay the debt $X - eC^f$. In equilibrium exports increase and imports decrease to pay the larger debt. The decline in domestic consumption with larger debt occurs precisely because such decline boosts exports. Domestic and imported consumption are lower with low productivity and they decline faster with debt with low productivity. The faster decline of consumption with debt arises because the bond price function is tighter with low productivity due to higher default risk. Panel (b) shows that that in contrast to consumption, output increase with debt. Output increase with debt for similar reasons: high debt coupled with restricted borrowing creates the need for larger



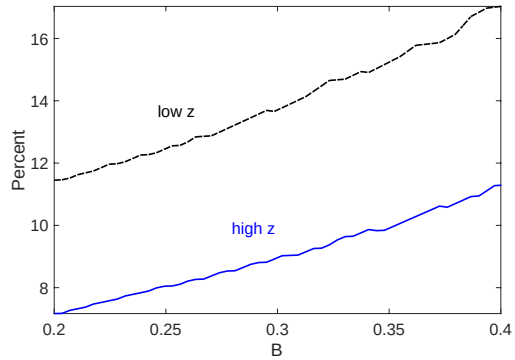
(a) Consumption



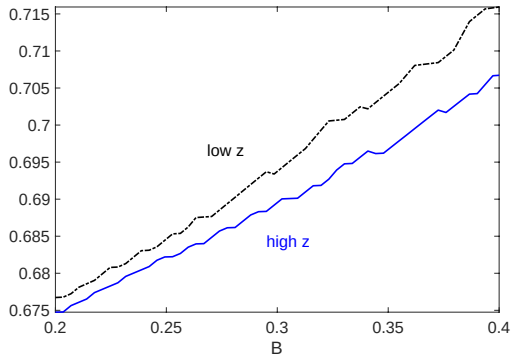
(b) Output



(c) Inflation

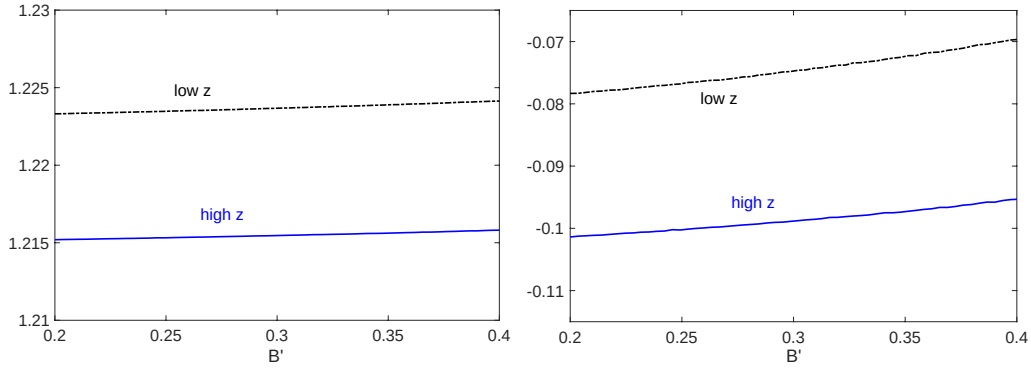


(d) Nominal Interest Rates



(e) Terms of Trade

Figure 3: Consumption, Output, Inflation, Nominal Rates, Terms of Trade



(a) Expected Marginal Utility (M) (b) Expected Inflation (F)

Figure 4: Expected Marginal Utility Functions and Expected Inflation Function

exports to pay the debt which leads to an increase in labor supply through a wealth effect. Output lower with low productivity and increases at a faster rate with debt.

In Panel (c) we plot inflation as a function of debt. Inflation increases with debt because unit costs for firms are higher. Higher unit costs reflect the larger labor used to produce exports to pay off the debt. In Panel (d) we see that nominal interest rates also increase with debt. In response to high inflation, interest rates increase because of the inflation target rule. Low productivity is associated with higher inflation and nominal interest rates. Unit costs are higher with low productivity leading to an increase in inflation.

Panel (e) shows that the terms of trade increase (depreciate) with debt. Higher debt is associated with smaller imported consumption and larger exports, both of which increase the price of imports relative to exports. Terms of trade are also high with low productivity.

In our model with lack of commitment, the government not only takes as given the bond price function $q(s, B')$ but also takes as given two additional functions that shape allocations, namely, the households expected marginal utility function $M(s, B')$ and the expected inflation function $F(s, B')$ in equations (21) and (23). In particular, the government understand that its choice of borrowing today can influence the the consumption and inflation that households and firms will choose tomorrow. Figure 4 plot the expected marginal utility and inflation functions as a function of the borrowing choice B' for the two levels of productivity. Expected marginal utility relative to inflation $M(s, B')$ is increasing in borrowing B' because the decrease in domestic consumption with debt is larger than the increase in inflation. Expected inflation times marginal utility is increasing with B' because inflation increases with debt and consumption decreases with debt. Expected marginal utility and expected inflation are decreasing in productivity.

4.5 Impulse Response Functions

We present the impulse response functions for key endogenous variables to i) a contractionary monetary shock m and ii) to a low productivity realization z. To highlight the mechanism of default risk and time-consistency problem of the government, we contrast our model with a reference model, a version of Gali and Monacelli (2005) with incomplete markets. The allocation in the reference model satisfies the same Private and Monetary Equilibrium conditions as the benchmark, equations (18) to (23). The key difference with the benchmark is that, in the reference model international, borrowing is governed by the undistorted Euler condition (34) while in the benchmark borrowing is governed by the Euler with wedges, equation (30). We close the reference model with a debt-elastic interest rate as in Schmitt-Grohé and Uribe (2003), to ensure stationarity. The bond price function depends the borrowing choice B' as follows:

$$\frac{1}{q^*(B')} = R + \phi_B \left[\exp(B' - \overline{B}) - 1\right]. \tag{39}$$

where ϕ_B, and $\overline{B}$ are constants governing the speed of return to steady state and the average level of debt, respectively. To parameterize the reference model, we set the coefficient ϕ_B to a small value in the range of 10^{-5}, essentially allowing for a near-constant interest rate. We also choose $\overline{\pi}$ and $\overline{B}$ to generate the same average inflation and debt-to-GDP as in the benchmark. All other parameters are the same as in the benchmark model.[7]

We construct the impulse response functions in our nonlinear model following Koop et al. (1996). We simulate a panel of $50,000$ time series for 5000 periods. For the first 4950 periods, the shocks follow their underlying Markov chains so that the cross-sectional distribution converges to the ergodic distribution of the model. In period 4951, the *impact period*, normalized to 0 in the plots, we alter shocks for all units by the same amount. From period 4952 onward, the shocks resume following their Markov processes. The impulse responses plot the average, across the time series.[8]

Monetary Shocks Figure 5 plot the responses to an increase in the i.i.d. monetary shock m of aggregate output, domestic consumption, imports, inflation, the nominal interest rate, terms of trade, debt, and spreads. The solid blue lines are for the benchmark model while

[7]We solve the reference model using Dynare 4.5 for MATLAB, with a first-order log-linear approximation of the equilibrium conditions.

[8]The impulse responses are computed over all 50,000 series, including those with defaults. Discarding defaults from the cross-section average does not alter the properties of the IRFs.

the red dashed lines are the reference model.

The monetary shock leads to an increase in the nominal interest rate of roughly 2.5% on impact, as shown in Panel (e). Due to this tightening of monetary policy, on impact, aggregate output in Panel (a) declines by 0.4%, domestic consumption drops by 1.3%, and imports shrinks almost 2.5%. The increased nominal interest rate leads to a reduction in inflation of about 1.2%. Terms of trade depreciates by about 0.6% and the spread declines 0.7% due to lower government borrowing, in Panel (g).

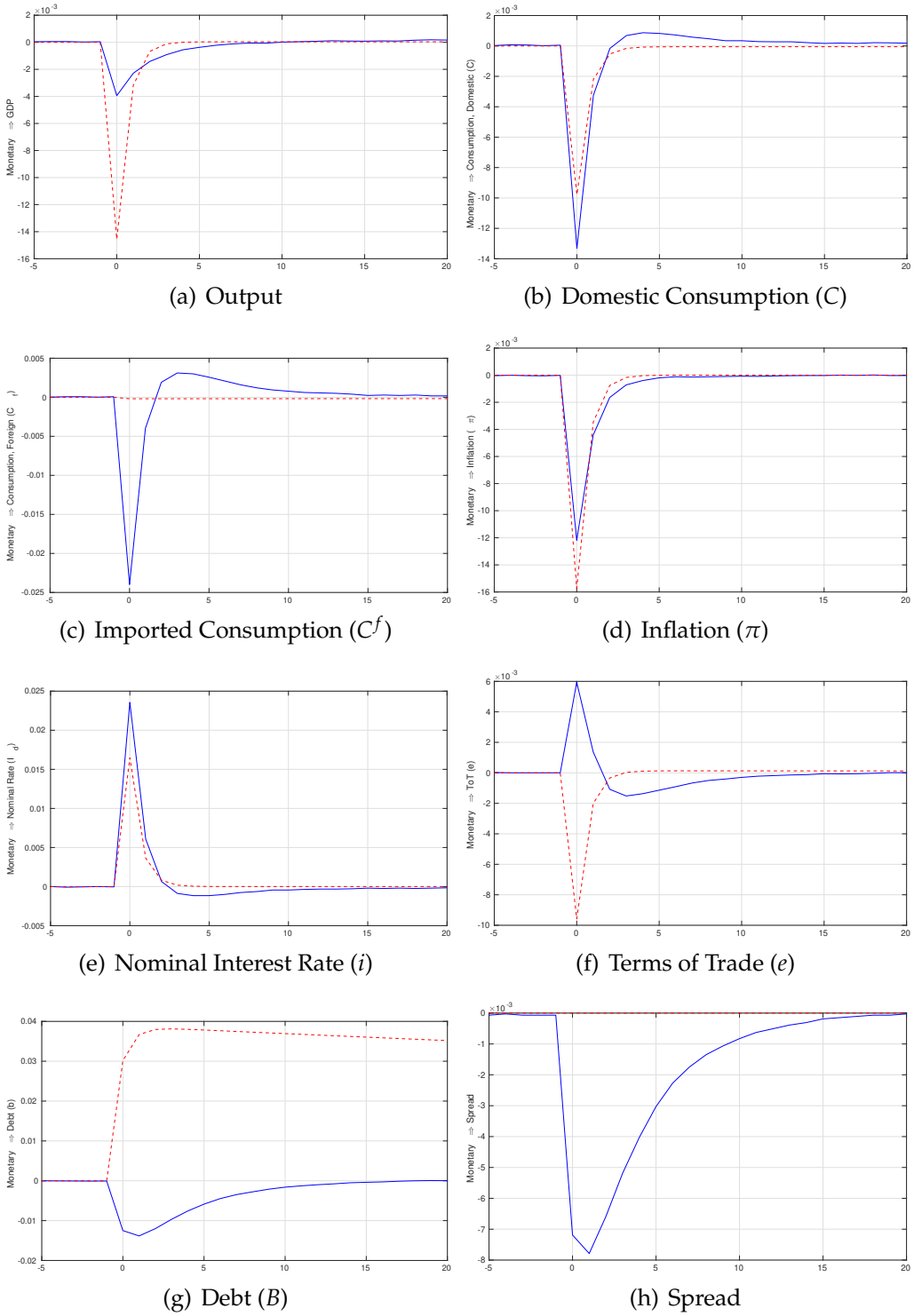
These responses of output, domestic consumption, inflation, and of the nominal rate are qualitatively similar to those in the standard New Keynesian small open economy models, e.g. Gali and Monacelli (2005), as seen from the comparison with the dashed lines of the reference model in Figure 5. In both models, tight monetary policy depresses consumption. The reduction in domestic consumption lowers the unit cost of production and leads to low inflation and output, in this demand-driven economy.

The two models differ in the behavior of imports, terms of trade, borrowing, and sovereign spread. The reference model provides almost perfect risk sharing. In this model, following a contractionary monetary shock, the country increases its debt dramatically, to smooth out imports. As shown in Panel (c), imports in the reference model are essentially flat. Higher inflows of foreign goods, supported by borrowing, lower the relative price of foreign goods and lead to an appreciation of the terms of trade.

In our benchmark model, borrowing also responds to the incentive to smooth imported consumption but the presence of sovereign debt with default risk restricts international borrowing possibilities and enables the government to impact the private equilibrium. In Panel (g), we show that government's borrowing and associated spread fall on impact. This is because, in addition to the standard forces in the reference model, the government understands the consequences of its borrowing decision on the terms of trade and future outcomes in the domestic economy. The government reduces debt in order to induce a depreciation to stimulate exports and production, at the expense of a much greater reduction in imported consumption.

Tight monetary policy will cause an increase in the monetary wedge τ_0, induced by pricing frictions. In the reference model output falls substantially (over 1.4%) yet imported consumption is essentially flat. In contrast, in the benchmark model the government partially mitigates the contraction through a reduction in debt. The net result is that output falls by only about one quarter of the fall in reference but imported consumption falls almost 2.5%.

The terms of trade present different dynamics across the two models. In the reference,



(a) Output (b) Domestic Consumption (C)

(c) Imported Consumption (C^f) (d) Inflation (π)

(e) Nominal Interest Rate (i) (f) Terms of Trade (e)

(g) Debt (B) (h) Spread

Figure 5: Impulse Responses to Monetary Rule Shock. Benchmark model in solid blue, Reference in dashed red.

the standard Unconvered Interest Rate Parity (UIP) logic calls for an appreciation on impact, followed by expected depreciation. In our benchmark, the monetary and default wedges in the UIP with sovereign default risk (36) invert the response. Terms of trade depreciate on impact, followed by expected appreciation.

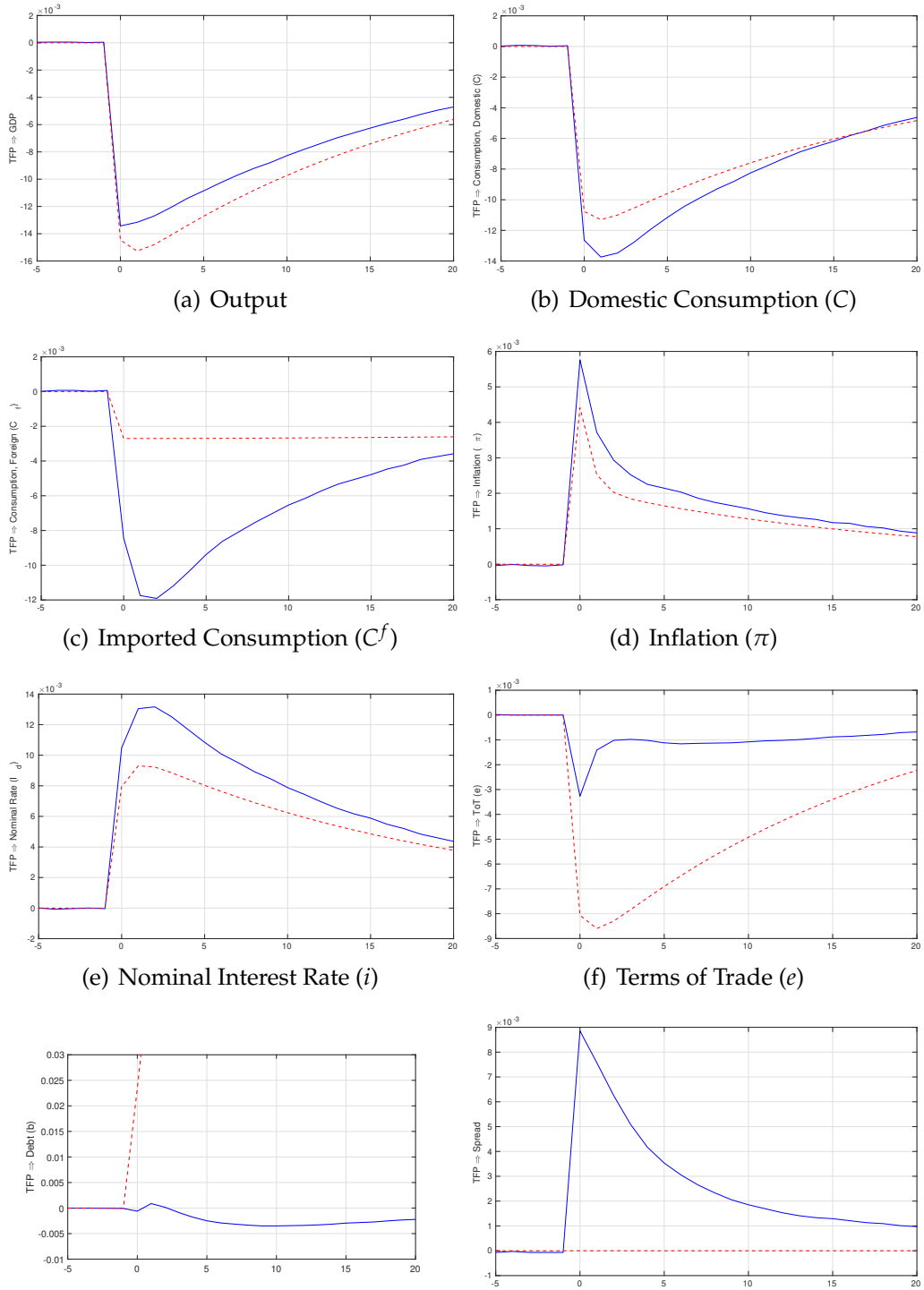
The decline of inflation is larger on impact and shorter-lived in the reference model relative to the benchmark. These different dynamics imply a higher resource cost from inflation in the reference. The additional smoothing of inflation in the benchmark is enabled by the reduction in borrowing, due to the increase in the future monetary wedge τ_1.

After the impact period, the allocations and prices recover in both models. Nevertheless, monetary shocks lead to more persistent dynamics in our model, because of the endogenous persistence of debt due to default risk and incentives to smooth inflation and output.

Productivity Shocks Figure 6 plots the impulse responses for the same variables to a productivity shock z, a decline of about 1.5%, for both the benchmark and reference model. We first discuss the responses in the reference model and then turn to the IRFs of the benchmark model.

The red dashed lines in Figure 6 plots the IRFs of the reference model. Panel (a), (b), and (c) plot aggregate output, domestic consumption, and imports respectively. As is standard, low productivity reduces output and consumption. In Panel (d) we see that, as is typical in New Keynesian models, inflation rises with low productivity, about 0.4%, since firms face higher unit costs. The response of the domestic nominal rate is in Panel (e). It rises about 0.8% due to the response called for by the interest rate rule, when facing higher inflation. Given the loose borrowing schedule, unaffected by the negative shock, the economy take on additional foreign debt (see Panel (g)), to smooth out imports. The inflow of foreign goods leads to an appreciation of the terms of trade, i.e. e decreases.

Qualitatively, the benchmark model has similar responses in output, consumption, domestic imports, inflation and nominal interest rate. The magnitudes, however, are different. In the benchmark, GDP falls less, 1.3% versus 1.5% for the reference. Imports falls by 0.6% more in the benchmark model. Inflation and the nominal rate increase more on impact, an extra 0.15% increase for inflation and an extra 0.2% for the nominal rate. Moreover, the terms of trade appreciation is much more modest in the benchmark, in contrast to the reference model. These dynamics reflect the presence of the default risk in the benchmark model. Low productivity tightens the borrowing schedule and leads to higher spreads even as the government reduces the debt, in the solid blue lines of Panels (g) and (h). The need to reduce indebtedness, contrasted with the expansion in debt in



(a) Output (b) Domestic Consumption (C)

(c) Imported Consumption (C^f) (d) Inflation (π)

(e) Nominal Interest Rate (i) (f) Terms of Trade (e)

(g) Debt (B) (h) Spread

Figure 6: Impulse Responses to Productivity Shock. Benchmark model in solid blue, Reference in dashed red.

the reference, will imply higher use of the labor input and thus a smaller drop in output. Such relative high labor usage in the benchmark drives up the unit cost, resulting in higher inflation. In response to the increased inflation in the benchmark, the monetary authority raises the nominal rate even higher than in the reference model.

The dynamics of the terms of trade in Panel (e) are also shaped by tight external financial conditions, due to default risk. With high spreads, the government cuts its debt with the understanding that the country will export more. An alternative way to view the pattern of appreciation is that with fewer foreign goods flowing into the country domestic goods are relatively abundant.

In summary, low productivity leads to a decline in output and consumption, both domestic and imported. Productivity shocks generate positive co-movements between inflation, the nominal interest rates and bond yields, as sovereign spreads are countercyclical. Moreover, the comparison of the two models highlights the transmission of sovereign spread to monetary policy. Sovereign default risk induces additional volatility in consumption and inflation, so that the interest rate rule calls for more aggressive monetary tightening, i.e. larger responses of the nominal rate to shocks.

4.6 Second Moments

Table 3 reports the mean and standard deviation of key variables as well as their correlations with sovereign spread for the data, the benchmark model, and the reference model.

Overall, our moment matches well the Brazilian data. The mean inflation, nominal interest rate and spread are close to the data. The volatility of inflation and the nominal rate are comparable but slightly higher in the model. The volatility of the spreads in the model is in line with the data. Both the model and the data exhibit inflation that is positively correlated with spread, the pattern is somewhat stronger in the model. The correlation between spread and nominal rate is 0.8 in the data and 0.4 in the model. The value for the model reflects the relative mix of productivity and monetary shocks, with productivity shocks inducing a higher correlation and monetary ones the opposite. As is usual in the sovereign default literature, spreads are countercyclical.

Table 3 also reports the moments from the reference model. Here without default risk, spreads are trivially zero. By construction, the reference model has the same average inflation and nominal rate as the benchmark. The volatility of nominal interest rate is, however, only about half of that in the benchmark. Inflation is also less volatile in the reference model than that in the benchmark. (Note that all parameters are kept the same

	Brazil	Benchmark	Reference
Means (%)			
Inflation	5.9	6.1	6.1
Nominal Rate	11.2	11.6	11.6
Spread	2.6	2.7	—
Standard Deviations (%)			
Output	1.9	2.0	1.9
Inflation	1.8	2.1	2.0
Nominal Rate	2.2	2.1	1.4
Spread	0.9	0.9	—
Correlations with Spread			
Output	-0.4	-0.5	—
Inflation	0.5	0.7	—
Nominal Rate	0.8	0.4	—

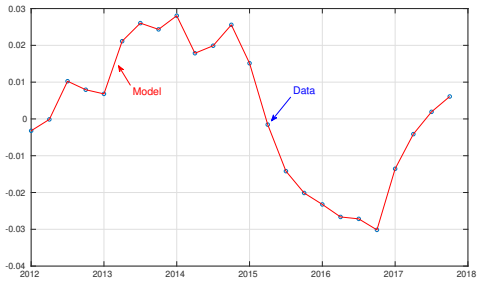
Table 3: Moments

between the two models.)

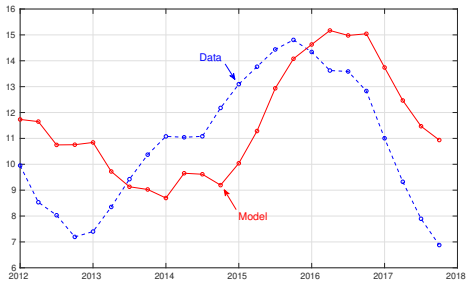
This comparison shows that in an environment with default risk, a central bank engaged in inflation targeting must need to implement a more aggressive interest rate policy. This is because inflation responds more to productivity shock with default risk, requiring a larger change in the nominal interest rate to keep inflation close to the target, as seen in the IRFs to productivity and monetary shocks.

5 Brazil Event

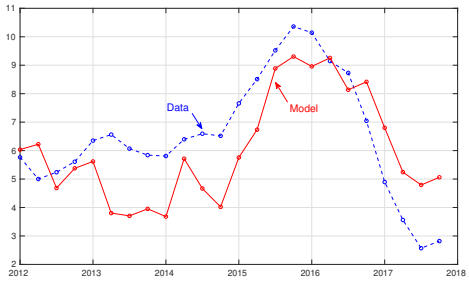
We now compare the quantitative implications of the model to the Brazil time series, during the 2012–2018 subsample, with a focus on the 2015 recession. The model produces similar increases of nominal rate, inflation, and spreads as in the data. To highlight the impact of monetary policy on sovereign default risk, we also conduct a counterfactual experiment where the nominal rate is fixed at its 2015 level through the recession. The experiment shows that if the central bank would have been more dovish during this episode, Brazil would have faced much higher inflation and spreads.



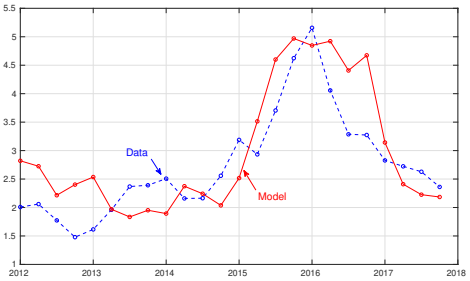
(a) Output



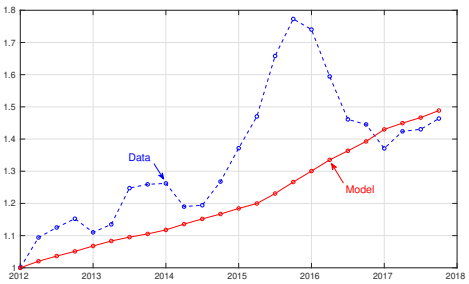
(b) Nominal Interest Rate



(c) Inflation

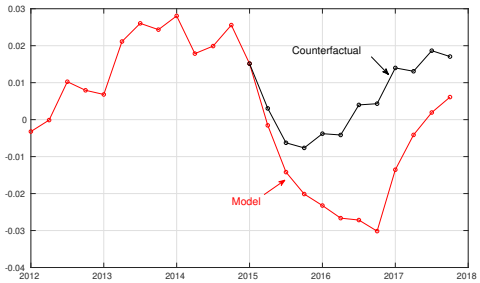


(d) Sovereign Spread

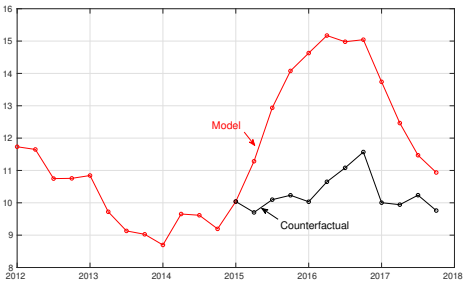


(e) Nominal Exchange Rate

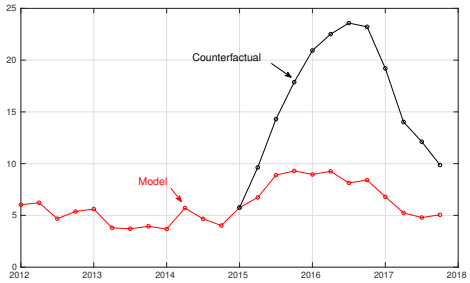
Figure 7: Event: Brazil



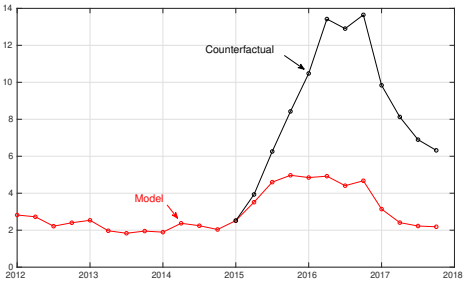
(a) Output



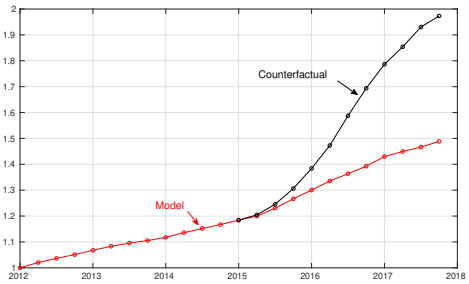
(b) Nominal Interest Rate



(c) Inflation



(d) Sovereign Spread



(e) Nominal Exchange Rate

Figure 8: Event: Counterfactual Experiment

To replicate the Brazilian event in the model, we feed in productivity shocks such that the time path of output in the model matches the one in the data. We choose an initial level of debt given by the mean of the limiting distribution and we hold the monetary shock m at its mean level. We then compare the predictions of the model for inflation, the nominal interest rate, government spreads, and nominal exchange rates to the data.

The blue lines with circle markers in Figure 7 represent the series in the data. Brazil experienced a recession from 2014 to mid 2016, and GDP declines from 3% above trend to 3% below trending, a 6% decline in total. It then recovers after 2016Q3. During this period, inflation increases by 4%, the nominal rate increases by 2%, and spreads rise from about 2% to 5%. When GDP recovers after 2017, inflation, nominal rate, and spread all fall.

The red lines in Figure 7 are the corresponding series in the model. To match the dynamics of output, the model requires that the underlying productivity shock first decreases from 2014 to late 2016 and then recovers. This implies that during the recession the unit cost of production increases, leading to an increase in inflation. Monetary policy responds to this high inflation with a hikes in the nominal rate. Low productivity also drives up the sovereign's default risk. Quantitatively, the model matches the peak of inflation and spreads during the recession, around 10% for inflation and 5% for the spread. The model also produces the observed recovery from 2016Q3 onward, inflation decreases about 6% and spread drops by about 2%.

The model delivers an overall nominal depreciation of the exchange rate comparable with the data, of about 50% but misses the high volatility of the data. Our model's Armington trade structure exhibits the real exchange rate puzzles widely discussed in the literature, e.g. Obstfeld and Rogoff (2001).

To understand the impact of monetary policy on fiscal policy and the resulting sovereign default risk, we conduct a counterfactual experiment with a dovish central bank. In this alternative scenario, instead of following the policy called for by its inflation target and the interest rate rule, the central bank keeps a low nominal interest rate, similar to its 2015 level at the start of the recession. We feed in a sequence of monetary shocks so keep nominal rates near this initial value. The counterfactual series are plotted in black in Figure 8. These expansionary monetary shocks induce lower nominal interest rates and thus stimulate consumption. This raises the unit cost of production, which in turn generates high inflation. In 2017, the inflation rate in the counterfactual scenario would be more than 10 percent higher than the benchmark case. Lower nominal interest rate from the dovish central bank also leads to a higher spread, since the government increases its borrowing, to shift future consumption to the current period. The exchange rate therefore depreciated even further.

In summary, our model matches well the Brazilian downturn around 2016 and 2017 in both real and nominal terms. Our model also provides an good laboratory to understand the interplay between monetary and fiscal policy. Had Brazil's central bank deviated from its pursue of price stability, the recession would have been milder but at a cost of much higher inflation and a deeper debt crisis.

6 Conclusion

[TODO]

References

Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2013, October). Crisis and commitment: Inflation credibility and the vulnerability to sovereign debt crises. Working Paper 19516, National Bureau of Economic Research. 1

Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics 69*(1), 64–83. 1, 1

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review 98*(3), 690–712. 1, 1

Arellano, C., Y. Bai, and S. Lizarazo (2017, November). Sovereign risk contagion. Working Paper 24031, National Bureau of Economic Research. 1

Arellano, C. and A. Ramanarayanan (2012). Default and the maturity structure in sovereign bonds. *Journal of Political Economy 120*(2), 187–232. 1

Bacchetta, P., E. Perazzi, and E. van Wincoop (2018). Self-fulfilling debt crises: What can monetary policy do? *Journal of International Economics 110*, 119 – 134. 1

Bianchi, J., P. Ottonello, and I. Presno (2018, March). Fiscal policy, sovereign risk and unemployment. 1

Bocola, L. (2016). The pass-through of sovereign risk. *Journal of Political Economy 124*(4), 879–926. 1

Calvo, G. A. (1988). Servicing the public debt: The role of expectations. *The American Economic Review 78*(4), 647–661. 1

Chatterjee, S. and B. Eyigungor (2012). Maturity, indebtedness, and default risk. *American Economic Review 102*(6), 2674–2699. 1, 4.3

Christiano, L. J., M. Trabandt, and K. Walentin (2011). Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics and Control 35*(12), 1999 – 2041. Frontiers in Structural Macroeconomic Modeling. 1

Clarida, R., J. Galí, and M. Gertler (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory*. *The Quarterly Journal of Economics 115*(1), 147–180. 4.2

Corsetti, G. and L. Dedola (2016). The mystery of the printing press: Monetary policy and self-fulfilling debt crises. *Journal of the European Economic Association 14*(6), 1329–1371. 1

Corsetti, G., L. Dedola, and S. Leduc (2010). Chapter 16 - optimal monetary policy in open economies. Volume 3 of *Handbook of Monetary Economics*, pp. 861 – 933. Elsevier. 1

Cuadra, G., J. M. Sanchez, and H. Sapriza (2010). Fiscal policy and default risk in emerging markets. *Review of Economic Dynamics 13*(2), 452 – 469. 1

Devereux, M. B., E. R. Young, and C. Yu (2018). Capital controls and monetary policy in sudden-stop economies. *forthcoming Journal of Monetary Economics*. 1, 4.3

Eaton, J. and M. Gersovitz (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *Review of Economic Studies 48*(2), 289–309. 1

Fanelli, S. (2017). Monetary policy, capital controls, and international portfolios. 1

Farhi, E. and I. Werning (2012, June). Dealing with the trilemma: Optimal capital controls with fixed exchange rates. Working Paper 18199, National Bureau of Economic Research. 1

Gali, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies 72*(3), 707–734. 1, 1, 3.1, 3.2, 4.5, 4.5

Hatchondo, J. C., L. Martinez, and C. Sosa-Padilla (2016). Debt dilution and sovereign default risk. *Journal of Political Economy 124*(5), 1383–1422. 1

Kondo, I., F. Perri, and S. Hur (2016). Inflation, Debt, and Default. 2016 Meeting Papers 1610, Society for Economic Dynamics. 1

Koop, G., M. H. Pesaran, and S. M. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics 74*(1), 119–147. 4.5

Lorenzoni, G. and I. Werning (2013, July). Slow Moving Debt Crises. NBER Working Papers 19228, National Bureau of Economic Research, Inc. 1

Na, S., S. Schmitt-Grohé, M. Uribe, and V. Z. Yue (2014). The Twin Ds: Optimal Default and Devaluation. NBER Working Paper 20314, National Bureau of Economic Research. 1

Neumeyer, P. A. and F. Perri (2004). Business cycles in emerging economies: the role of interest rates. *Journal of Monetary Economics 52*(2), 345–380. 4.1

Nuno, G. and C. Thomas (2018). Monetary policy and sovereign debt vulnerability. 1

Obstfeld, M. and K. Rogoff (2001, January). *The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?*, pp. 339–412. MIT Press. 5

Pitchford, R. and M. L. Wright (2012). Holdouts in sovereign debt restructuring: A theory of negotiation in a weak contractual environment. *The Review of Economic Studies 79*(2), 812–837. 1

Roger, S. (2009, October). Inflation Targeting at 20 - Achievements and Challenges. IMF Working Papers 09/236, International Monetary Fund. 6

Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy 90*(6), 1187–1211. 1, 2.3

Schmitt-Grohé, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*(1), 163–185. 4.5

Schmitt-Grohé, S. and M. Uribe (2007). Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics 54*(6), 1702 – 1725. 1

Sunder-Plassmann, L. (2018). Inflation, default and sovereign debt: The role of denomination and ownership. 1

Yue, V. Z. (2010). Sovereign default and debt renegotiation. *Journal of International Economics 80*(2), 176 – 187. 1

A Deriving Uncovered Interest Parity with Default

We can derive the uncovered interest parity under default (UIPD) through the Euler equation for domestic bond (21) and the one for foreign bond (30). We can rewrite foreign bond Euler (30) as

$$1 = \frac{1}{q}\beta\mathbb{E}_{s'}\left[\frac{u_{cf}(s',B')}{u_{cf}}\tau^{\text{UIP}}(s',B')\right] \tag{40}$$

where the composite wedge τ^{UIP} includes all the three wedges in the Euler/

$$\tau^{\text{UIP}}(s,s',B') = \frac{qu_{cf}\left(1-\tau_0(s',B')\right)}{qu_{cf}\left(1-\tau_0(s)\right)\tau_1 - \tau_2}$$

We can also rewrite the domestic bond Euler (21) as

$$1 = \beta i(S)\mathbb{E}\left[\frac{u_c(S')}{u_c(S)\pi(S')}\right]. \tag{41}$$

Equating equation (40) and (41), we have

$$\beta i(S)\mathbb{E}\frac{u_c(S')}{u_c(S)\pi(S')} = \frac{1}{q}\beta\mathbb{E}_{s'}\left[\frac{e(s',B')u_c(s',B')}{eu_c}\tau^{\text{UIP}}(s',b')\right].$$

Let $i^f = 1/q$ and $\hat{x}$ be the percentage deviation of x from a steady state. We log-linearize the above equation and get

$$\hat{i}_t = \hat{i}_t^f + \mathbb{E}_t[\hat{\pi}_{t+1} + \hat{e}_{t+1} - \hat{e}_t] + \mathbb{E}_t\hat{\tau}_{t+1}^{\text{UIP}}$$

B Numerical Implementation

We solve the model with discrete value function iteration methods. The state variables are the two shocks (z, to TFP, and m, to the Taylor rule), the nominal rate in the previous period i_-, and the level of debt B. The two shocks constitute a VAR(1) process that we discretize into a Markov chain with 21 and 11 support points respectively (covering ± 3 and ± 4 standard deviations). The grid for the nominal rate consists of 11 points set to cover the ergodic distribution of the nominal rate in equilibrium. The grid for debt has 144 points, equally spaced over $[\underline{B}, \underline{B} + 0.1]$. The iid shock to the cost of default ν is discretized over a grid with 301 points, spanning ± 3 standard deviations, so as to evaluate fast its CDF and the conditional expectation at the end of (29).

The algorithm proceeds as follows

1. We start with initial guesses for the value function W_0 and the bond price schedule q_0, together with guesses for the F_0 and M_0 functions.

2. For each point in the state space, (z, m, i_-, B), we consider all potential choices $B' \in [\underline{B}, \overline{B}(z)]$ where $\overline{B}(z)$ is the peak of the bond sale proceeds schedule, in states with TFP z, i.e.
$$\overline{B}(z) = \arg\max_{B'} \left\{ \arg\min_{i} q_0(z, i, B') B' \right\}$$
(Note that the bond price schedule is not a function of the interest rate rule shock m since the shock is iid and therefore its current value provides no information about default behavior in the future.)

3. For each B' choice considered in state (z, m, i_-, B), we solve for the behavior of the private sector, characterized by the system of equations (15)-(23). For each $< C^f, i >$ guess we compute
$$\mathcal{B} \equiv B - q_0^{\text{interp}}(z, \mathbf{i}, B') B'$$
$$\pi = \overline{\pi} \left[\frac{\mathbf{i}}{m R^{1-\kappa} i_-^{\kappa}} \right]^{1/(1-\kappa)\alpha_p}$$
consumption of domestic goods (C),
$$C = \frac{1}{\beta \, \mathbf{i} \, M_0^{\text{interp}}(z, \mathbf{i}, B')}$$

the terms of trade (e), exports (X), and labor supply (N).

$$e = \left(\frac{\mathbf{C^f} + \mathcal{B}}{\xi}\right)^{1/(\rho-1)}$$

$$X = e^{\rho}\xi = \left(\frac{\mathbf{C^f} + \mathcal{B}}{\xi}\right)^{\rho/(\rho-1)} \xi = \left(\mathbf{C^f} + \mathcal{B}\right)^{\rho/(\rho-1)} \xi^{1/(\rho-1)}$$

$$N = \frac{C + X}{z\left[1 - \frac{\varphi}{2}\left(\pi - \overline{\pi}\right)^2\right]}$$

We then look for the pair $< C^f, i >$ associate with private sector choices that are consistent with the Phillips Curve (23) under F_0, and the relative demand for foreign goods, using the Powell hybrid method.

$$\begin{cases} \frac{\chi}{\theta} N^{1+\mu} + F^{\text{interp}}(s, \mathbf{i}, B') = \frac{zN}{C}\left[1 + \frac{\varphi}{\eta - 1}\left(\pi - \overline{\pi}\right)\pi\right] \\ \frac{C}{\mathbf{C^f}} = \frac{\rho}{\rho - 1}\frac{\theta}{1-\theta} e \end{cases}$$

Finally, using this private sector responses, we compute the value associated with each potential B'. The government maximizes over these and the maximum is stored in W_1, at the appropriate coordinates.

4. Using the new function W_1 we compute a new default threshold for the ν shock, $\nu^*(z, m, i_-, B)$, according to equation (28), and the associated default probabilities for each state.

5. We construct a new bond price schedule, q_1, by iterating on equation (17); then update the F_1 and M_1 functions according to equations (25) and (24) respectively. Dampening the update of the q schedule is never necessary but possible, yet seldom yields a faster convergence, for dampening factors in the 0.01–0.3 range.

6. If $||W_1 - W_0||_\infty < 10^{-6}$, $||q_1 - q_0||_\infty < 10^{-5}$, $||F_1 - F_0||_\infty < 10^{-5}$, and $||M_1 - M_0||_\infty < 10^{-5}$ we stop; otherwise we iterate ($W_0 \leftarrow W_1$, $q_0 \leftarrow q_1$, $F_0 \leftarrow F_1$, $M_0 \leftarrow M_1$) and move to step 2 above. Convergence, as defined here, is achieved within 700–1000 iterations, largely due to the relatively high value of β used in the calibration.

Whenever a function is superscripted with the "interp" label, we use linear interpolation (and extrapolation, whenever necessary) over the i_- dimension of the state space. In equilibrium, extrapolation is never necessary.

C Parameter Robustness

This appendix documents the robustness of the main results to alternative assumptions about the Frisch elasticity of labor supply, the persistence of the TFP shock, and the volatility of the iid shock to the value of default. Finally, we explore the role of the interest rate smoothing parameter κ for equilibrium dynamics.

[TODO]