Sovereign Default Resolution
Through Maturity Extension

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Abstract

Sovereign default episodes are eventually resolved by restructuring the debt, through negotiations with the lenders, and implemented by bond swaps and resumption of debt service payments. This process partially compensate lenders for their losses and provide debt relief for the sovereign. The empirical literature studying these events emphasizes that the bulk of debt relief is implemented by lengthening the maturity of debt, rather than changing face value. Countries exit renegotiation with less debt but with a greater share of long-term debt in total, compared to the maturity structure at the time of default. We augment a standard maturity choice model with a post-default renegotiation phase and study whether it can replicate this observed maturity extension in the data. The model is successful in generating this and other key features of renegotiations and maturity choice, but critically only when we assume that countries continue to be temporarily excluded from financial markets after renegotiation, as in the data. A version of the model where the sovereign can immediately resume borrowing following renegotiation features instead a counterfactual reduction in maturity. We interpret these findings in terms of the tension between the sovereign’s preference for consumption smoothing and the inefficiency of debt dilution inherent in long-term debt.

(JEL F34, G11, G15, H63)

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1 Introduction

Emerging markets are vulnerable to protracted debt crises which often result in outright default. Recently, similar dynamics proved relevant for the European Union “periphery” including Euro Area members such Spain and Greece. One option in the policy-makers’ toolkit is the active management of the maturity structure of the debt, over the business cycle and in the run-up of crises. Broner et al. [2013] document that emerging markets issue shorter-term debt than average when financial conditions worsen. A relatively recent quantitative literature on maturity choice (e.g. Arellano and Ramanarayanan [2012], Hatchondo et al. [2016], Sánchez et al. [2018]) studies the trade-offs faced under endogenous default risk, yet much less is known about the maturity structure determined as part of the bond swaps used to resolve default episodes. These bond swaps implements new terms for the lenders, by replacing (“swapping”) the old bonds on which the sovereign stopped making payments with new obligations, with a new face value and maturity structure. This restructuring process is a necessary but not sufficient first step towards regaining international financial market access. In parallel, a body of largely empirical work has focused on the haircuts applied to lenders as part of this process. It relates bond-level and aggregate haircuts to debt relief, the length of market exclusion, and exit spreads. These papers stress the importance of taking a present-value approach to haircuts, rather than considering reductions in face value alone, given the substantial changes in the timing of payments that are commonly observed. The comprehensive account of Cruces and Trebesch [2013] concludes that maturity extension is the main mechanism for debt relief: bond swaps reduce overall indebtedness, yet a greater share of the debt takes the form of long-term bonds compared to the composition of debt at the time of default. A key implication of their observation is that understanding maturity choice during restructuring is of vital importance for understanding the resolution of default episodes, the nature of debt relief, and lenders’ eventual compensation.

We provide a quantitative-theoretic analysis of the resolution of default crises via bond swaps and the incentives that lead country to negotiate maturity extensions. To the best of our knowledge, we are the first to provide a joint, quantitative account of maturity choice both during issuance and as part of the post-default restructuring process. In so doing, we bring together separate but inherently related strands of the literature on sovereign debt. Our model embeds Yue [2010]’s Nash bargaining setup, for the resolution of default episodes, in the sovereign debt model with maturity choice of Arellano and Ramanarayanan [2012]. The sovereign can issue both short- and long-term debt, while in good credit standing, and the eventual renegotiations, following default, alter both the maturity structure of the debt as well as its overall level. Lenders are assumed risk-neutral and competitive, resulting in bond prices that are actuarially
fair, reflecting equilibrium default probabilities and recovery rates alone. As standard in this
class of models, we focus on a Markov equilibrium, so that the sovereign lacks commitment
over future default decisions and future debt issuance behavior. Default is potentially desirable
because it support higher levels of consumption, due to the associated suspension of debt service
payments, but costly because it leads to temporary financial autarky and an exogenous loss
of income, which captures in reduced-form any real costs or disruptions caused by default.
The model’s success in generating the maturity extension in the data hinges critically on the
inclusion of a post-renegotiation market exclusion spell. This exclusion is in line with the data.
Using the dataset of Cruces and Trebesch [2013] we find that, on average, countries are excluded
for 3 year prior to restructuring and an additional 4 afterwards. These estimates are robust to
different definition of market re-access and the exclusion of less informative cases.

The post-renegotiation exclusion is essential for inducing maturity extension because it calls
for the sovereign to resume debt service payments before being able to tap again international
markets. The maturity structure of these renegotiated payments will impact the degree of
consumption smoothing and create strong incentives for the use of long-term debt, i.e. many
smaller payments over a longer time horizon, rather than relatively large payments at the time of
the renegotiation or shortly thereafter. In contrast, in a version of the model where renegotiations
are followed by immediate market access, the sovereign will use additional borrowing to roll
over the newly renegotiated payments, but in so doing dilute the claims of its lenders. In this
alternative specification, maturity is reduced rather than extended, first because this avoid
the inefficiency associated with dilution and, second, because market access means that the
sovereign can avoid a costly temporary drop in consumption. These results support two main
takeaways. First, the explicit inclusion of a post-renegotiation exclusion spell is both realistic
and critical for generating model behavior consistent with the data. Second, the issues raised
by the debt dilution of long-term debt and its hedging benefit, e.g. Hatchondo et al. [2016] and
Arellano and Ramanarayanan [2012], play a significant role in the choice of maturity not only at
issuance, during normal market access, but also in the restructuring process that follows default.

Our project is related to all work on emerging market long-term debt and maturity choice but
most closely to two recent papers. In terms of theory, Aguiar et al. [Forthcoming] lay out a
maturity choice model with a tractable shock structure and prove that inaction in secondary
markets as well as no further issuance of long-term debt are optimal in their environment, where
the sovereign is paying down an exogenous initial stock of debt. Their result rationalize why
buy-backs in secondary markets are uncommon and why they are unlikely to improve the
country’s overall position. Quantitatively, Dvorkin et al. [2018] also study maturity extension in
renegotiation, and argue that lenders’ aversion to reduction in the face value of debt, plausibly

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due to regulatory or balance-sheet concerns, together with post-renegotiation exclusion are needed to replicate the maturity extension in the data. The structure of their model differs from the one studied here: their maturity choice mechanism follows Sánchez et al. [2018], a payment schedule with finite maturity, and they model bond swaps using a non-cooperative bargaining game. Their emphasis on lenders’ concerns over the face value of the renegotiated debt is complementary to our results, since we have restricted attention to the reference case of risk-neutral lenders who only value the fair present value of the debt rather than its faced value or timing directly.

In generating the maturity extension behavior found in the data, we have abstracted from several recent developments in the sovereign default literature, chiefly production and an endogenous cost of default as in Mendoza and Yue [2012], the possibility of self-fulfilling crises following Cole and Kehoe [2000], lender risk-aversion or otherwise richer stochastic discount factors for the pricing of debt, e.g. Lizarazo [2013], capital and investment, e.g. Gordon and Guerron-Quintana [2018], the accumulation by emerging markets of reserves, e.g. Bianchi et al. [2018], or models with finite maturity and flexible cash flow structure, e.g. Sánchez et al. [2018] and Bai et al. [2015]. These extensions and mechanisms likely have first-order consequences for the choice of maturity at issuance and during renegotiation. We cannot rule out that one or some combination of them could induce the maturity extension behavior observed in the data. Another difference with some recent work concerns the modeling of negotiations, we follow Yue [2010] and model renegotiation using a cooperative game theory solution concept rather than the literature using non-cooperative bargaining as in Benjamin and Wright [2009] and more recently Asonuma and Joo [2017] and Dvorkin et al. [2018]. One final noteworthy omission from the model is abstracting from involvement by international financial institutions (IFIs) and conditionality, as discussed by Boz [2011] and more recently by Fink and Scholl [2016]. Emerging markets borrow from IFIs intermittently but these often play an important role in renegotiations and during debt crises. In the case of the Greece 2012 event, as Greece is a member of the European Union, the European Commission was actively involved with the restructuring process and the negotiation of the bond swap, as discussed in Zettelmeyer et al. [2013]. Conditionality would plausibly reduce default incentives in the aftermath of restructuring, bringing the model closer to the benchmark case, with post-renegotiation exclusion, and away from the case with immediate market re-access and dilution.

One key challenge in the study of maturity choice and long-term debt more broadly is the poor convergence properties of this class of models, across a wide range of numerical methods and strategies. Here we follow Dvorkin et al. [2018] and Gordon [2018] and use discrete choice methods. These are an attractive alternative to Chatterjee and Eyigunog [2012]’s iid income
shocks with continuous support, as they induce a qualitatively comparable “randomization” across issuance choices in a way that loosens the tight interconnection between bond prices and issuance policies, but do so in a more tractable way, with more modest computational requirements. Appendix A reviews the method and details its use here.

We calibrate the model to the Greece and the 2012 restructuring event there, as well as key findings in a broader sample of events, detailed in Section 2. The behavior we aim to replicate is robustly observed across most if not all default episodes, since the early 1990s. Greece 2012 is in many way an idiosyncratic case, e.g. 3rd party involvement by the European Commission and the International Monetary Fund, yet it has the advantage of readily available high quality data on cash flows prior to and following the bond swap as well as haircuts at the instrument level, as provided by Zettelmeyer et al. [2013]. The parameter values resulting from the calibration are comparable with their magnitudes reported in reference papers on maturity choice, suggesting that the results here carry over to a broader set of countries and circumstances.

The calibrated model mirrors the behavior of debt maturity in the data, both in terms of choice at issuance and regarding the lengthening of maturity during restructuring. In the data, maturity increases from 6.4 to 8.2 years (measures in terms of Macaulay duration) while the benchmark model has maturity increasing, on average, from 6.4 to 10 years. The key assumption supporting this positive result is the inclusion of a post-renegotiation exclusion spell. Absent this feature, we find that a version of the model with immediate market reentry would instead induce a reduction in the maturity of debt, from 7.6 to 5.4 years. With immediate market reentry, the sovereign has the option to roll over and restructure the portfolio resulting from the bond swap, in particular it can and does dilute the long-term obligations. This makes compensating lenders with long-term debt relatively costly. This is consistent with the comprehensive analysis of the dilution of long-term debt in Hatchondo et al. [2016], who find that the bulk of default risk, up to 78% of it, can be attributed to debt dilution. In contrast, in the benchmark model where countries remain temporarily excluded from borrowing after renegotiation and the resumption of payments, the main concern in choosing maturity is the degree to which consumption volatility can be mitigated. Rolling over the restructured debt is not immediately possibly, so that the timing and relative size of payments directly impact consumption. From this point of view, short-term debt is particularly costly, since it calls for an immediate reduction in consumption, followed by little or no further payments in the subsequent periods. Instead, using long-term debt enables smaller reductions in consumption, both immediately and over future periods, inducing a much smoother time path of consumption, in a way that enhances sovereign’s welfare.

In addition to replicating the maturity extension, the model is consistent with the data more broadly, on several key dimensions: the haircuts applied to lenders following renegotiation are
close to the values observed in the Greece 2012 event, an overall 65% haircut, with substantial heterogeneity, e.g. 75% haircut for short-term debt, debt-to-GDP in the 50% range, a countercyclical trade balance, and consumption slightly more volatile than income. Moreover, model spreads are low and have little volatility, compared to standard emerging market values, as observed in the Greece case. The model has difficulty in matching the data on one dimension in particular, the average share of short-term debt in total (i.e. the average maturity structure). To address this, in the benchmark specification we introduce a small adjustment cost that penalizes deviations of maturity from a target, average value. This cost breaks indifference over maturity choice in states of the world in which borrowing is essentially risk-free. We document that the maturity extension and most of the quantitative success of the model does not rely on the presence of this adjustment cost, by reporting calibrations of the benchmark model and the model with immediate reentry where this cost has been eliminated.

The rest of the paper proceeds as follows: Section 2 will summarize key stylized facts about restructuring of sovereign debt and maturity choice, Section 3 lays out the structure of the model and the definition of equilibrium, Section 4 contains the quantitative analysis of the model, both the benchmark version with post-renegotiation exclusion and a version with immediate market re-access, and Section 5 concludes.

2 Maturity and Debt Restructuring in the Data

We summarize the key features of recent debt restructuring events, drawing on the relevant literature. We identify 4 main stylized facts, with the aim of using them for the evaluation of the quantitative and qualitative properties of the model.

<table>
<thead>
<tr>
<th></th>
<th>Duration (Years)</th>
<th>Haircut (%)</th>
<th>Face Value Cut (%)</th>
<th>Δ Average Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before</td>
<td>After</td>
<td>Δ</td>
<td></td>
</tr>
<tr>
<td>All Episodes</td>
<td>4.9</td>
<td>10.4</td>
<td>5.9</td>
<td>36.1</td>
</tr>
<tr>
<td>Excl. Preemptive</td>
<td>5.7</td>
<td>11.4</td>
<td>5.8</td>
<td>42.4</td>
</tr>
<tr>
<td>Greece 2012</td>
<td>6.7</td>
<td>17.7</td>
<td>11.0</td>
<td>64.6</td>
</tr>
</tbody>
</table>

Table 1: Maturity Extension. Average duration in years, before and after restructuring, haircuts, changes in the face value of debt, and the change in average maturity, across 23 cases (1994–2015). Source: Reinhart et al. [2016] and Fang et al. [2016]. Cases marked as “preemptive” are bond swaps without prior default. “Average Maturity” is weighed using principal face value (n.b. not Macaulay duration).

I. Maturity Extension. We start with our main motivating observation, that virtually all cases involve a robust increase in the maturity of debt. Table 1 summarizes data from Fang et al. [2016] on all cases during the most recent 20 years. We find that debt is relatively short-term prior to restructuring and that after the exchange average maturity increases by roughly 6 years.
Sturzenegger and Zettelmeyer [2006] confirm the importance of changes in maturity, as part of the rescheduling plan, for events in the ’90s and early ’00s, in their comprehensive account of debt and default during this era.

The fact that the maturity structure of the debt is altered during bond swaps is important for the nature and level of haircuts suffered by creditors. Their implementation can be understood in terms of a mix of outright reduction in face value and rescheduling of payments. Cruces and Trebesch [2013] report that many episodes feature no reduction in face value but rather implement haircuts through rescheduling exclusively.

Figure 1 compares the maturity structure of debt before and after the bond swap in the Greece 2012 event. Macaulay duration increases from 6.4 to 8.2 years while the average decay rate of payments decreased from about 18% to 4%. This latter measure can be understood as a data counterpart to the decay rate of payments used to model long-term debt in quantitative model of sovereign default, including in Section 3.

<table>
<thead>
<tr>
<th>Average Length of Exclusion (Years)</th>
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<tbody>
<tr>
<td>Before Restructuring</td>
</tr>
<tr>
<td>All Episodes</td>
</tr>
<tr>
<td>Excluding Preemptive</td>
</tr>
<tr>
<td>Excluding S&amp;M 2004</td>
</tr>
<tr>
<td>After Restructuring</td>
</tr>
<tr>
<td>All Episodes</td>
</tr>
<tr>
<td>Excluding HIPC</td>
</tr>
</tbody>
</table>

Table 2: Exclusion Duration. The average number of years between default and renegotiation (“before”) and between renegotiation and market access (“after”). Based on the dataset of Cruces and Trebesch [2013], Table 2 and Table A2 in the Online Appendix. “S&M 2004” is the Serbia and Montenegro restructuring of 2004, listed only as having defaulted since the 1990s. HIPC = “highly indebted poor country.”

II. Market Exclusion, Pre- and Post-Restructuring. A second feature of the data, of particular interest to our quantitative exercise, is the nature and length of market exclusion following default and after restructuring. Table 2 reports on this dimension of the data, using the dataset in Cruces and Trebesch [2013]. We note that sovereign are excluded both prior the restructuring event, having previously defaulted, and following the event. During this latter exclusion, the sovereign is making payments on the restructured debt but is unable to tap international private markets. We find that countries are excluded on average about 2 to 2.5 years prior to their restructurings, as well as roughly 4 to 5 years in their aftermath. These magnitudes are robust to the exclusion of preemptive restructurings (which are coded with a “before” exclusion of 0) or highly indebted poor countries (HIPC). Cruces and Trebesch [2013] compare the features of this more recent data to the results of Gelos et al. [2011] who consider a sample ending in 2000, in
Figure 1: Greece 2012 Cashflows. Cashflows scheduled by outstanding, defaulted debt (“before”) and resulting from the bond swap (“after”), in levels in the top panel and in log in the bottom one. The bond swap reschedules a substantial subset of payments after 2020 while reducing payments due within the first 5-10 years. The slope of the cashflows in logs are counterparts to the model’s decay rate of payments: we find $\delta_{\text{before}} = 18\%$ and $\delta_{\text{after}} = 4\%$. Payments are decaying over time at a much slower rate in the “after” cashflow, indicative of a longer maturity. Source: Zettelmeyer et al. [2013]
particular with respect to the evidence on whether harsher haircuts are followed by lengthier exclusions. Indeed, one of their main finding is that higher haircuts are associated with higher spreads upon, and longer delay before, return to markets.

<table>
<thead>
<tr>
<th>SZ Haircut (%)</th>
</tr>
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<tbody>
<tr>
<td>Residual duration</td>
</tr>
<tr>
<td>Constant</td>
</tr>
</tbody>
</table>

Table 3: Haircut Heterogeneity, Greece 2012. Regression of haircut on residual duration, at the level of individual bonds, in the Greece 2012 restructuring case. Source: Zettelmeyer et al. [2013]. Heteroskedasticity-robust standard errors in parentheses.

III. Haircut Heterogeneity. Asonuma et al. [2017] study the recent 1999–2015 sample and conclude that short-term debt suffers higher haircuts compared to long-term instruments. They report that a 10-year instrument will receive a haircut that is 3-11% smaller than a 1-year instrument, depending on the empirical specification used. Haircut heterogeneity is particularly striking in the Greece 2012 case, as documented by Zettelmeyer et al. [2013] and summarized by Table 3. The overall haircut applied was roughly 65% yet debt due within one year suffered haircuts in excess of 78-80%, while instruments due in 10+ years received haircuts of 50% or less. In the case of Greece, this robust, monotonic relation between residual maturity and haircuts is induced by the “one-size-fits-all” nature of the swap, a format in which lenders receive shares of the new portfolio proportional to the face value of the bonds they hold, irrespective of their maturity. Section 3.3 in the paper by Zettelmeyer and coauthors provides examples of other cases where such a format was used.

IV. Countercyclical Maturity. Rodrik and Velasco [1999] and Broner et al. [2013] are among the first to document the systematic response of maturity to deteriorating conditions. They find that issuance of new debt shifts into short-term debt in response to increases in the cost of borrowing, during crises and weak fundamentals. Arellano and Ramanarayanan [2012] confirm this pattern by estimating yield curves and documenting the response of maturity and the term premium to worsening financial conditions.

Evidence on default events and their resolution, beyond the 4 stylized facts emphasized here, is provided by Sturzenegger and Zettelmeyer [2006, 2007] and Tomz and Wright [2013]. Of particular importance to the empirical literature is the question of whether harsher haircuts cause or are associated with further delayed market access or higher cost of borrowing upon
reentry and evidence on the costs of default and haircuts more broadly.

3 Model

We develop a quantitative sovereign default model with a choice of maturity both at issuance and as part of the post-default debt swap process. The model features two distinct types of market exclusion: first, after default and prior the renegotiation, and second, following renegotiation, while making arrears payments on the restructured debt.

There are two types of agents: a risk-averse sovereign and a continuum of competitive, risk-neutral international lenders. In the tradition of Eaton and Gersovitz [1981], markets are incomplete in that the sovereign can borrow by issuing state-uncontingent instruments only, under lack of commitment. In any period, the sovereign can choose not to service its outstanding debt, in which case it enters a default state to be described below. Finally, market access is eventually regained following negotiations of a debt swap. We compare the mix of short- and long-term debt prior to default with the one resulting from the swap.

3.1 Debt Maturity and Payment Schedule

The sovereign can issue long-term bonds as in Hatchondo and Martinez [2009], Arellano and Ramanarayanan [2012], Chatterjee and Eyigungor [2012], or Hatchondo et al. [2016] with a choice of two maturities, Short and Long. A unit of a bond of maturity $i \in \{S, L\}$ issued this period schedules an infinite stream of debt service payments, starting next period, given by

$$\kappa_i (1 - \delta_i) \kappa_i (1 - \delta_i)^2 \kappa_i (1 - \delta_i)^3 \kappa_i, \ldots$$

where $\kappa_S$ and $\kappa_L$ are scaling parameters, and the short term bond has a faster decay rate than the long term bond, $\delta_S > \delta_L$. Let $r$ be the risk-free interest rate at which lenders can borrow or lend internationally, assumed constant throughout. Then, the risk-free price for bond of maturity $i$ is

$$q_{i}^{rf} = \kappa_i \sum_{\tau=0}^{\infty} (1 + r)^{-(1+\tau)} (1 - \delta_i)^{\tau} = \frac{\kappa_i}{\delta_i + r}$$

and the risk-free Macaulay [1938] duration is

$$D_i = \frac{1}{q_i^{rf}} \sum_{\tau=0}^{\infty} (1 + \tau) (1 + r)^{-(1+\tau)} (1 - \delta_i)^{\tau} \kappa_i^{\tau} = \frac{1 + r}{\delta_i + r}.$$  

We use risk-free duration throughout, both for the data and model, in order to focus on the changes in quantities, size and timing of payments, rather than movements in bond prices. We
normalize the risk-free bond prices for both maturities to unity, \( q_i^{rf} = 1 \), by setting \( \kappa_i = \delta_i + r \).

This leaves each bond’s duration unchanged. This normalization facilitates comparisons of debt magnitudes across maturities and simplifies expressions characterizing the overall debt stock.

For a portfolio consisting of \( b_S \) units of the short bond and \( b_L \) units of the long bond, the Macaulay duration can be computed as

\[
D (b_S, b_L) = \frac{1}{q_S^{rf} b_S + q_L^{rf} b_L} \sum_{\tau=0}^{\infty} (1 + \tau) (1 + r)^{-(1+\tau)} \left[ (1 - \delta_S)^{\tau} \kappa_S^{\tau} b_S + (1 - \delta_L)^{\tau} \kappa_L^{\tau} b_L \right] = \frac{b_S}{b_S + b_L} D_S + \frac{b_L}{b_S + b_L} D_L = D_S + (D_L - D_S) \frac{b_L}{b_S + b_L}.
\]

(3)

For the computation of the duration of new issuance (excluding buy-backs), we follow Arellano and Ramanarayanan [2012] and define

\[
D^{\text{issue}} (\ell_S^+, \ell_L^+) = D_S + (D_L - D_S) \frac{\ell_L^+}{\ell_S^+ + \ell_L^+}
\]

(4)

where \( \ell_i^+ = \max \{0, \ell_i\} \) so that buy-backs (negative issuance) are not considered. Finally, for a bond trading a market price \( q_i \), the yield-to-maturity spread \( s_i \) is implicitly defined by

\[
q_i = \frac{\kappa_i}{\delta_i + (r + s_i)}
\]

(5)

where the term in parentheses in the denominator is the yield-to-maturity of the bond.

3.2 The Sovereign

The sovereign starts each period with \( b_S \) and \( b_L \) outstanding units of the short and long bond respectively and an endowment realization \( y \), drawn from a Markov process with support \( Y \). We use the convention that positive \( b \) is debt. The state is given by the tuple \( \langle y, b_S, b_L \rangle \in Y \times \mathbb{R} \times \mathbb{R} \).

The government may decide to exert its default option and receive state-contingent value \( V^d \).

Alternatively, it can continue making debt service payments and achieve value \( V^p \), in which case we say that the country is in good credit standing.

\[
V (y, b_S, b_L) = \max_{d \in \{0,1\}} dV^d (y, b_S, b_L) + (1 - d) V^p (y, b_S, b_L)
\]

(6)

In each period with good credit standing, the sovereign will auction \( \ell_S \) units of the short bond and \( \ell_L \) units of the long bond. Whenever \( \ell_i < 0 \) the country is retiring, buying back some of its outstanding bonds. The stock of debt at the start of the period \( \langle b_S, b_L \rangle \) and the new issuance \( \langle \ell_S, \ell_L \rangle \) determine the stock of debt at the start of the next period \( \langle b'_S, b'_L \rangle \) via the stock-flow
identity \( b_i' = (1 - \delta_i) b_i + \ell_i, \) so that we can think of the government as either choosing issuance or the new debt stock.

The sovereign makes debt service payments and chooses consumption together with new issuance, subject to the budget constraint

\[
c + \kappa_S b_S + \kappa_L b_L = y + q_S (y, b'_S, b'_L) \ell_S + q_L (y, b'_S, b'_L) \ell_L - \phi(b'_S, b'_L),
\]

(7)

taking the bond price schedules \( q_S \) and \( q_L \) as given. \( \phi(\cdot) \) is an adjustment cost function that penalizes deviations from the targeted average maturity of debt, as in Bocola and Dovis [2016]. The role of this function is to anchor average maturity to the value in the data by breaking the near-indifference over maturity structure in the risk-free region. We report results with and without this cost function. The value achieved under continuing debt service satisfies

\[
V^p(y, b_S, b_L) = \max_{c, b'_S, b'_L} (1 - \beta) u(c) + \beta \mathbb{E}_{y'} | y \{ (1 - \eta) V^d(y', b_S, b_L) + \eta V^a(y', \gamma_S, \gamma_L) \}
\]

(8)

and (7)

The choice of default triggers an endowment penalty and temporary exclusion from world markets. For an endowment realization of \( y, \) the country’s actual income is \( h(y) \leq y \) whenever the sovereign is in default. Afterwards, the country will bargain with its creditors with probability \( \eta \) each period. Following this restructuring process, the country resumes debt service payments but only reenters financial markets with probability \( \eta^a. \) This implies that on average the country will be excluded from financial markets \( 1/\eta \) periods before restructuring and an additional \( 1/\eta^a \) periods following the resumption of debt service payments.

Consistent with these assumptions, the value achieved while in default, prior to restructuring satisfies

\[
V^d(y, b_S, b_L) = (1 - \beta) u(h(y)) + \beta \mathbb{E}_{y'} | y \{ (1 - \eta) V^d(y', b_S, b_L) + \eta V^a(y', \gamma_S, \gamma_L) \}
\]

(9)

where \( \langle \gamma_S, \gamma_L \rangle \) is the new debt owed to the creditors, following the bond swap, to be characterized shortly.

After restructuring, the country resumes payments, while still unable to tap international market. We allow the country to pay down its debt faster than required by the decay rate of each maturity,
but not to borrow \((\ell_i \leq 0)\). Then, the value achieved following the renegotiation is given by

\[
V^a (y, b_S, b_L) = \max_{c,b'_S,b'_L} \{ \eta^a V (y', b'_S, b'_L) + (1 - \eta^a) V^a (y', b'_S, b'_L) \}
\]

s.t. \(\ell_i = b'_i - (1 - \delta_i) b_i \leq 0\), for all \(i \in \{S, L\}\)

\[
c + \kappa_S b_S + \kappa_L b_L = h(y) + q^a_S (y, b'_S, b'_L) \ell_S + q^a_L (y, b'_S, b'_L) \ell_L
\]  

The bond price schedules \(q^a_i\) reflect both the country's temporary exclusion from borrowing but also its eventual return to market and any future defaults. We will characterize all bond price schedules once we lay out the structure of renegotiation.

For reference, we denote \(B_i(y, b_S, b_L)\) the borrowing policies while in good credit standing and \(B^a_i(y, b_S, b_L)\) the policies while excluded, following the renegotiation.

### 3.3 Debt Renegotiation and Haircuts

The government and its bondholders agree to swap the defaulted debt \(\langle b_S, b_L \rangle\) for a new portfolio \(\langle \gamma_S, \gamma_L \rangle\). We model the debt swap negotiation using the Generalized Nash Bargaining solution. We assume that all creditors are represented by a single committee, that prefers to maximize the overall value of the new bonds. We discuss shortly the allocation of the new bonds to the old bond holders.\(^1\)

We set the threat point for the government to the value it can achieve in permanent autarky while subject to the output loss penalty. The corresponding threat point for the creditors is 0, i.e. no recovery. This is standard but not without loss of generality.

\[
V^{aut} (y) = (1 - \beta) u (h (y)) + \beta_y |y| \mathbb{V}^{aut} (y')
\]  

The sovereign’s surplus from successfully renegotiating, with resulting bonds \(\langle \gamma_S, \gamma_L \rangle\), is given by the difference between the value associated with resuming payments and the one for autarky,

\[
\Delta_{Sov} (y, \gamma_S, \gamma_L) = V^a (y, \gamma_S, \gamma_L) - V^{aut} (y)
\]  

The surplus of international creditors is given by the the market value of the new debt,

\[
\Delta_{Cre} (y, \gamma_S, \gamma_L) = [\kappa_S + (1 - \delta_S) q^a_S (y, \gamma'_S, \gamma'_L)] \gamma_S + [\kappa_L + (1 - \delta_L) q^a_L (y, \gamma'_S, \gamma'_L)] \gamma_L
\]  

\(^1\)It can be shown that the same bargaining outcome, in terms of maturity composition and haircuts, can be achieved if we were to allow short and long debt holders to form separate committees, with distinct bargaining powers, as long as the sovereign’s bargaining power is fixed. This result reflects the homogeneity of lenders in terms of attitude towards risk and outside options, as assumed here.
This expression reflects the fact that payments resume immediately, in the renegotiation period, and that the sovereign will eventually resume borrowing and potentially default again, so that the value of the tail of payments is priced at $q^a_i$. The tail is evaluated at $\gamma'_i = B^a_i(y, \gamma_S, \gamma_L)$ since the sovereign has the option to pay down debt immediately.

The Nash program for a sovereign that defaulted with outstanding debt $\langle b_S, b_L \rangle$ and current endowment realization of $y$ is

$$\arg\max_{\gamma_S, \gamma_L} \left[ \Delta_{\text{Sov}}(y, \gamma_S, \gamma_L) \right]^a \left[ \Delta_{\text{Cre}}(y, \gamma_S, \gamma_L) \right]^{1-a}$$

subject to $\Delta_{\text{Sov}} \geq 0$ and $\Delta_{\text{Cre}} \geq 0$ (14)

where $a$ is the sovereign’s bargaining power parameter. Note that the defaulted debt portfolio $\langle b_S, b_L \rangle$ does not enter the Nash program. $\gamma_S$ and $\gamma_L$ are functions of the endowment realization $y$ alone. This further implies that the value of default is independent of the defaulted portfolio. This is Yue [2010]’s “bygones are bygones” result. Nash bargaining maximizes and splits the total surplus, an inherently forward-looking object. The value created by resolving the default is independent of pre-default debt levels, as the lenders have the ability to “hold up” the sovereign regardless of whether it defaulted on more or less debt.

A final remark on the bargaining setup here is that the joint surplus can always be positive, in all states of the world, so that conditional on negotiating, the program (14) always has a solution and the threat point is never visited in equilibrium. This is related to the observation that because lenders have an outside option of zero, any strictly positive but arbitrarily small recovery will induce positive surpluses for both parties.

The policies resulting from the solution to (14) are denoted $\Gamma_i(y)$. The value of default can be reduced to

$$V^{d}(y) = (1 - \beta)u(h(y)) + \beta E_{y'|y}\{(1 - \eta)VV^{d}(y') + \eta V^{d}(y', \Gamma_S(y'), \Gamma_L(y'))\}.$$ (15)

The new bonds are allocated to the old bond holders using the following rule, motivated by the evidence on heterogeneous haircuts in Asonuma et al. [2017] and the one-size-fits-all offer in the Greece 2012 event: the old short bonds receive a share

$$S_S = \frac{\mu_S b_S}{\mu_S b_S + \mu_L b_L}$$

of the new portfolio, made up of units of both maturities, while the rest are assigned to the old long bonds. On a per-unit basis, each old unit of the short bond is swapped for $S_S \gamma_S / b_S$ new units of short and $S_S \gamma_L / b_S$ units of long. We normalize $\mu_L = 1$ so that $\mu_S$ is a parameter.
controlling the “priority” of a short bond units relative to long units. In the calibration section below we explore its quantitative role. Here, we focus on its role in determining the recovery rates and haircuts.

We define the aggregate haircut as the percent change in risk-free, present face value

\[ H_{\text{Agg}} = 1 - \frac{\Delta_{\text{Cre}}^\text{rf}}{q_S b_S + q_L b_L} = 1 - \frac{(1 + r)(\gamma_S + \gamma_L)}{b_S + b_L}. \]  

(16)

or, alternatively, using market prices

\[ H_{\text{Agg}}^\text{Mrkt} = 1 - \frac{\Delta_{\text{Cre}}}{q_S(y, \gamma_S, \gamma_L) b_S + q_L(y, \gamma_S, \gamma_L) b_L} \]

Similarly, we define the haircut applied to maturity \( i \in \{S, L\} \) as

\[ H_i = 1 - S_i \frac{\Delta_{\text{Cre}}^\text{rf}}{q_i^S b_i^S} = 1 - S_i \frac{(1 + r)(\gamma_S + \gamma_L)}{b_i} \]

(18)

and a corresponding version based on market prices,

\[ H_i^\text{Mrkt} = 1 - S_i \frac{\Delta_{\text{Cre}}}{q_{i}^S(y, \gamma_S, \gamma_L) b_i} \]

(19)

The \( 1 + r \) term in the numerator of the risk-free expressions reflects the timing of the renegotiation: payments are resumed within period, rather than starting next period.

3.4 Lenders and Bond Prices

We assume international investors are competitive, risk-neutral, and that they can borrow or lend freely at a constant risk-free rate. Then, in order for them to break even, the bond price for maturity \( i \) must satisfy

\[ q_i(y, b_S^i, b_L^i) = \frac{1}{1 + r} \mathbb{E}_{y'|y} \{ d(y', b_S^i, b_L^i) \chi_i(y', b_S^i, b_L^i) \}
+ (1 - d(y', b_S^i, b_L^i)) (\kappa_i + (1 - \delta_i) q_i(y', b_S^i, b_L^i)) \} \]

(20)

here \( b_i'' = B_i(y', b_S', b_L') \) is the stock of maturity \( i \) that the sovereign will choose next period, conditional on not defaulting, \( d \) is the sovereign’s default policy function and \( \chi \) is the expected recovery rate, implied by the debt swap procedure described in the previous section, which
satisfies

\[ \chi_i(y, b'_S, b'_L) = \frac{1}{1+r} \mathbb{E}_{y'|y}\left\{ (1 - \eta) \chi_i(y', b'_S, b'_L) + \eta S_i \frac{\Delta_{\text{Cre}}(y', \gamma_S, \gamma_L)}{b_i} \right\} \]  

(21)

where \( \gamma_i = \Gamma_i(y') \) is the new debt of maturity \( i \) bargained in renegotiation. Finally, the debt serviced while still excluded, prior to the eventual return to market, is priced using bond price schedules that satisfy

\[ q^a_i(y, b'_S, b'_L) = \eta^a q_i(y, b'_S, b'_L) + (1 - \eta^a) \frac{1}{1+r} \{\chi_i + (1 - \delta_i) \mathbb{E}_{y'|y} q^a_i(y', b''_S, b''_L)\} \]  

(22)

where \( b''_i = B_i^a(y', b'_S, b'_L) \), using the relevant borrowing policies. Note that with probability \( \eta^a \) the sovereign accesses markets, can default again, and the relevant value of the bond is the one under good credit standing (\( q_i \)).

### 3.5 Equilibrium

Let \( S = \mathbb{Y} \times \mathbb{R} \times \mathbb{R} \) denote the state. A Recursive Markov Equilibrium consists of

(a) Value functions \( V, V^a, V^p : S \rightarrow \mathbb{R}, V^d : \mathbb{Y} \rightarrow \mathbb{R} \)

(b) Default \( d : S \rightarrow \{0, 1\} \) and borrowing policies \( \ell_S, \ell_L, B_S, B_L, B^a_S, B^a_L : S \rightarrow \mathbb{R} \)

(c) Nash solution policies \( \Gamma_S, \Gamma_L : \mathbb{Y} \rightarrow \mathbb{R} \)

(d) Bond price schedules and recovery rates \( q_S, q_L, q^a_S, q^a_L, \chi_S, \chi_L : S \rightarrow \mathbb{R} \)

such that

1. The value functions satisfy equations (6), (8), (9), and (10),

2. The sovereign’s policies solve programs (6), (8), and (10),

3. The Nash solution solves the program (14),

4. International investors break even and bond prices satisfy (20), (22), and (21).

### 3.6 Model without Post-Restructuring Exclusion

In the analysis of our results, we will stress the role of the post-restructuring exclusion for maturity extension. To emphasize this point, we will compare our benchmark model with an alternative version in which renegotiations result in immediate return to markets.
In this case, default is followed by market access directly, so that
\begin{equation}
V^d(y, b_S, b_L) = (1 - \beta)u(h(y)) + \beta \mathbb{E}_{y'|y} \left\{ (1 - \eta) \ V^d(y', b_S, b_L) + \eta V^p(y', \gamma_S, \gamma_L) \right\}. \tag{23}
\end{equation}
and the relevant surpluses during bargaining are, for the sovereign
\begin{equation}
\Delta_{Sov}(y, \gamma_S, \gamma_L) = V^p(y, \gamma_S, \gamma_L) - V^{aut}(y) \tag{24}
\end{equation}
and for the lenders
\begin{equation}
\Delta_{Cre}(y, \gamma_S, \gamma_L) = [\kappa_S + (1 - \delta_S)q_S(y, \gamma_S', \gamma_L')] \gamma_S + [\kappa_L + (1 - \delta_L)q_L(y, \gamma_S', \gamma_L')] \gamma_L \tag{25}
\end{equation}
Since the post-restructuring exclusion regime is never visited, its associated value function ($V^a$), bond prices ($q^i$), and policies ($B^i$) are now redundant. The structure of the model remains otherwise unaltered.

4 Quantitative Analysis

4.1 Calibration

We calibrate the model to a yearly frequency. We set $\delta_S = 1$ and $\delta_L = 0.071$ so that they have risk-free Macaulay durations of 1 and 10 years respectively. The short-bond is thus one-period debt. The risk-free rate is set to 3.2%, the widely used, conventional value in the literature including by Arellano and Ramanarayanan [2012]. We will compare model spreads with yields for Greece, relative to comparable German instruments. Over the sample period German yields are trending downwards and near zero, therefore we use a reference value for the risk-free rate in order to avoid having to augment the model with risk-free rate dynamics consistent to the ones in the data. Hatchondo et al. [2016] study a related version of the model with risk-averse lenders and a time-varying risk-free rate, but restrict attention to an exogenous recovery rate, without a change in maturity.

The endowment process is assumed AR(1), its parameters (autocorrelation $\rho$ and innovation standard deviation $\sigma_\varepsilon$) are estimated using OECD National Accounts data and the process is discretized using the standard Tauchen [1986] method.
\begin{equation}
\log y_t = \rho \log y_{t-1} + \sigma_\varepsilon \varepsilon, \quad \varepsilon \sim \text{iid } \mathcal{N}(0, 1) \tag{26}
\end{equation}
All annual data is detrended using the Hodrick-Prescott filter, with a parameter value of 100. The data counterpart for the endowment process is GDP minus Gross Capital Formation, given
that we abstract from investment and production. The utility function has a constant coefficient of relative risk aversion $\sigma$, set to 2 in line with the literature,

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} = 1 - \frac{1}{c} \quad (27)$$

The default output cost follows Chatterjee and Eyigungor [2012],

$$h(y) = y - \max \{0, \lambda_0 y + \lambda_1 y^2\} \quad (28)$$

with $\lambda_0 < 0$, $\lambda_1 > 0$ and no output cost for endowments draws lower than $-\lambda_0/\lambda_1$. The adjustment cost for deviations from the average maturity structure is given by

$$\phi(b'_S, b'_L) = \iota \left( \frac{b'_S}{b'_S + b'_L} - \bar{S} \right)^2 \quad (29)$$

and the parameter $\iota$ controls the magnitude of the cost.

The remaining parameters, related to discounting ($\beta$), the output cost of default ($\lambda_0, \lambda_1$), length of exclusions ($\eta, \eta''$), bargaining parameters ($\alpha, \mu_S$) and mean maturity ($\iota, \bar{S}$) are picked to match key moments: total debt to GDP, the degree to which the trade balance is counter-cyclical, the relative volatility of consumption to endowment, the volatility of short-term spreads, the lengths of exclusion pre- and post-swap, overall haircut, the haircut applied to short-term debt, and the average share of short-term debt in total. Tables 4 and 5 include the calibrated parameter values while Table 6 reports the model’s fit over the targeted and untargeted moments.

We follow the same strategy in the calibration of the model with immediate market access. The
Table 4: Calibration I. Parameters calibrated outside the model.

<table>
<thead>
<tr>
<th>Value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$r$</td>
<td>3.2%</td>
</tr>
<tr>
<td>$\delta_S$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\delta_L$</td>
<td>0.071</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon}$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The main difference is due to the calibration of the pre-renegotiation exclusion, which is targeted to the overall exclusion in the data, an expected exclusion of 7 years.

Table 5: Calibration II. Parameters set as to match select moments.

<table>
<thead>
<tr>
<th>Benchmark Model</th>
<th>Immediate Reentry</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.94</td>
<td>0.935</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>-0.85</td>
<td>-0.7</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>1.0</td>
<td>0.78</td>
</tr>
<tr>
<td>$\iota$</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.33</td>
<td>0.14</td>
</tr>
<tr>
<td>$\eta^a$</td>
<td>0.25</td>
<td>—</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.945</td>
<td>0.9</td>
</tr>
<tr>
<td>$\mu_S$</td>
<td>0.5</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 5: Calibration II. Parameters set as to match select moments.

4.2 Ruling Out Certain Default

We further restrict problems (8) and (14), for good credit standing issuance and the Nash bargaining respectively (the latter only for the case of immediate return to markets) with a minimum bond price for long-term debt. This prevents, as discussed in Hatchondo et al. [2016], booms in consumption prior to default in states of the world with low endowment draws, in which the country would want to receive from lenders the discounted present value of recovery today, and default next period with probability 1. This happens whenever the maximum amount of resources lenders would be willing to transfer to the country ($q_{SB}b_S' + q_{LB}b_L'$) is weakly lower than the expected recovery value ($\chi_SB_S' + \chi_Lb_L'$), but equal to it for arbitrarily high borrowing. In these low endowment states, the output cost of default $y - h(y)$ is low or null, due to its concave shape so that default would be deterred by exclusion alone. Quantitatively, for values of $y$ that are 2.5–3 standard deviations below mean, the benefits from promising certain default
and receiving the recovery value now outweighs the cost of financial market exclusion, hence the necessity of implementing such a limit. We set the floor on $q_L$ to 0.7, following Hatchondo et al. [2016], corresponding to a maximum long-term spread of about 17%. In equilibrium this constraint is not binding, default now dominating as an option in the relevant states.

4.3 Results

We compute the model using the methods details in Appendix A, following Dvorkin et al. [2018] and Gordon [2018]. Table 6 reports key moments, for two cases, with and without post-renegotiation exclusion, both with and without the adjustment cost for deviations from average maturity. We approximate the model’s ergodic distribution by simulating the model for a large number of periods (150,000) and drop an initial segment (5,000). We then report moments computed over all periods such that the country is in good credit standing, in the periods in question and over the 3 periods prior. This additional requirement over prior periods is imposed in order to eliminate times associated with recent return to markets: the country re-accesses markets with little debt and reentry is associated with a small number of periods of unusually high borrowing, as the country quickly returns to levels of debt closer to average. Without excluding these outlier periods, the computed correlation of net exports to GDP and GDP is slightly higher (closer to zero or positive values). Outside of these particular circumstances, the trade balance is counter-cyclical, as induced by the standard bond pricing mechanism. The other moments are largely unaffected by this selection criterion.

Table 6 documents that the model can generate targeted moments in line with the data while also inducing a maturity extension comparable with the broad evidence reported in Cruces and Trebesch [2013] and the Greece 2012 episode in particular. At the same time, the model features spreads (level and volatility, short- and long-term debt) comparable with Greece data. The statistics of spreads reported in the Greece 2012 column are computed relative to analogous instruments for Germany, over 2000–2012 due to data limitations. The right panel of Figure 2 plots 1-year and 10-year spreads for Greece from Global Financial Data [2015]. The values are low by historic standards due to the 2001–2008 sub-sample, during which Greece could borrow at roughly the same terms as Germany. The table reports 4 calibrations: the benchmark model, with post-renegotiation exclusion and the adjustment cost $\iota$, a version of the model with immediate reentry and the adjustment cost, and versions of the two models in which the adjustment cost has been set to zero. In the data, the case of Greece 2012, maturity was extended from 6.4 to 8.2 years while the benchmark model generates an increase from 6.4 to 10.0 years. This means that the Nash solution calls for lenders to be compensated with long-term debt only. In contrast, with immediate market reentry, maturity is counterfactually reduced, from 7.6 to
Table 6: Moments. Comparison of key moments in the data with model ergodic distribution statistics. Data column is Greece, except values marked with † which are computed with the Cruces and Trebesch [2013] dataset in Section 2. Business cycle moments are based on yearly OECD data over 1980–2010, spreads are computed over 2000–2010 at a yearly frequency, Debt to GDP is External Debt from Global Financial Data [2015], averaged over 2004–2015. See Figure 2 for time paths of these series. †† The maturity extension is from 6.4 to 8.2 years for Greece 2012 and from 4.9 to 10.4 in the full sample of recent events (with a slightly different measure). Model numbers are sample statistics from a 150,000 period simulation, after dropping the initial 5,000 observations/years. Observations are included in the calculation if the country is in good credit standing in the current period and in 3 periods prior.

Table 7: Maturity Choice. Regressions of the duration chosen on either the short-term spread or on state variables (fundamentals), with simulated data from the Benchmark model. Stock measures reflect duration of the stock of debt \((b_{S,t}, b_{L,t})\) while the Issue(ance) measures capture issuance excluding buy-backs \((l_{S,t}^*, l_{L,t}^*)\).
5.4 years. The columns reporting the versions of the models without adjustment cost were not recalibrated, but rather $\rho$ was set to zero in the corresponding calibration. In these cases, the model does not match average maturity, the main reason for the inclusion of the adjustment cost, yet the findings concerning the maturity implemented in the swap are unaffected: the benchmark model with post-restructuring exclusion features maturity extension while the one with immediate reentry behaves counterfactually on this dimension.

Figure 3 summarizes the model’s maturity choice behavior. It plots average duration chosen at various level of endowment, both during good credit standing, with normal market access, and during the bond swap. The behavior is consistent with the data in two key dimension: first, the model generates a reduction of maturity in bad times, low endowment realizations induce portfolio choices with a greater share of short-term debt and therefore lower duration, and second, the bond swap compensates lenders using long-term debt only, so that maturity is extended in renegotiation. Table 7 reports regressions of the chosen duration (for either the stock of debt or issuance alone) on short-term spreads or the state variables, using model-generated data. Consistent with the data, our forth stylized fact in Section 2, maturity is shortened during times of high spreads and when the endowment realization is low.

![Figure 3: Maturity Choice and Spreads.](image)

**Figure 3: Maturity Choice and Spreads.** *Left Panel:* Average Macaulay duration chosen during good credit standing, stock (blue) and new issuance (red), and as part of the bond swap when debt is renegotiated (black), across the level of the endowment, averaged over the simulation and 0.1 and 0.9 percentiles (dashed lines). For the same endowment level ($y$), different maturity mixes are chosen at different times due to variation in the maturity structure of outstanding debt and due to the discrete choice shocks used in computation. See Appendix A for a discussion of the method. *Right Panel:* Average Short- (blue) and Long-term (red) debt spread, versus endowment, conditional on good credit standing.

Figure 4 replicates a key finding in the maturity choice literature. It plots the two bond price schedules, for Short- and Long-term debt against the corresponding own quantity, keeping the debt of the other maturity constant at zero. The Short bond price schedule offers better prices
(lower yields) than the corresponding Long schedule, but in a way that is more elastic (steeper) to the amount borrowed. Aguiar et al. [Forthcoming] discuss the implications of these elasticity differences for incentives to borrow and the maturity structure of debt.

The use of long-term debt in restructuring reflects the incentives for consumption-smoothing while in post-renegotiation exclusion. Since short-term debt is one-period, its use requires an immediate payment to the lenders, at the time of the swap, without the country having access to additional borrowing through financial markets. Given the targeted level of haircuts, using short-term debt would call for a substantial reduction in consumption at the time of the swap. In contrast, the use of long-term debt requires small losses of consumption, not only at the time of the renegotiation but in all subsequent periods. Moreover, since the country continues to be excluded for markets, these payments are safe from additional dilution, at least until the country’s eventual access to new issuance. In practice, these forces are reflected in the sovereign’s surplus in renegotiation $\Delta_{\text{Sov}}(y, \gamma_S, \gamma_L)$, in particular its steeper slope in $\gamma_S$ than in $\gamma_L$, as displayed in Figure 5. The net result is that the use of exclusively long-term debt is an effective way to deliver value to lenders while at the same time avoiding large, costly jumps in consumption around the swap. Behavior in the model with immediate market re-access is markedly different: in this setting the sovereign will dilute the lenders’ claim the same period, by borrowing in excess of the newly renegotiated debt. This additional source of inefficiency for long-term debt, as discussed by Hatchondo et al. [2016], leads the Nash bargaining to select short-term debt as the preferred instrument with which to compensate lenders. This
will not require a sizable consumption drop on the part of the country as it can and does use financial markets to borrow, unlike in the full model. This finding highlights the role of dilution not only at the time of issuance, during good credit history, but also in terms of evaluating post-restructuring behavior and lenders’ recovery.

Figure 5: Sovereign’s Value in Renegotiation. The value achieved by the sovereign in post-renegotiation exclusion $V^a$ is substantially steeper in short-term debt compared to long-term obligations, reflecting the inability to tap international markets for roll-over and therefore the need for a larger consumption reduction.

5 Conclusion

We have argued that a model of maturity choice for sovereign debt, once augmented with a post-default renegotiation process, can replicate major features of bond swaps in the data, including haircut heterogeneity across maturities and an extension of maturity as part of the restructuring process. We focused on the maturity extension feature of the data and argued that the continued exclusion of the country from financial markets after restructuring plays a key role in rationalizing the use of long-term debt. In contrast, if countries were to immediately resume borrowing, the resulting dilution of the long-term debt would render it an unattractive instrument during the renegotiation, and the country would instead repay lenders largely with a one-time, lump-sum payment, corresponding to the short-term bond. Lengthening the maturity of debt strikes a balance between the country’s consumption-smoothing concerns, in light of continued lack of access to markets, and the need to compensate lenders as to eventually regain good credit standing.
References


Global Financial Data. GF Database, October 2015.


### A Numerical Solution

Models of long-term debt and of maturity choice raise substantial computational challenges, in part due to the role of dilution in bond pricing. With long-term debt, the bond price schedule reflects the market value of the tail of payments, evaluated at equilibrium issuance policies. In parallel, issuance policies are determined in response to the bond price schedules. This interdependence induces divergent behavior during computation. Several approaches in the
recent literature have addressed this issue by creating some residual uncertainty in issuance policies by means of small iid shocks. Chatterjee and Eyigungor [2012] use such a shock for income with continuous support, weakening the tight link between issuance and prices. Unfortunately, their method requires additional computational burden due to the need to find income shock values that leave the sovereign indifferent between discrete borrowing options, potentially many such levels. For our application, we will instead follow Dvorkin et al. [2018] and Gordon [2018] and use standard discrete choice methods. We require that the borrowing policies take values in a discrete set and subject each potential choice to an iid draw from a Gumbel distribution, as in the multinominal logit model. In each state \( \langle y, b_S, b_L \rangle \) we associate to each choice \( \langle b'_S, b'_L \rangle \) a shock \( \varepsilon_{b'_S, b'_L} \) so that now the ex-ante value achieved under repayment is

\[
V^p (y, b_S, b_L) = E_{\varepsilon_{b'_S, b'_L}} \max \left\{ W(y, b_S, b_L, b'_S, b'_L) + \rho \varepsilon_{b'_S, b'_L} \right\} \tag{30}
\]

where \( \rho \) is a parameter controlling the precision of the taste shocks. Under standard assumptions concerning the distribution of the shocks, choice probabilities conditional on repayment are given by

\[
G (b'_S, b'_L | y, b_S, b_L) = \frac{\exp \left[ W(y, b_S, b_L, b'_S, b'_L) / \rho \right]}{\sum_{x'_S, x'_L} \exp \left[ W(y, b_S, b_L, x'_S, x'_L) / \rho \right]} \tag{31}
\]

In the model we treat the default option symmetrically with the borrowing choices but nest it out in order to address the “blue bus red bus” problem associated with changing the number of grid points for bonds.

\[
V (y, b_S, b_L) = E_{\varepsilon_d, \varepsilon_p} \left\{ V^p (y, b_S, b_L) + \rho_D \varepsilon_p, V^d (y, \varepsilon_p) + \rho_D \varepsilon_d \right\} \tag{32}
\]

so that each state is associated with a default probability given by

\[
d (y, b_S, b_L) = \frac{\exp \left[ V^d (y) / \rho_D \right]}{\exp \left[ V^d (y) / \rho_D \right] + \exp \left[ V^p (y, b_S, b_L) / \rho_D \right]} \tag{33}
\]

Finally, we augment the Nash problem (14) with analogous taste shocks, one for each \( \langle \gamma_S, \gamma_L \rangle \) option. As a result, the level of the endowment at the time of the swap does not fully determine the debt level and maturity mix of the exit portfolio, as some uncertainty remains, associated with the taste shocks. The precision parameter associated with the Nash taste shocks is \( \rho_N \).

In the numerical exercise, convergence is achieved under 1,000 iterations, with full updating of the \( q \) schedules each iteration, for precision parameters in the 1.0e-5 – 3.0e-5 range. We compute the model under the tightest taste shocks consistent with convergence within 1,000 iterations to at most a 1.0e-5 change in the bond price schedules and 1.0e-6 change in value.
functions. The reported results are for $\rho$ equal to 1.0e-5 for all precision parameters, for both models (with and without post-swap exclusion). An alternative approach would be to use these precision parameters to target additional quantitative features of the data. We use precision parameter values as small as possible, given convergence criteria, and confirm that the main takeaway (the role of post-swap exclusion in generating maturity extension) is not sensitive to these parameters.

Figure 6: **Discrete Choice Method.** This figure illustrates the choice probabilities in the model’s equilibrium, in a representative state: we pick average endowment and an initial portfolio with an average short-to-long mix (red circle). The heat map represents choice probabilities over $\langle b'_S, b'_L \rangle$.

Figure 6 plots representative choice probabilities from the computed model, for a given state. The bulk of the probability mass is tightly centered around the option with the highest associated payoff. The taste shocks induce some degree of additional uncertainty over the maturity composition of the new portfolio and less over the overall level of debt. On average, the sovereign will increase indebtedness and shift the portfolio into a shorter-term structure. The taste shocks generate some uncertainty about the magnitude of this change but not its sign.

For the main results, both models, the endowment support is discretized over 21 points and the two bonds are restricted to grid of 120 points each, equally spaced between 0 and 60% of average endowment (normalized to $\overline{y} = 1$).