Accounting for the Corporate Cash Increase

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Abstract

Why do U.S. firms hold much more cash now than they did 30 years ago? I construct an industry equilibrium model of firm dynamics where cash provides a buffer against cash-flow shortfalls in the presence of costly external finance. My model finds that 63% of the increase in corporate cash holdings can be accounted for by the increase in cash flow volatility. The increase in cash flow volatility observed in the data arises from a decrease in the correlation between revenue and operating expenses. The model has a corresponding correlation parameter between the shocks on revenue and operating expenses and only this parameter is changed in the primary experiment. The decomposition of revenue and operating expenses is important and I show that other ways of modeling the cash flow volatility increase are both counterfactual and dampening.

KEYWORDS: Corporate cash increase; correlation decrease; firm dynamics

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1 Introduction

In the last 30 years, the cash-to-assets ratio of U.S. firms has increased significantly. Bates et al. (2009) report that the cash-to-assets ratio more than doubles from 10.5% in 1980 to 23.2% in 2006 and has risen in every major industry. When firms are categorized by size as in Figure 1, it can be seen that the cash¹ buildup of small firms is even greater. For firms with less than 1 billion 2010 dollars in total assets, the cash ratio almost triples from 1980 to 2010. This upward trend in the cash ratio is clearly an important and compelling feature of the data. The trend also appears to be remarkably linear and has little correlation with the aggregate fluctuations in the business cycle.² For instance, cash increases during the recessions of the early 1980s and early 2000s, while cash decreases during the recession of the late 2000s. The cash ratio change is then indeterminate across firm size categories for the early 1990s recession.



Figure 1: This figure plots the cash-to-assets ratio of firms categorized by size. Small (large) firms have less (more) than 1 billion 2010 dollars in total assets.

In this same time period, cash flow volatility has also increased substantially as

¹The data definition of cash used in this paper is the cash and short-term investments variable (CHE) in Compustat.

 $^{^{2}}$ More precisely, the correlation between cash ratio growth and real GDP growth is -0.147 in the last 30 years.

demonstrated in Figure 2. However, it should be emphasized that no direct causal link has been established in the empirical literature between cash and cash flow volatility. The correlation between cash and cash flow volatility suggests that a causal relationship is possible, but it is difficult to rule out confounding variables or even determine a causal direction, i.e. high cash holdings may instead cause high cash flow volatility. In particular, it is hard to find good instruments which exogenously shift cash flow volatility but not cash holdings and vice versa. A regression of cash on cash flow volatility almost always produces a significant coefficient, but at the same time, fails the Durbin-Wu-Hausman test for endogeneity with large F-statistics. The form of endogeneity of particular concern is simultaneity bias. It is also straightforward to argue that cash flow volatility affects, in a causal way, most of the other regressors in standard cash regressions such as market value and firm size. These other regressors can also deliver significant coefficients if they are included. Therefore, even though the cash flow volatility coefficient has a large t-statistic, its magnitude is relatively small, and Bates et al. (2009) do not predict that the increase in cash is mainly due to the increase in cash flow volatility. They state, "holding all other variables constant, we infer that the average cash ratio increased by 2.1 percentage points from the 1980s to 2006 because of the increase in cash flow volatility." One of the primary advantages of a structural approach is that it is possible to clearly determine the direction of causation and disentangle the mechanisms behind the cash increase under the context of a dynamic, rational expectations model. The endogeneity pitfalls can then be carefully pinned down and analyzed.

While the cash flow volatility has increased substantially over the last 30 years, it is interesting that revenue volatility and operating expenses volatility have not risen as seen in Figure 3. Rather, the correlation between revenue and operating expenses has actually declined as I find in Figure 4.³ The decrease in the correlation between revenue and operating expenses is a possibly salient and important fact that has not been well investigated. In my paper, the decrease in correlation is exogenous but plays an important role in how the model is constructed and estimated.⁴ The correlation

 $^{^{3}}$ Cash flow is mostly determined by revenue minus operating expenses. While there are other components such as interest, taxes, and depreciation, the variances and covariances contributed by these sources to cash flow variance is negligible.

⁴Otherwise, the cash flow volatility increase would either have to come through a reduced-form and somewhat *ad hoc* cash flow shock, or through a counterfactual revenue or operating expenses volatility increase.



Figure 2: This figure plots the mean standard deviation of cash flow ((IB + DP)/AT) in 5 year panels where IB, DP, and AT refer to the income before extraordinary items, depreciation, and total assets variables in Compustat. The last year of the 5 year rolling panel is graphed, and small (large) firms have less (more) than 1 billion 2010 dollars in total assets.

decrease occurs in every major industry as well and is an independently interesting phenomenon which is explored in the appendix.

I construct a buffer stock model of cash holdings with financing frictions where firms make dynamic capital, cash, equity flow, and exit decisions. The model is then taken to the data to determine that 63% of the increase in corporate cash holdings can be accounted for by the increase in cash flow volatility which arises from the decrease in correlation between revenue and operating expenses. The model has a corresponding correlation parameter between the shocks on revenue and operating expenses and only this parameter is changed in the primary experiment. A regression using the model data then produces a coefficient on cash flow volatility similar to what was found in previous studies which indicates that standard cash regressions underestimate the true impact of volatility on cash holdings.

The key mechanism is that, with a correlation decrease, revenue no longer acts as a strong natural hedge for operating expenses. In the past, when revenue fell, costs also fell, but now, when revenue falls, costs are less likely to fall. Therefore, this natural hedge occurs at a lesser degree which then translates to both more frequent



Figure 3: This figure juxtaposes the mean standard deviation of revenue (REVT/AT) and operating expenses (XOPR/AT) in 5 year panels where REVT, XOPR, and AT refer to the revenue, operating expenses, and total assets variables in Compustat. The last year of the 5 year rolling panel is graphed, and small (large) firms have less (more) than 1 billion 2010 dollars in total assets.

and more severe negative cash flow events. Negative cash flow is especially harmful if cash is exhausted since the only options left to the manager are to sell off capital and/or raise costly external finance. Cash consequently acts as a buffer against cash flow shocks. Most other structural models in the corporate finance literature do not even have the possibility of negative cash flow. However, management of negative cash flow and the implications of negative cash flow for default and exit are cited as central financial concerns by real world managers.⁵ Other papers, in particular, do not decompose revenue and operating expenses and they have difficulty generating the observed rise in corporate cash holdings with volatility alone. I show that increasing cash flow volatility through an increase in revenue volatility produces counterfactual moments and creates a dampened effect on cash holdings.

In essence, my paper shows that the corporate cash increase can be mostly attributed to rational behavior in response to the idiosyncratic cash flow volatility increase. Cash holdings are much less puzzling once the cash flow structure and shocks are modeled in a more comprehensive fashion. Using my model, I also demonstrate that policy attempts to motivate firms to invest or distribute their cash might have unintended consequences. Lowering the corporate tax rate or the real interest rate for example increases investment and firm value but cash holdings increase as well.

⁵Lindsey and Carfang (April 4, 2013) conducted a quarterly survey of chief financial officers. In the survey, CFOs reported that paying down negative cash flows and financing capital expenditures are the two major uses of cash.



Figure 4: This figure displays the decrease in the mean correlation between revenue (REVT/AT) and operating expenses (XOPR/AT) in 5 year panels where REVT, XOPR, and AT refer to the revenue, operating expenses, and total assets variables in Compustat. The last year of the 5 year rolling panel is graphed, and small (large) firms have less (more) than 1 billion 2010 dollars in total assets. The shaded area is two times the standard error above and below the mean.

Finally, I show that cash restrictions can reduce firm value considerably.

The rest of the paper is organized as follows: Section 2 discusses my contribution relative to the literature, Section 3 presents the model, Section 4 provides intuition on the optimal cash policy, Section 5 details the results, Section 6 analyzes several policy experiments, Section 7 concludes, and the appendix contains the computational and normalization procedures as well as an analysis of the correlation decrease phenomenon.

2 Literature

There are four traditional motives for firms to hold cash - namely the transaction motive, the agency motive, the precautionary motive, and the tax motive. With the large increase in financial innovation in the last 30 years, it is actually quite surprising that corporate cash holdings have not decreased due to the reduced importance of the transaction motive. Nikolov and Whited (2014) argue however that agency costs are relevant under certain assumptions on managerial incentives and contracts. Empirical papers such as Bates et al. (2009) show that the precautionary motive has an important but relatively small effect on the increase in the cash-to-assets ratio. On the other hand, they do not find evidence that agency conflicts make a definitive contribution to the rise in cash. Opler et al. (1999) also do not observe significant agency costs, but they do "find evidence that firms that do well tend to accumulate more cash than predicted by the static tradeoff model where managers maximize shareholder wealth." This indicates that the static tradeoff model is not rich enough to understand firm cash behavior and/or that there are other explanations for the cash increase. They also noticed that derivative usage is quite rare (less than 10% of the observations) among S&P 500 firms. Derivative usage is naturally rarer still among the small firms that I study in this paper. So perhaps the benefits of financial innovation are largely experienced by a small subset of firms.

Han and Qiu (2007) construct a two-period model to study the role of the precautionary motive. They find that an increase in cash flow volatility increases the cash holdings of constrained firms but has no systematic effect on the cash holdings of unconstrained firms. In an infinite horizon structural model, every state in the ergodic distribution can be reached with nonzero probability in the future, so all firms are constrained to some degree. Therefore, it is more pertinent to think about the impact of a continuum of "constrainedness" in regards to the precautionary motive.

The effect of repatriation taxes on the cash holdings of multinational corporations was studied by Foley et al. (2007). Their paper concludes that repatriation taxes have a significant effect on multinational companies with big foreign tax spreads, but they cannot explain the cash buildup of other large firms or especially of small domestic firms. However, recall that small firms with under 1 billion 2010 dollars in total assets experience the largest increase in the cash ratio and they comprise 76.3% of Compustat firms. Also, well over 90% of income for firms under 1 billion 2010 dollars in total assets come from domestic sources. Besides repatriation taxes, it is possible that the dramatic lowering of the corporate tax schedule over the postwar period is a factor for the cash increase, but few papers on corporate cash holdings have directly investigated the quantitative effect of the tax rate decrease on the cash ratio. The tax effect is particularly ambiguous because of the different forces involved. A fall in the corporate tax rate diminishes the precautionary motive, which would reduce cash holdings. However, firms also tend to save out of increased cash flows. I find

that the latter force dominates and so the drop in corporate taxes in the last 30 years contributes a small amount to the overall cash increase as well.

The main structural focus in the corporate finance literature so far has been on the motivation to hold cash. For instance, Gamba and Triantis (2008) create a model where firms make dynamic debt and liquidity decisions. They find that financing frictions can cause firms to simultaneously borrow and lend which implies that cash is not just negative debt. On the other hand, Riddick and Whited (2009) focus on the cash flow sensitivity of cash, i.e. whether a firm tends to save or dissave out of cash flows. They find a negative propensity to save out of cash flow in contrast to Almeida et al. (2004) since firms in their model have large positive cash flows when they receive favorable profit shocks. The marginal value of capital increases with high profit shocks so that firms dissave to purchase more capital. The saving propensity is therefore not necessarily a good proxy to measure financial constraints and the costs of external finance. Although my model is similar to the one used in Riddick and Whited (2009), it does not exhibit strongly predictive propensities due to the presence of a transitory shock. Bolton et al. (2011) highlight the importance of the ratio of marginal q to the marginal value of liquidity for the analysis of the investment and cash management problems.

Armenter and Hnatkovska (2013), Boileau and Moyen (2013), and Falato et al. (2013) also investigate the increase in cash holdings but with different factors and mechanisms. Armenter and Hnatkovska (2013) state that firms hold more cash now because equity has become cheaper relative to debt. Boileau and Moyen (2013) look at the precautionary and transaction motives with a cash-in-advance structure which drives firm liquidity needs. In contrast, Falato et al. (2013) assume that only tangible capital is pledgeable and they cite the rise in intangible capital usage as the primary explanation for the cash increase.

However, the dynamic firm decision in response to shocks is arguably the most fundamental problem. The precautionary motive is widely conjectured as a first order concern for firms but it is difficult to establish a large precautionary incentive to hold cash under standard structural models of firm dynamics. My model is based on the framework developed by Hennessy and Whited (2005) and Gomes (2001) which are in turn related to and influenced by Cooley and Quadrini (2001), Hopenhayn and Rogerson (1993), and Hopenhayn (1992). The specifics of my model are of course tailored to study corporate cash holdings. To reiterate, decomposing revenue and operating expenses and considering the correlation between them is key to form a model which maps better to the data and which produces a stronger precautionary motive.

Finally, my model is able to generate a reduction in investment due to an increase in cash flow volatility as in Minton and Schrand (1999), and it is able to produce a wide cross-sectional distribution for the marginal value of cash as in Faulkender and Wang (2006) and Dittmar and Mahrt-Smith (2007) due to the rich shock structure and external finance costs.

3 Model

3.1 Firm's problem

Assume that time is discrete and infinite, and that firms in the economy are riskneutral. Firms are assumed to be owned by a representative risk-neutral agent that is not explicitly modeled. Firms in the economy are also heterogeneous but they face the same decision problems - therefore, I can refer to a single firm from now on without loss of generality. Let $k \in \mathbb{R}_+$ denote the capital stock and $m \in \mathbb{R}_+$ denote the cash holdings of the firm.⁶ The firm comes into each period with these control variables as well as with revenue shock state variable z. The revenue shock $z \in [\underline{z}, \overline{z}] \equiv Z \subset \mathbb{R}_{++}$ is strictly positive, bounded, and has Markov transition function Γ .

The firm's production technology is assumed to exhibit decreasing returns to scale $\alpha < 1$ which implies that there exists a well-defined upper bound \bar{k} on the optimal level of capital stock, where \bar{k} will be defined later in the section. The firm's capital is therefore selected from the compact set $k \in [0, \bar{k}] \equiv K$.

Production is performed by the firm each period by using its capital to generate revenue. Operating expenses are proportional to the amount of capital used. This parsimonious specification encapsulates various costs the firm faces such as production costs, research and development costs, and selling and administrative expenses

⁶Previous versions of this paper contained debt as an additional continuous state variable and priced it competitively. This feature of the model engendered a great deal of complexity and the results were not that different numerically. Corporate debt is unmodeled now and it is assumed that debt is rolled over each period without any frictions. On the other hand, I do not use net debt since the structure and assumptions on the collateral constraint can play a large role on the results.

without modeling them separately.⁷ The (operating) profit function is then,⁸

$$\pi(k, z, \eta_1, \eta_2; P) = P\eta_1 z k^{\alpha} - C(k, \eta_2)$$
(1)

where the cost function $C(k, \eta_2)$ has the form,

$$C(k,\eta_2) = \eta_2 c_v k + c_f. \tag{2}$$

The price $P \in \mathbb{R}_+$ can be thought of as the relative price of the homogenous consumption good to the price of capital. Note that the cost function has both variable and fixed components $c_v \in \mathbb{R}_+$ and $c_f \in \mathbb{R}_+$ respectively. In addition, the pair (η_1, η_2) is an i.i.d. random vector drawn from the truncated bivariate normal distribution,

$$\xi(1, 1, \sigma_1, \sigma_2, \rho, \underline{\eta}_1, \overline{\eta}_1, \underline{\eta}_2, \overline{\eta}_2) \sim \mathcal{N}\left[\begin{pmatrix} 1\\ 1 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2\\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \right] \text{ in } [\underline{\eta}_1, \overline{\eta}_1] \times [\underline{\eta}_2, \overline{\eta}_2]$$

$$(3)$$

with mean (1,1) and where $\underline{\eta}_1 = \underline{\eta}_2 = 0 = \underline{\eta}$ are the left and bottom truncation lines and $\overline{\eta}_1 = \overline{\eta}_2 = 2 = \overline{\eta}$ are the right and top truncation lines.⁹ The truncations have virtually no numerical effect and are only needed to ensure that the revenue and costs are not negative.¹⁰ To save space on notation, I will write $\xi(\sigma_1, \sigma_2, \rho)$ for the truncated bivariate normal distribution from now on. This correlated i.i.d. shock is introduced so that the model can mimic the decrease in the correlation between revenue and operating expenses observed in the data. If the firm has low operating margins, i.e. when mean revenues and expenses are much larger than mean profit, small changes in ρ can have powerful effects on the profit volatility and hence the cash flow volatility. The fundamental assumption here is that both persistent and transitory shocks may have important implications for real world firm dynamics.¹¹ The

⁷The estimation section will show that this way of modeling costs can approximate real-world cost dynamics reasonably well.

⁸The data analogue of the profit function is corporate earnings before interest, taxes, depreciation, and amortization (EBITDA).

 $^{^9{\}rm This}$ bivariate normal shock is a transitory shock with a specific structure. The right and top truncations are imposed simply for the sake of symmetry.

¹⁰In fact, σ_1 and σ_2 are always estimated to be small enough where the probabilities of $\eta_1 = 0$, $\eta_2 = 0$, $\eta_1 = 2$, or $\eta_2 = 2$ are numerically zero.

 $^{^{11}\}mathrm{Section}~5.2$ details the identification of the persistent and transitory shocks.

magnitude of the persistent or transitory component is then determined numerically. For example, the estimation may very well discover that $\sigma_1 = \sigma_2 = 0$ which would indicate that the transitory shock is an extraneous model feature. Of course, since the transitory shock is highlighted as an important part of the model, $\sigma_1 = \sigma_2 = 0$ is not what I find.

To recapitulate, the state vector of the firm at the beginning of the period is $\{k, m, z\}$ and profit $\pi(k, z, \eta_1, \eta_2; P)$ is generated after the realization of the transitory shocks $\{\eta_1, \eta_2\}$.

The firm also faces corporate taxes where taxable income,

$$y(k, m, z, \eta_1, \eta_2; P) = \pi(k, z, \eta_1, \eta_2; P) - \delta k + r_f m$$
(4)

includes depreciation and interest, and δ is the capital depreciation per unit of time and r_f is the risk-free real interest rate.¹² Therefore, net income is,

$$n(k, m, z, \eta_1, \eta_2; P) = (1 - \phi_c \tau_c) y(k, m, z, \eta_1, \eta_2; P)$$
(5)

where τ_c is the corporate tax rate and ϕ_c is the shorthand notation for the indicator $\mathbf{1}_{\{y(\cdot)\geq 0\}}$.¹³ Cash flow,

$$f(k, m, z, \eta_1, \eta_2; P) = n(k, m, z, \eta_1, \eta_2; P) + \delta k$$
(6)

simply adds back depreciation. Cash flow along with the current and next period capital and cash choices determine the equity flow of the firm. Therefore, the period equity flow to or from shareholders if the firm chooses to continue to operate and adjusts its capital to k' and its cash holdings to m' is,

$$e_I(k, k', m, m', z, \eta_1, \eta_2; P) = (1 - \phi_d \tau_d + \phi_\lambda \lambda) \{f(\cdot) - [k' - (1 - \delta)k] - [m' - m]\}$$

where τ_d is the tax rate on a positive distribution (dividends) and λ is the equity

¹²Compustat firms hold most of their cash in interest bearing accounts or treasuries. Therefore, there is a small positive interest rate on cash which is well approximated by r_f .

¹³In previous versions of this paper, the tax function was a more complicated arctangent function to emulate real-world tax brackets. However, the complication was numerically unimportant because Compustat firms are large enough where they essentially face the top tax bracket whenever they have positive taxable income.

flotation cost incurred per unit of negative equity flow (equity issuance). The function ϕ_d is the shorthand notation for the indicator $\mathbf{1}_{\{f(\cdot)-[k'-(1-\delta)k]-[m'-m]\geq 0\}}$ and ϕ_{λ} is the shorthand notation for the indicator $\mathbf{1}_{\{f(\cdot)-[k'-(1-\delta)k]-[m'-m]<0\}}$. Also, the law of motion for capital is given by $[k' - (1 - \delta)k]$ and the law of motion for cash is given by [m'-m]. In this class of models, it is straightforward to prove that the firm would never simultaneously distribute dividends and issue equity. One of the most important features of the profit function defined above is that profit can be negative.¹⁴ When the firm encounters negative cash flow, it must tap into its cash reserve or issue equity to maintain the same level of capital in the next period. Equity issuance is costly, so therefore, cash acts as a buffer stock against both transitory and persistent shocks even though the firm is risk neutral. Distributing dividends in the current period and then issuing equity in the next period is particularly expensive. The shareholders have to pay the distribution tax τ_d in the current period and then pay the per unit equity issuance cost λ in the next period if this occurs. On the other hand, the firm could have just retained the earnings without incurring additional taxes and equity issuance costs. The balance between the benefit and cost of holding cash is analyzed in the section on optimal cash policy.

If the firm instead chooses to exit, the equity flow is,

$$e_X(k, m, z, \eta_1, \eta_2; P) = (1 - \tau_d) \max \{ \pi(\cdot) - \phi_c \tau_c y(\cdot) + s(1 - \delta)k + (1 + r_f)m, 0 \}$$

where shareholders receive a positive distribution after exiting if the cash plus the proceeds from selling the capital at the fire-sale price is more than enough to offset any negative cash flow. The coefficient $s \in [0, 1)$ is the fire-sale value of capital.

Finally, the equity flow for a potential entrant that chooses initial capital of k'and initial cash of m' is,

$$e_E(k', m') = (1 + \lambda) [-k' - m'].$$

Notice that the firm is purely equity financed at entry and pays per unit equity issuance cost λ .

¹⁴Most other profit functions in the structural literature are weakly positive such as $\pi(k, z) = zk^{\alpha}$.

3.2 Recursive formulation

The precise bound on capital $\bar{k} = \left(\frac{\bar{\eta}\bar{z}\alpha}{1+r_f + \bar{\eta}c_v + \delta}\right)^{\frac{1}{1-\alpha}}$ can be constructed directly from the first order condition on k' by assuming that the firm will receive the best possible shocks next period. The boundedness of the cash choice must also be proven. Let $\beta = \frac{1}{1+r_f(1-\tau_i)}$ be the discount factor where τ_i is the individual tax rate. So all that is needed for cash holdings to be bounded is $\tau_c > \tau_i \implies (1+(1-\tau_c)r_f) < (1+(1-\tau_i)r_f)$ which is a maintained assumption throughout the paper. That is, cash needs to be more valuable outside the firm than inside the firm at some level of holdings. Therefore m' is selected from the compact set $[0, \bar{m}] \equiv M$.

The value function of an incumbent that continues to operate is,

$$V_{I}(k, m, z, \eta_{1}, \eta_{2}; P) = \max_{k', m'} \left\{ e_{I}(k, k', m, m', z, \eta_{1}, \eta_{2}; P) + \beta \int \int V(k', m', z', \eta'_{1}, \eta'_{2}; P) d\xi(\sigma_{1}, \sigma_{2}, \rho) d\Gamma(z'|z) \right\}$$

subject to,

$$e_I(k, k', m, m', z, \eta_1, \eta_2; P) = (1 - \phi_d \tau_d + \phi_\lambda \lambda) \{f(\cdot) - [k' - (1 - \delta)k] - [m' - m]\}.$$

The value function of an incumbent that exits is,

$$V_X(k, m, z, \eta_1, \eta_2; P) = e_X(k, m, z, \eta_1, \eta_2; P)$$

subject to,

$$e_X(k, m, z, \eta_1, \eta_2; P) = (1 - \tau_d) \max \left\{ \pi(\cdot) - \phi_c \tau_c y(\cdot) + s(1 - \delta)k + (1 + r_f)m, 0 \right\}.$$

Therefore the value function of an incumbent is just the maximum value of either continuing to operate or exiting the economy, i.e.

$$V(k, m, z, \eta_1, \eta_2; P) = \max_{x'} \{ V_I(k, m, z, \eta_1, \eta_2; P), V_X(k, m, z, \eta_1, \eta_2; P) \}$$

The next period capital, cash, and exit decision rules for the incumbent are de-

noted $k' = \mathcal{K}(k, m, z, \eta_1, \eta_2; P), m' = \mathcal{M}(k, m, z, \eta_1, \eta_2; P)$, and $x' = \mathcal{X}(k, m, z, \eta_1, \eta_2; P) \in \{0, 1\}$ respectively. The exit decision rule for the incumbent $\mathcal{X}(k, m, z, \eta_1, \eta_2; P)$ is a discrete choice in $\{0, 1\}$ where x' = 0 implies that the firm continues to operate and x' = 1 implies that the firm exits.

The value function of a potential entrant is,

$$V_{E}(z;P) = \max_{k',m',x'} \left\{ e_{E}(k',m') + \beta \int \int V(k',m',z',\eta'_{1},\eta'_{2};P)d\xi(\sigma_{1},\sigma_{2},\rho)d\Gamma(z'|z),0 \right\}$$

subject to,

$$e_E(k', m') = (1 + \lambda) [-k' - m'].$$

The next period capital, cash, and entry decision rules for the potential entrant are denoted $k' = \mathcal{K}_E(z; P)$, $m' = \mathcal{M}_E(z; P)$, and $x' = \chi_E(z; P) \in \{0, 1\}$ respectively. Similarly, the entry decision rule for the potential entrant $\chi_E(z; P)$ is a discrete choice in $\{0, 1\}$ where x' = 0 implies that the potential entrant chooses to invest in capital and cash, and x' = 1 implies that the potential entrant does not choose to invest in capital and cash.¹⁵ The important assumption here is that the potential entrant determines next-period capital and cash after z is realized. The potential entrant can also discover that the expected firm value is negative after the realization of z. This causes the potential entrant to not invest in capital and cash and not enter the economy.

3.3 Free entry

Assume that the potential entrant receives an independent z draw from the stationary distribution of the Markov process with transition Γ . It does not know the value of z before becoming a potential entrant. Therefore, the free entry condition is,

¹⁵To economize on notation, I use χ and χ_E to refer to the exit and entry decision rules and $\mathcal{X}(k, m, z, \eta_1, \eta_2; P) = \chi_E(z; P) = 1$ always denotes that the firm or potential entrant leaves the economy.

$$\int (1 - \chi_E(z; P)) \Big\{ (1 + \lambda) [-\mathcal{K}_E(z; P) - \mathcal{M}_E(z; P)] \\ + \beta \int \int V(\mathcal{K}_E(z; P), \mathcal{M}_E(z; P), z', \eta'_1, \eta'_2; P) d\xi(\sigma_1, \sigma_2, \rho) d\Gamma(z'|z) \Big\} d\Gamma_E(z) \le c_E$$

$$\tag{7}$$

where $\Gamma_E(z)$ is the stationary distribution of z and c_E is the entry cost. The left side of the inequality is the expected value of the potential entrant prior to the knowledge of z. Recall that the potential entrant has the option of choosing $\chi_E(z; P) = 1$ which implies that it does not become an operational firm since there is no investment in capital and cash. This means that a potential entrant which does not invest in capital and cash never enters the economy and disappears immediately. However, every potential entrant pays the entry cost and it may be sunk. More precisely, the "entry cost" is the cost paid to receive the z draw since a potential entrant can pay this cost and not enter. The fixed costs of production are not incurred until the period after entry but the fixed costs nonetheless discourage potential entrants to enter with low z draws.

Again, the exact value of z is learned only after entry. This assumption induces larger firms to enter with a wide range of firm sizes and is a realistic model of entry into Compustat.¹⁶ In contrast, the standard entry condition assumed in Hopenhayn (1992) and Gomes (2001) where the shock is learned after entry would cause all firms to enter with the same capital and cash. Entering firms tend to be smaller as well under this type of entry assumption and firms with low initial shock draws would immediately exit in the next period due to strong shock persistence.¹⁷

3.4 Distribution

The distribution μ can be computed by the following equation,

 $^{^{16}\}mathrm{Newly}$ listed Compustat firms have a similar average size and size dispersion in comparison to existing firms.

¹⁷However, only 1% of Compustat firms under 1 billion 2010 dollars in total assets exit within 1 year of their IPO date.

$$\mu'(k',m',z') = \int \int \int \int I(k,m,z,\eta_1,\eta_2;P) d\xi(\sigma_1,\sigma_2,\rho) d\Gamma(z'|z) d\mu(k,m,z) + M' \int \int (1-\chi_E(z;P)) \mathbf{1}_{\mathcal{K}_E(z;P)=k'} \mathbf{1}_{\mathcal{M}_E(z;P)=m'} d\Gamma(z'|z) d\Gamma_E(z)$$
(8)

where

$$I(k, m, z, \eta_1, \eta_2; P) \equiv (1 - \mathcal{X}(k, m, z, \eta_1, \eta_2; P)) \mathbf{1}_{\mathcal{K}(k, m, z, \eta_1, \eta_2; P) = k'} \mathbf{1}_{\mathcal{M}(k, m, z, \eta_1, \eta_2; P) = m'}$$

is a combined indicator function and M' is the mass of potential entrants every period. Another way of writing the law of motion of μ is to define an operator T^* such that

$$\mu' = T^*(\mu, M'; P).$$
(9)

The T^* operator maps distributions to distributions and in equilibrium, $\mu=\mu'=\mu^*.$

3.5 Industry demand

Assume that the relative price of the homogenous consumption good to the price of capital is determined by,

$$P = \frac{1}{Q^d}$$

where Q_d is the quantity demanded. Therefore the demand function is,

$$Q^d = \frac{1}{P}.$$

The specific form the demand function takes is unimportant as long as $\lim_{P\to 0^+} (Q^d) = \infty$ and $\lim_{P\to\infty} (Q^d) = 0$. The total quantity supplied by the firms in the economy is,

$$Q^{s} = \int \int \eta_{1} z k^{\alpha} d\xi(\sigma_{1}, \sigma_{2}, \rho) d\mu(k, m, z).$$

And so the product market clears when $Q^d = Q^s$.

3.6 Incumbent timing

- 1. The firm comes into the period with state vector $\{k, m, z\}$.
- 2. The transitory shocks $\{\eta_1, \eta_2\}$ are realized and profit $\pi(k, z, \eta_1, \eta_2; P)$ is generated.
- 3. The firm chooses whether or not to exit. If the firm exits, there is possibly one last dividend distribution. If the firm continues to operate, then k' > 0 and m' are chosen.
- 4. Dividend is distributed or equity is issued to shareholders depending on the sign of the equity flow.
- 5. The next period revenue shock z' is realized.

3.7 Potential entrant timing

- 1. The potential entrant draws z from the stationary distribution and pays entry cost c_E .
- 2. If x' = 1, then the firm never invests in capital and cash and does not enter into the economy. Otherwise, the firm chooses k' > 0 and m' and it is purely equity financed.
- 3. The next period revenue shock z' is realized.

3.8 Equilibrium

Definition 1. A stationary recursive competitive industry equilibrium is: a set containing (i) value functions $V_I(k, m, z, \eta_1, \eta_2; P)$, $V_X(k, m, z, \eta_1, \eta_2; P)$, and $V_E(z; P)$, (ii) decision rules for incumbents $k' = \mathcal{K}(k, m, z, \eta_1, \eta_2; P)$, $m' = \mathcal{M}(k, m, z, \eta_1, \eta_2; P)$, and $x' = \mathcal{X}(k, m, z, \eta_1, \eta_2; P)$, (iii) decision rules for potential entrants $k' = \mathcal{K}_E(z; P)$, $m' = \mathcal{M}_E(z; P)$, and $x' = \chi_E(z; P)$, (iv) a price P, and (v) a stationary distribution μ^* such that,

- 1. The decision rules solve the value functions,
- 2. The free entry condition (7) is satisfied,

- 3. The stationary distribution $\mu^* = \mu = \mu'$ solves (8),
- 4. And the product market clears $Q^d = Q^s$.

4 Optimal cash policy

The intuition behind the cash decision rule is explored in this section. The value function is not everywhere differentiable due to the equity issuance cost, dividend distribution tax, and the discrete choice of exit. However, assuming differentiability of the value function and deriving the optimal cash policy under this assumption can still offer some important insights. The optimal cash policy is dependent on the state of the firm and the marginal value of a unit of cash to an incumbent that will continue to operate in the next period is,

$$\frac{\partial V(k,m,z,\eta_1,\eta_2;P)}{\partial m} = (1 - \phi_d \tau_d + \phi_\lambda \lambda)(1 + r_f - \phi_c \tau_c r_f).$$
(10)

Therefore, the marginal value of cash can vary greatly and cash is more valuable if the firm issues equity than if the firm pays out dividends. In fact, there are six different values of current-period cash. Cash is the most valuable with marginal value $(1 + \lambda)(1 + r_f)$ when the firm has negative taxable income and issues equity. As long as $(1 + \lambda)(1 + (1 - \tau_c)r_f) > (1 + r_f)$,¹⁸ the next most valuable state for cash occurs when the firm has positive taxable income and issues equity, and the marginal value is $(1 + \lambda)(1 + (1 - \tau_c)r_f)$. Then, the third (fourth) most valuable state for cash occurs when the firm has negative (positive) taxable income and retains all earnings, and the marginal value is $(1 + r_f)$ or $(1 + (1 - \tau_c)r_f)$ respectively. Finally, cash is the least valuable with marginal value $(1 - \tau_d)(1 + r_f)$ or $(1 - \tau_d)(1 + (1 - \tau_c)r_f)$ when the firm has negative (positive) taxable income and distributes dividends. This ordering is consistent with intuition and generates a wide range of values for current-period cash.

The marginal value of a unit of next-period cash depends on the probability of ending up in the states just described. The first order condition with respect to m' is,

¹⁸This inequality holds for all of the parameterizations in the paper.

$$(1 - \phi_d \tau_d + \phi_\lambda \lambda) = \beta \int \int \frac{\partial V(k', m', z', \eta'_1, \eta'_2; P)}{\partial m'} d\xi(\sigma_1, \sigma_2, \rho) \partial \Gamma(z'|z).$$
(11)

Plugging in envelope condition 10 gives,

$$(1 - \phi_d \tau_d + \phi_\lambda \lambda) = \beta \int \int (1 - \phi'_d \tau_d + \phi'_\lambda \lambda) (1 + r_f - \phi'_c \tau_c r_f) d\xi(\sigma_1, \sigma_2, \rho) \partial \Gamma(z'|z).$$
(12)

The left side of Equation 12 is the marginal value of shareholder distributions, retained earnings, or external finance while the right side of the equation is the shadow value of next-period cash. If the cash would otherwise be distributed, then the expected value of next-period cash only requires marginal value $(1 - \tau_d)$. On the other hand, if the firm retains all earnings or needs equity, the expected benefits of nextperiod cash require higher marginal values of more than $(1 - \tau_d)$ up to and including $(1 + \lambda)$. If the firm needs equity, cash is very valuable today and it must be just as valuable tomorrow in expectation for the firm to hold on to the amount of cash given by m'.



Figure 5: This figure graphs the optimal cash policy for different marginal cost functions.

Figure 5 illustrates the optimal cash policy given three different marginal cost functions of next-period cash. It can be seen that next-period cash holdings increase as the marginal cost decreases. The vertical lines indicate the values of m' where the firm switches from paying out dividends to issuing equity. The marginal benefit function can intersect the marginal cost functions at the horizontal lines or at the vertical line. When the intersection is at the horizontal lines, marginal benefit equals marginal cost and must have value $(1-\tau_d)$ or $(1+\lambda)$. In contrast, when the intersection is at the vertical line, the firm is at an inaction region where all earnings are retained and the marginal benefit of next-period cash can be anything between $(1 - \tau_d)$ and $(1 + \lambda)$. That is, the firm neither finds it worthwhile to distribute dividends nor issue equity.

5 Results

5.1 Parameterization

The z shock used for the estimated model is assumed to follow an AR(1) process in logs, i.e.

$$\log(z') = \phi \log(z) + \theta + \epsilon' \tag{13}$$

where $\phi \in (0, 1)$, $\theta \in \mathbb{R}$, and $\epsilon' \sim N(0, \sigma_{\epsilon}^2)$. Some parameters are first set to values taken directly from the data or from the related literature as shown in Table 1. The risk-free real interest rate is found by using the 3 month treasury rate minus the rate of inflation and then averaged for the 1980-1984 period. The depreciation rate is set at the value found in Cooper and Haltiwanger (2006). The fire-sale value of capital and distribution tax are then set to the values used in Hennessy and Whited (2005) and both are within the range commonly used in the literature. Finally, the top marginal U.S. corporate tax rate was 46% for the entire 5 year period and the price is initially normalized to 1.¹⁹

¹⁹This is a convenient trick since entry cost is assumed to be unobservable. Later on, entry cost will be set to the value found when P = 1. The important element is the change in price to satisfy the free entry condition and not the original normalization.

Out	Value	
r_{f}	Risk-free real interest rate	0.05
δ	Depreciation rate	0.069
s	Fire-sale value of capital	0.75
$ au_i$	Individual tax rate	0.296
$ au_d$	Distribution tax rate	0.12
$ au_c$	Corporate tax rate	0.46
P	Price	1

Table 1: This table lists the parameters taken from outside the model corresponding to the 1980-1984 time period.

5.2 Identification

There are 10 parameters that need to be estimated in the model, namely, the revenue returns to scale α , AR(1) in logs scale parameter θ , AR(1) in logs persistence parameter ϕ , AR(1) in logs standard deviation parameter σ_{ϵ} , standard deviations and correlation of the bivariate normal shock $\{\sigma_1, \sigma_2, \rho\}$, production costs $\{c_v, c_f\}$, and per unit equity issuance cost λ .

The identification is relatively straightforward for several initial parameters. The revenue returns to scale α is identified by the standard deviation of capital. The AR(1) in logs scale parameter θ is identified by mean revenue while the variable cost parameter c_v is identified by mean operating expenses. Finally, the fixed cost c_f is identified by the exit rate and the equity floatation cost λ is identified by mean equity issuance.

Revenue can be decomposed into $\log(\text{rev}) = \log(P\eta_1 z k^{\alpha}) = \log(P) + \log(\eta_1) + \log(z) + \alpha \log(k)$ and variable operating expenses can be decomposed into $\log(\text{vxp}) = \log(\eta_2 c_v k) = \log(\eta_2) + \log(c_v) + \log(k)$. Taking the appropriate variances and covariances of the revenue and variable operating expenses produces Table 2. With some algebra, it can be shown that the moments listed in the table can identify both persistent and transitory shock parameters.

Moment	Components	Row
$\operatorname{cov}(\widetilde{\operatorname{rev}}_t, \widetilde{\operatorname{rev}}_{t-1})$	$\cos(\tilde{z}_t, \tilde{z}_{t-1}) + \alpha \cos(\tilde{k}_{t-1}, \tilde{z}_t)$	1
$\operatorname{cov}(\tilde{\operatorname{vxp}}_t, \tilde{\operatorname{vxp}}_{t-1})$	$+\alpha \operatorname{cov}(k_t, z_{t-1}) + \alpha^{-} \operatorname{cov}(k_t, k_{t-1})$ $\operatorname{cov}(\tilde{k}_t, \tilde{k}_{t-1})$	2
$\operatorname{cov}(\tilde{\operatorname{rev}}_{t-1}, \tilde{\operatorname{vxp}}_t)$	$\operatorname{cov}(\tilde{k}_t, \tilde{z}_{t-1}) + \alpha \operatorname{cov}(\tilde{k}_t, \tilde{k}_{t-1})$	3
$\operatorname{cov}(\widetilde{\operatorname{rev}}_t, \widetilde{\operatorname{vxp}}_{t-1})$	$\operatorname{cov}(\tilde{k}_{t-1}, \tilde{z}_t) + \alpha \operatorname{cov}(\tilde{k}_t, \tilde{k}_{t-1})$	4
$\operatorname{var}(\widetilde{\operatorname{rev}}_t)$	$\operatorname{var}(\tilde{\eta}_{1,t}) + \operatorname{var}(\tilde{z}_t) + \alpha^2 \operatorname{var}(\tilde{k}_t)$	5
$\operatorname{var}(\widetilde{\operatorname{vxp}}_t)$	$\operatorname{var}(\tilde{\eta}_{2,t}) + \operatorname{var}(\tilde{k}_t)$	6
$\operatorname{cov}(\widetilde{\operatorname{rev}}_t, \widetilde{\operatorname{vxp}}_t)$	$\operatorname{cov}(\tilde{\eta}_{1,t}, \tilde{\eta}_{2,t}) + \operatorname{cov}(\tilde{k}_t, \tilde{z}_t) + \alpha \operatorname{var}(\tilde{k}_t)$	7

Table 2: This table decomposes all the variances and covariances needed for the identification of persistent and transitory shock parameters. The tildes indicate log variables.

First, the variance and autocovariance of the AR(1) in logs shock process can also be written as $\operatorname{var}(\tilde{z}_t) = \frac{\sigma_{\epsilon}^2}{1-\phi^2}$ and $\operatorname{cov}(\tilde{z}_t, \tilde{z}_{t-1}) = \phi \frac{\sigma_{\epsilon}^2}{1-\phi^2}$ respectively. The covariances $\operatorname{cov}(\tilde{k}_t, \tilde{z}_{t-1})$ and $\operatorname{cov}(\tilde{k}_{t-1}, \tilde{z}_t)$ in turn can be found by subtracting α times Row 2 from Row 3 and Row 4. Next, note that $\operatorname{cov}(\tilde{k}_{t-1}, \tilde{z}_t) = \operatorname{cov}(\tilde{k}_t, \tilde{z}_{t+1})$ and the persistent shock $\tilde{z}_{t+1} = \phi \tilde{z}_t + \theta + \epsilon_{t+1} = \phi^2 \tilde{z}_{t-1} + (\phi + 1)\theta + \phi \epsilon_t + \epsilon_{t+1}$ can be rewritten by direct iteration. Therefore, $\operatorname{cov}(\tilde{k}_{t-1}, \tilde{z}_t) = \phi \operatorname{cov}(\tilde{k}_t, \tilde{z}_t) = \phi^2 \operatorname{cov}(\tilde{k}_t, \tilde{z}_{t-1})$ because the choice of k_t does not depend on ϵ_t or ϵ_{t+1} . Row 1 then pins down σ_{ϵ} , after the autocovariance of the shock process is expressed in terms of ϕ and σ_{ϵ} and the latter three terms are eliminated using Rows 2 through 4. The identification of σ_1 , σ_2 , and ρ in the end comes from Rows 5 through 7 respectively since the terms not related to the bivariate normal shock are terms which have already been ascertained.

While the clean identification strategy outlined above may seem to suggest that the model does not need to be fully solved to find the parameters, in reality, each parameter has effects on multiple moments and everything is jointly determined. Identification simply comes from the fact that each parameter has stronger effects on certain moments.

5.3 Data

The data source is the Compustat North America Fundamentals Annual from 1980 to 2010. The focus of the paper is on industrial firms, and therefore regulated firms with

Standard Industrial Classification (SIC) codes between 4,900 and 4,999 and financial firms with SIC codes between 6,000 and 6,999 are dropped. In addition, firms with under 10 million 2010 U.S. dollars in total assets and firms with missing or negative revenue, operating expenses, capital, cash, or assets are dropped from the sample. Firms with missing income are also dropped. There are a total of 129,507 firm-year observations remaining after the data is processed. The small firms which are the focus of this paper comprise 76.3% of the sample.

The large firms are only used to generate the initial graphs which provide a more comprehensive picture of the general patterns exhibited in the Compustat data. The moments used in the estimation just contain small firms and are then normalized by the mean total assets A_T of small firms in the cross-section for each year. This form of normalization preserves the relative magnitudes of the variables while removing the real growth trend. A detailed analysis of the normalization procedure is presented in the appendix.

I compute the cross-sectional statistics of the moments used in the identification by considering each firm-year as a data point. For example, a firm that was in Compustat for the first 4 years of the sample would contribute 4 data points. The cross-sectional covariances are also computed in this way where each firm-year revenue-expense pair is a data point. In the model, there is only one type of firm and the cross-sectional distribution is the same as if a single firm is simulated for a large number of periods. Ideally, the model would perform well along both panel and cross-sectional dimensions which is true in this case.

Finally, the data definition of revenue, operating expenses, cash flow, capital, cash, and equity issuance are the REVT, XOPR, (IB + DP), PPENT, CHE, and (SSTK – PRSTKC) variables in Compustat respectively. Equity issuance is defined to be equity issuance net of repurchases (SSTK–PRSTKC) and cash flow is defined to be income before extraordinary items plus depreciation (IB + DP) as used in Riddick and Whited (2009).²⁰

 $^{^{20}}$ Extraordinary items do not actually contribute much to the idiosyncratic cash flow volatility of firms (the difference of the means is less than 1% and the difference of the standard deviations is less than 2% when extraordinary items are included). I used the definition of cash flow from Riddick and Whited (2009) which was (IB + DP) but the moments would almost be the same if I used net income plus depreciation (NI + DP) instead.

5.4 Estimation

The estimation was done using simulated method of moments and the estimated parameters and moments matched are presented in Table $\frac{3}{2}$ and Table $\frac{4^{21}}{2}$ The parameters are quite reasonable overall. For example, the returns to scale parameter α is close to 1 since there is no labor in the model and low values of α generate counterfactually compressed distributions. There is also high persistence ϕ in the AR(1) process which then requires the scale and standard deviation parameters θ and σ_{ϵ} to be low. The bivariate normal i.i.d. shock estimate finds nonzero values for σ_1 and σ_2 which indicates that the transitory shock is important to matching the variance and covariance moments. In particular, σ_1 and σ_2 are needed to match the broad distribution of revenue and operating expenses respectively. The variable cost parameter c_v encapsulates many costs such as production costs, research and development costs, and selling and administrative expenses. Therefore c_v is estimated to be greater than 1. Finally, there is always some fear that the equity issuance cost must be unreasonably high to match the cash level in this class of models. Fortunately, λ is determined to have a sensible value of 4.22% which is within the range commonly found in the literature.

Overall, the moments are matched very closely and the only moment that is off by more than 5% is mean equity issuance. It is especially reassuring that the variances and covariances are matched well since the transitory shock and its identification are central to the results in this paper. Mean equity issuance is hard to match due to complex interactions. While the mean equity issuance is sensitive to a lowering of the per unit equity issuance cost λ , other moments such as the exit rate are also somewhat sensitive to changes in λ .

²¹One of the classical references for simulated method of moments is McFadden (1989). The estimator has asymptotic distribution $\sqrt{N}(\hat{\mathbf{b}} - \mathbf{b}) \xrightarrow{d} \mathcal{N}(0, V)$ where $\hat{\mathbf{b}}$ is the estimated vector of parameter vector $\mathbf{b}, V = (1 + \frac{1}{S}) \left[\hat{d'} W \hat{d} \right]^{-1}$ and $\hat{d} = \frac{\Delta E[g(x_t, \hat{\mathbf{b}}]}{\Delta \hat{\mathbf{b}}}$ is the numerical derivative of moment condition g, the vector of differences between model and data moments. The weighting matrix W is the inverse of the variance-covariance matrix $(\Omega = W^{-1})$ calculated from S = 100 repetitions of N = 3000 firms on average per year (approximately the size of the Compustat sample).

Inside parameters (1980-1984)		Estimate	Std Error
α	Revenue returns to scale	0.939	0.0003
θ	AR(1) in logs scale parameter	0.0175	0.0024
ϕ	AR(1) in logs persistence parameter	0.984	0.0001
σ_{ϵ}	AR(1) in logs standard deviation parameter	0.0459	0.0075
σ_1	Stdev of bivariate shock on revenue	0.238	0.0102
σ_2	Stdev of bivariate shock on operating expenses	0.218	0.0077
ρ	Correlation of bivariate shock	0.967	0.0055
c_v	Variable cost	3.735	0.0014
c_f	Fixed cost	0.0146	0.0204
λ	Equity floatation cost	0.0422	0.0833
c_E	Entry cost	0.103	-

Table 3: This table lists the parameters estimated using the model corresponding to the 1980-1984 time period.

Moments (1980-1984)	Data	Model
Revenue mean	1.51	1.51
Revenue standard deviation	2.36	2.42
Operating expenses mean	1.37	1.37
Operating expenses standard deviation	2.21	2.16
Cash flow mean	0.090	0.086
Cash flow standard deviation	0.159	0.161
Capital mean	0.367	0.363
Capital standard deviation	0.567	0.562
Cash mean	0.0988	0.0983
Revenue - operating expenses covariance	5.20	5.22
Revenue autocovariance	5.19	5.42
Operating expenses autocovariance	4.58	4.36
Revenue - operating $expenses_{-1}$ covariance	4.86	4.83
$Revenue_{-1}$ - operating expenses covariance	4.87	4.89
Equity issuance mean	0.021	0.006
Exit rate	0.05	0.051

Table 4: This table lists the data moments from the 1980-1984 time period and the model moments which attempt to match them.

5.5 Decision rules

Equity flows implied by the capital and cash decision rules are graphed in Figure 6, Figure 7, and Figure 8. First, equity flow is plotted along the capital and cash dimensions in Figure 6. For low values of capital and cash, the firm will issue equity, for medium values of capital and cash the firm will retain all earnings, and for high values of capital and cash the firm will distribute dividends. Next, along the persistent shock and capital dimensions, the firm will exit for low shock and capital values as seen in Figure 7. The empty locations on the surface plot are where the firm is better off exiting the economy. But the most interesting feature about this graph is that dividend distributions peak around the middle persistent shock value. The reason is that, as the shock becomes higher, there is also the tendency for the firm to invest more. In this case, the investment propensity dominates the dividend distribution propensity for high values of z. Finally in Figure 8, the equity flow behavior along the η_1 and η_2 dimensions is very intuitive. A high (low) revenue transitory shock combined with a low (high) operating expenses transitory shock values form the inaction region.



Figure 6: This figure graphs equity flow along the capital and cash (k, m) dimensions.



Figure 7: This figure graphs equity flow along the revenue shock and capital (z, k) dimensions.



Figure 8: This figure graphs equity flow along the transitory shock (η_1, η_2) dimensions.

The investment policies $[k' - (1 - \delta)k]$ are then graphed in Figure 9, Figure 10, and Figure 11. Note that investment does not depend much on cash for low or high values of current capital while investment increases with cash for moderate amounts

of current capital as seen in Figure 9. On the other hand, along the persistent shock and capital dimensions in Figure 10, investment monotonically decreases with current capital. Since the conditional distribution of the AR(1) process in logs is lognormal and has a fat right tail, investment is considerably higher for the high values of z. Figure 11 demonstrates that the investment behavior along the transitory shock dimensions also conforms well with intuition. Investment increases (decreases) with a high (low) η_1 shock and a low (high) η_2 shock.

Finally, the Compustat capital and cash distributions are juxtaposed with the model capital and cash distributions in Figure 12 and Figure 13. The general shapes of the Compustat distributions are captured nicely but of course the discreteness in the model does not allow for such a smooth decrease in proportion. In particular, the model capital distribution has a mass point just past 1.6 which the firms with the highest z value tend to choose. Also, a small fraction of the Compustat distributions an extremely diverse set of firms. The model capital and cash grids in contrast are set so that no firms are at the right endpoints.



Figure 9: This figure graphs investment along the capital and cash (k, m) dimensions.

Investment along the (z,k) dimensions



Figure 10: This figure graphs investment along the revenue shock and capital (z, k) dimensions.



Figure 11: This figure graphs investment along the transitory shock (η_1, η_2) dimensions.



Figure 12: This figure compares the Compustat and model capital distributions for 1980-1984.



Figure 13: This figure compares the Compustat and model cash distributions for 1980-1984.

5.6 Correlation decrease

The main experiment in this paper is performed in Table 5. It should be emphasized that the moments in the table are steady state moments and the model contains no aggregate shocks. Only the correlation parameter ρ is decreased from the estimated value of 0.967 to a lower amount. The correlation parameter ρ is decreased to 0.862 so that the cash flow volatility in the model is matched exactly to the volatility increase observed in the data. All other parameters are kept at the originally estimated values for the 1980-1984 data. This experiment is very clean because the effect of a change in ρ is completely isolated.²²

The results are quite good. In fact, almost every moment moves in the correct direction. However, while the standard deviation of revenue and operating expenses and the various covariances correctly move downward, these moments are significantly higher in the model experiment than in the data. The mean cash flow generated by the model is also somewhat higher than the mean cash flow observed in the data. These moments behave in this fashion because P adjusts upward to 1.014 in equilibrium to clear the goods market and the average quantity produced by each firm drops by 12%. On the other hand, if there was no equilibrium response and the price was kept at 1, the standard deviation of revenue and operating expenses, the covariances, and the mean cash flow would be lower and closer to the data. The entry/exit rates in the model would be higher as well.

Again, the fact that my model is an industry equilibrium means that there is an equilibrium response of relative price and the firm size distribution to the change in the stochastic process. I find that the relative price of capital (the inverse of P) falls by 1.4%, entry/exit falls by 9%, and that firm size, as measured by mean capital, decreases by 12%, while the coefficient of variation of capital rises by 2.3%.

Table 6 summarizes the cash increase resulting from the correlation decrease experiment. When ρ decreases from 0.967 to 0.862, 63% of the increase in cash in the last 30 years can be accounted for. The reasonable performance of the other moments acts as a test of the model and provides confidence in the validity of the correlation induced mechanism.

 $^{^{22}}$ I also do a full estimation in Section 5.10 where I allow all the parameters to change and the 2006-2010 ρ is similarly estimated to be substantially lower than the 1980-1984 ρ .

Moments (2006-2010)	Data	$\rho = 0.862$
Revenue mean	1.05	1.34
Revenue standard deviation	1.55	2.20
Operating expenses mean	0.95	1.20
Operating expenses standard deviation	1.44	1.94
Cash flow mean	0.043	0.081
Cash flow standard deviation	0.241	0.241
Capital mean	0.257	0.318
Capital standard deviation	0.447	0.504
Cash mean	0.2204	0.1750
Revenue - operating expenses covariance	2.21	4.23
Revenue autocovariance	2.04	4.50
Operating expenses autocovariance	1.76	3.45
Revenue - operating $expenses_{-1}$ covariance	1.87	3.90
$\operatorname{Revenue}_{-1}$ - operating expenses covariance	1.89	3.98
Equity issuance mean	0.021	0.008
Exit rate	0.07	0.047

Table 5: This table presents the correlation decrease experiment where ρ is lowered from 0.967 to 0.862 to match the bolded cash flow standard deviation moment.

Statistic	Data	$\rho = 0.862$
Cash in 1980-1984	0.0988	0.0983
Cash in 2006-2010	0.2204	0.1750
Percentage increase	123%	78%
Percentage accounted for	-	63%

Table 6: This table summarizes the behavior of cash from the correlation decrease experiment.

Table 7 shows that the decrease in correlation reduces investment and firm size where firm size is measured by the amount of capital holdings. Both investment and firm size drop because the increased volatility induces firms to substitute cash for capital for precautionary reasons. Cash flow and equity flow also decrease due to the reduction in firm size, and the coefficient of variation of size increases since volatility increases.

The first moment that behaves somewhat counterintuitively is firm value which remains roughly the same when the correlation decreases. The increased volatility increases firm value for struggling firms at the margin (low assets and/or shocks) since equity holders are residual claimants in good states of the world and have limited liability in bad states of the world. On the other hand, the increased volatility decreases firm value for decently performing firms (medium assets and/or shocks) since higher volatility just increases the chance that they will need costly external finance. The increased volatility increases firm value however for very successful firms (high assets and/or shocks) since the best firms are even better now. To be precise, the firms in the bottom size tercile experience a 2.6% rise in value, the firms in the middle size tercile experience a 9.3% fall in value, and the firms in the top size tercile experience a 4.4% rise in value on average. The effect of volatility on mean firm value is mostly neutral once the price of the consumption good relative to the price of capital adjusts upward to clear the goods market. If there is no equilibrium response and the price does not adjust upward, the value drops for firms in any state, although, some firms are still more affected than others.

Finally, the entry/exit rate becomes lower which is also a bit counterintuitive. Recall that the price rises in order to clear the goods market. This benefits the firms operating in the economy, and while the entry/exit rate is lower now, the firms that exit are worse than before. That is, the firms which choose to exit have lower expected value if they are forced (counterfactually) to stay in the economy when volatility is higher. Potential entrants in contrast have the same expected discounted value since the free entry condition must be satisfied.

Remember that positive equity flow is the same as dividend distribution and negative equity flow is the same as equity issuance. Table 8 tracks the change in average dividend distribution and equity issuance in an economy with low and high volatility. I break down the change for all firms, below median size firms, and above median size firms. Overall, firms distribute less dividends and issue more equity when the volatility rises.

The response across firm sizes is quite different however. For firms below (above) median size, the mean dividend distribution falls (rises) significantly. On the other hand, equity issuance increases for both firm size categories but increases more for large firms.

	1980-1984 ($\rho=0.967)$	2006-2010 ($\rho = 0.862$)	% Change
Cash (m)	0.0983	0.1750	78.0%
Size (k)	0.3628	0.3177	-12.4%
CV of size	1.5499	1.5855	2.3%
Investment $(k' - (1 - \delta)k)$	0.0206	0.0178	-13.6%
Cash flow (f)	0.0857	0.0812	-5.3%
Equity flow (e_I)	0.0565	0.0545	-3.5%
Price (P)	1.0000	1.0137	1.4%
Value (V_I)	1.0068	1.0077	0.1%
Entry/exit rate	0.0511	0.0465	-9.0%

Table 7: This table highlights the differences in various other important moments for high and low correlation economies.

	1980-1984 ($\rho=0.967)$	2006-2010 ($\rho = 0.862$)	% change
Dividend (all firms)	0.0638	0.0636	-0.3%
Dividend (below median)	0.0139	0.0129	-7.2%
Dividend (above median)	0.0943	0.1042	10.5%
Equity (all firms)	0.0055	0.0076	38.2%
Equity (below median)	0.0047	0.0056	19.1%
Equity (above median)	0.0060	0.0092	53.3%

Table 8: This table highlights the differences in the dividend distribution and equity issuance policies for high and low correlation economies.

5.7 Revenue volatility increase

Given the results in the previous subsections, one might ask, why shouldn't a revenue volatility increase be used to increase the cash flow volatility? In particular, what is the advantage of decomposing revenue and operating expenses? The intuition of revenue acting as a natural hedge for operating expenses was already outlined in earlier sections. Table 9 then addresses the numerical concerns. By just increasing the revenue volatility with a mean preserving spread on z to match the cash flow volatility increase, most of the moments are shown to be counterfactual. Some moments in fact are wildly counterfactual such as the standard deviations and covariances. For the

mean preserving spread, each shock z is transformed to $\hat{z} = (1 + \omega)z - \omega \bar{z}$ where $\omega \geq -1$ is the spread parameter and \bar{z} is the mean of z. The ω needed to obtain the desired level of cash flow volatility increase is 0.85 and the implied equilibrium price is 0.820 in this experiment.

Cash does increase a small amount to 0.116, but clearly, achieving a rise in cash flow volatility with a revenue volatility increase is both counterfactual and dampening. The cash increase in the $\rho = 0.862$ correlation decrease experiment is thus more than quadruple the cash increase in the revenue volatility increase experiment.

Moments (2006-2010)	Data	$\omega = 0.85$
Revenue mean	1.05	1.93
Revenue standard deviation	1.55	3.33
Operating expenses mean	0.95	1.75
Operating expenses standard deviation	1.44	2.97
Cash flow mean	0.043	0.111
Cash flow standard deviation	0.241	0.241
Capital mean	0.257	0.464
Capital standard deviation	0.447	0.773
Cash mean	0.2204	0.1157
Revenue - operating expenses covariance	2.21	9.87
Revenue autocovariance	2.04	10.40
Operating expenses autocovariance	1.76	8.26
Revenue - operating $expenses_{-1}$ covariance	1.87	9.19
$Revenue_{-1}$ - operating expenses covariance	1.89	9.35
Equity issuance mean	0.021	0.011
Exit rate	0.07	0.068

Table 9: This table demonstrates the counterfactual and dampening nature of a cash flow volatility increase through an increase in revenue volatility.

5.8 Transition simulation

The cash transition simulation is plotted in Figure 14 and the price along the transition path is plotted in Figure 15. The simulation uses backwards induction from 2010 assuming a linear decrease of ρ from 0.967 to 0.862 over the last 30 years. The model economy experiences a steadier and more tempered increase in cash than the real world economy. Also, the rise in price P implies that the relative price of capital has declined in the last 30 years.



Figure 14: This figure plots the cash transition simulation which was computed using backwards induction.



Figure 15: This figure plots the relative price of the homogenous consumption good to the price of capital along the transition path. The upward trend implies that the model predicts a decline in the relative price of capital.

5.9 Regressions

Model data can then be produced to imitate Compustat data by using the transition computed in Section 5.8. A simplified regression using the Bates et al. (2009) regressors which have a model analogue can be performed on Compustat data and on model data. The results are detailed in Table 10. First note that all the signs of the coefficients are the same. The coefficients for cash flow volatility are also of similar magnitude for all three regressions. However, the coefficients for the other regressors are of larger magnitude in the model regressions. Since the model is a parsimonious description of the real world, these regressors naturally contain more information about the dynamics in the model than the dynamics in the real world.

Cash	Data	Model	Model w/ ρ
Cash flow volatility	0.188***	0.191***	0.142***
Capital expenditure	-0.045***	-0.232***	-0.232***
Dividend dummy	0.017^{***}	0.100^{***}	0.102^{***}
Market value	0.047***	0.009^{***}	0.012^{***}
ho	-	-	-0.903***
R^2	0.276	0.258	0.296

Table 10: This table presents a simplified Bates et al. (2009) regression on the Compust data and on the model generated data for the correlation decrease transition experiment. The last column includes unobservable ρ as a regressor for the regression on the model data. All coefficients are significant at the 1% level.

In the Compustat data, the cash flow volatility in 1980-1984 is 0.078 and in 2006-2010 is 0.103. So the regression using the Compustat data predicts a 4.8% increase in cash holdings over the last 30 years if taken literally. Similarly, in the model data for the $\rho = 0.862$ transition experiment, the cash flow volatility is 0.083 in the first 5 year period and is 0.121 in the last 5 year period. So the regression using the model data predicts a 7.4% increase in cash holdings over the last 30 years. However, it known that only ρ is changed in the model from 0.967 to 0.862 and this change in ρ then increases cash flow volatility which ultimately generates the increase in cash. Therefore the regressions severely underpredict the contribution of cash flow volatility to the increase in cash. Simultaneity bias is the specific endogeneity issue at play here. An increase in cash flow volatility also reduces capital expenditure and raises market value. Therefore the regression is picking up these effects as well even though the increase in cash flow volatility is the true source of causation. If unobservable ρ is added into the model regression, the regression predicts a 96.5% increase in cash holdings which then accounts for 78.4% of the total cash increase!

5.10 Real interest rate and corporate taxes

During the 30 year time period, the real interest rate has also decreased substantially as illustrated in Figure 16. A decrease in the real interest rate increases the discount rate from $\beta = \frac{1}{1+r_f(1-\tau_i)}$ to $\hat{\beta} = \frac{1}{1+\hat{r}_f(1-\tau_i)}$. Recall that the real return on cash is also pegged to r_f which means that $\frac{1}{1+r_f(1-\tau_i)}(1+r_f) > \frac{1}{1+\hat{r}_f(1-\tau_i)}(1+\hat{r}_f)$ if $r_f > \hat{r}_f$, i.e. the marginal benefit of cash decreases when it is isolated from the rest of the model dynamics. However firms actually tend to hold more cash when the real interest rate decreases because they also place a higher weight on the future cost of equity issuance.



Figure 16: This figure plots the real interest rate over the last 30 years.

Table 13 was obtained by performing an estimation on the last 5 year period where the real interest rate r_f is decreased from 0.05 to 0.034, the individual tax rate τ_i is decreased from 0.296 to 0.25, the corporate tax rate τ_c is decreased from 0.46 to 0.35, and the entry cost c_E is kept at 0.103. Note that the interest rate was lowered to 3.4% instead of to the mean value observed in the last 5 years of the sample period. Compustat firms actually report the expectation of the risk-free rate and the mean expectation is 3.4% in the last 5 year period. This expectation is significantly higher than the observed rate for 2006-2010. However, the expectation is arguably a better approximation of the return and discount rate used in the firm decision. In the model, the realized return on cash for a few periods has very little numerical significance while the expectation on the future return and discount rate is very important. Only starting in 2002 was the expectation tracked in Compustat and so the realized real interest rate had to be used for the 1980-1984 period.

Out	Value	
r_{f}	Risk-free real interest rate	0.034
δ	Depreciation rate	0.069
s	Fire-sale value of capital	0.75
$ au_i$	Individual tax rate	0.25
$ au_d$	Distribution tax rate	0.12
$ au_c$	Corporate tax rate	0.35
c_E	Entry cost	0.103

Table 11: This table lists the parameters taken from outside the model corresponding to the 2006-2010 time period.

Inside parameters (2006-2010)		Estimate	Std Error
α	Revenue returns to scale	0.960	0.0005
θ	AR(1) in logs scale parameter	0.0230	0.0060
ϕ	AR(1) in logs persistence parameter	0.978	0.0002
σ_{ϵ}	AR(1) in logs standard deviation parameter	0.0408	0.0226
σ_1	Stdev of bivariate shock on revenue	0.244	0.0419
σ_2	Stdev of bivariate shock on operating expenses	0.243	0.0380
ρ	Correlation of bivariate shock	0.804	0.0140
c_v	Variable cost	3.253	0.0017
c_f	Fixed cost	0.0081	0.0625
λ	Equity floatation cost	0.0318	0.3198
P	Price	1.024	-

Table 12: This table lists the parameters estimated using the model corresponding to the 2006-2010 time period.

Moments (2006-2010)	Data	Model
Revenue mean	1.05	0.87
Revenue standard deviation	1.55	1.61
Operating expenses mean	0.95	0.79
Operating expenses standard deviation	1.44	1.46
Cash flow mean	0.043	0.051
Cash flow standard deviation	0.241	0.224
Capital mean	0.257	0.242
Capital standard deviation	0.447	0.433
Cash mean	0.2204	0.2293
Revenue - operating expenses covariance	2.21	2.32
Revenue autocovariance	2.04	2.37
Operating expenses autocovariance	1.76	1.88
Revenue - operating $expenses_{-1}$ covariance	1.87	2.08
$\operatorname{Revenue}_{-1}$ - operating expenses covariance	1.89	2.15
Equity issuance mean	0.021	0.012
Exit rate	0.07	0.073

Table 13: This table lists the data moments from the 2006-2010 time period and the model moments which attempt to match them.

A full estimation on the last 5 years finds that the correlation parameter decreases to 0.804. This is exciting because the estimation predicts a decline in correlation between revenue and operating expenses similar to the value used in the simple correlation decrease experiment. The full estimation result also lends further credibility to the correlation decrease since a lower correlation is exactly what the model estimation pushes towards even given the freedom to alter any of the other parameters. The other parameters do not change much in fact while the correlation decreases considerably.

6 Policy experiments

6.1 Corporate tax and real interest rate

The Obama administration has proposed that the top marginal corporate tax rate should be lowered to 28% as reported in Landler and Calmes (July 30, 2013). The prevailing idea is that a tax reduction along with a foreign tax holiday would propel

firms to invest more and possibly hold less cash. Average investment does increase by 7.3% if this policy change is enacted. However, the model also predicts that average cash holdings rise by 11% as perhaps an unintended consequence.

Also, the real interest rate has hovered around 1% in the last few years. If the drop in the real interest rate suggests that there is a long-term shift in monetary policy, then the expectation would adjust as well. Thus, if the expected real interest rate drops from 3.4% to 1%, the model predicts that cash rises by 19%. Investment, on the other hand, increases by 6.8%.

6.2 Cash restrictions

Suppose that there are restrictions on cash. These restrictions may come from the government or from activist shareholders. For example, a real estate investment trust (REIT) is a type of corporate organization which is required to distribute at least 90% of its taxable income to shareholders. Well-meaning policymakers can possibly impose a REIT-like structure on existing firms if they believe that firms hold far too much cash. First, starting from the parameter estimates for the 2006-2010 period, I can look at the mean firm value when the option to hold cash is removed. As a baseline comparison, the mean firm value drops by 25% when no corporate cash holdings are allowed.

In the real world, a popular refrain is that firms should distribute excess cash. But in my model, no cash is excess since all choices are fully rational. However, I can still run an experiment where firms are forced to distribute the cash that would not be necessary to cover any possible negative cash flow in the next period. When firms must distribute "excess" cash in this manner, mean firm value drops by 11% and mean cash drops by 35%. The period after the next period may require even more cash but accounting for the fact that the firm may need additional cash for many periods afterwards would entail not having a restriction at all at some point. The key takeaway here is that cash restrictions can be quite harmful to firms and my model can actually provide a prediction of how harmful.

7 Conclusion

The corporate cash increase is a phenomenon that has attracted a large amount of recent attention. This paper is an attempt to understand the phenomenon using an industry equilibrium model of firm dynamics. My model finds that 63% of the increase in corporate cash holdings can be accounted for by the increase in cash flow volatility which arises from a decrease in the correlation between revenue and operating expenses. The correlation decrease observed in the data may be easily overlooked - however, careful attention to this issue might uncover other important insights.

In addition, I show that the standard regressions of cash on cash flow volatility may face endogeneity problems, and building a model to explain the data can provide a deeper understanding of firm behavior. Policies to induce firms to spend their cash such as lowering the corporate tax rate or the real interest rate increases firm value and investment but cash holdings increase as well. Finally, I argue that restrictions on cash can reduce firm value considerably.

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8 Appendix

8.1 Computational algorithm

- 1. Set the grid to 25 points along the capital dimension where $k \in [0, 2]$, and 20 points along the cash dimension where $m \in [0, 1]$. Let the persistent shock z have 10 points and the transitory shock η_1 and η_2 have 5 points along each dimension. The persistent shock is discretized using the Adda-Cooper method and the transitory shock is discretized using the Tauchen method.²³
- 2. Set an initial value for the price P.
- 3. Solve for the decision rules and value functions.
- 4. Find the entry cost for the economy. Then use bisection and repeat Step 3 to find the P which generates entry cost c_E .
- 5. Set an initial value for the mass of entry M'.
- 6. Solve for the stationary distribution.
- 7. Find the quantity supplied for the economy. Then use bisection and repeat Step 6 to find the M' which generates quantity supplied $Q_s = Q_d$.
- 8. Finally, the simulated method of moments estimation is another outside loop which essentially minimizes the mean squared distance between data moments and model moments.

 $^{^{23}}$ See Adda and Cooper (2003) and Tauchen (1986) for details on the discretization.

Estimation of the model requires a large amount of computational resources because the model contains two continuous state variables, a persistent shock, a twodimensional transitory shock, and an industry equilibrium. To solve this high dimensional problem, parallelization is performed on the estimation loop. Such coarse parallelization allows the estimation to be very efficient due to infrequent message passing and to be almost perfectly scalable. More specifically, the minimization routine employs a multistart derivative-free local optimization method with the trust region determined globally.²⁴ A full estimation of the model takes approximately 100,000 CPU hours.

8.2 Normalization

Recall that the profit function is,

$$\pi(k, z, \eta_1, \eta_2; P) = P\eta_1 z k^\alpha - \eta_2 c_v k - c_f.$$

Let A denote the mean total assets of firms in the economy and then normalize by dividing through by A to get,

$$\frac{\pi}{A} = \frac{P\eta_1 z k^\alpha}{A} - \frac{\eta_2 c_v k}{A} - \frac{c_f}{A}$$

Let $\hat{z} = \frac{z}{A^{1-\alpha}}$ and rewrite the previous equation as,

$$\frac{\pi}{A} = P\eta_1 \hat{z} \left(\frac{k}{A}\right)^{\alpha} - \eta_2 c_v \left(\frac{k}{A}\right) - \frac{c_f}{A}.$$

Now assume that there is real growth in the economy up to time T which can be represented by,

$$G_T = \prod_{t=0}^T (1+g_t)$$

where t is the time index, g_t is the per period growth rate, and $g_0 = 0$. Also assume that $\hat{z}_T = G_T^{1-\alpha} z$ so that the profit function with real growth is,

$$G_T \pi = P \eta_1 \hat{z}_T (G_T k)^\alpha - \eta_2 c_v (G_T k) - G_T c_f.$$

Finally assume that the mean total assets of firms in the economy also grows at

 $^{^{24}}$ Derivative-free methods and implementations are surveyed in Rios and Sahinidis (2013).

the same rate such that $A_T = G_T A$ is the mean total assets at time T. Therefore, the normalization now gives,

$$\frac{G_T\pi}{G_TA} = \frac{P\eta_1 \hat{z}_T (G_T k)^{\alpha}}{G_T A} - \frac{\eta_2 c_v (G_T k)}{G_T A} - \frac{G_T c_f}{G_T A}$$

which transforms to,

$$\frac{G_T\pi}{G_TA} = P\eta_1 \hat{z} \left(\frac{G_Tk}{G_TA}\right)^{\alpha} - \eta_2 c_v \left(\frac{G_Tk}{G_TA}\right) - \frac{G_Tc_f}{G_TA}.$$

8.3 Decomposing the data

Why did the correlation between revenue and operating expenses fall so much in the last 30 years? This phenomenon is arguably just as puzzling as the cash increase. But there are fortunately several ways to decompose the data to obtain a better understanding of the issue.

First, the cost of goods sold (COGS) has become less correlated with revenue while the research and development expenses (RD) have become more correlated with revenue over the last 30 years (see Figure 17). At the same time, the correlation between revenue and selling, general, and administrative expenses (SGA) have fluctuated with no general trend.

COGS compose around 70% of operating expenses (see Figure 18). Therefore, the decline in the revenue-COGS correlation is the primary source of the decrease in the correlation between revenue and operating expenses. Note that while the revenue-RD correlation has gone up, it is still substantially lower than the revenue-COGS and revenue-SGA correlations. A larger share of expenses are attributed to research and development now so that the change in the operating cost structure also contributes somewhat to the overall correlation decline.



Figure 17: This figure breaks down the correlation between revenue and various types of operating expenses.



Figure 18: This figure breaks down the proportion of the various types of operating expenses.

Table 14 shows that the cash increase and correlation decrease occurred in every major Standard Industry Classification (SIC) industry. In fact, the industries which

experienced the greatest cash increases also had the most significant correlation decreases between revenue and operating expenses.

The decoupling of revenue and operating expenses can happen due to many different reasons. Suppose that the bivariate normal shock had the following structure instead,

$$\eta_1 = \omega \eta_e + (1 - \omega) \eta_u$$

$$\eta_2 = \nu \eta_e + (1 - \nu) \eta_w$$

where $\omega \in [0, 1]$ and $\nu \in [0, 1]$. This would imply that there are regional components, namely east and west, to the shocks on revenue and operating expenses. The data suggests that revenue has become more global while operating expenses have remained relatively local. Table 14 also indicates that the industries which have a higher proportion of global income now experienced the more substantial correlation declines. To be clear, this explanation is different from a cash increase due to repatriation taxes - rather, it is about the regional nature of the shocks. Pinkowitz et al. (2012) find that foreign tax holidays do little to reduce cash holdings which would imply that repatriation taxes do not have as large of an effect as found in Foley et al. (2007). The small firms considered in this paper also receive the vast majority of their income from domestic sources (still well over 90% in the last 5 years) and repatriation taxes are therefore unlikely to have a sizable impact.

Another simple way to decompose the data is to construct dummy variables for firms with non-zero exports or foreign income, research and development expenses, intangible assets, and inventory. Table 15 has the breakdown of the correlation between revenue and operating expenses, cash-to-assets ratio, and assets in 2010 dollars for the dummy variables just described. I find that firms with non-zero research and development expenses and firms with no inventory have especially low correlations and high cash ratios.

In this breakdown, firms with non-zero exports or foreign income does not exhibit strong cash differences in comparison to firms with no reported exports or foreign income. This "globalization" dummy variable is the only one which reverses the firm size ordering for the first 5 years versus the last 5 years of the sample. That is, firms with non-zero exports or foreign income used to be smaller on average while now they

		1980-	1984			2006-2	2010	
Industry	# Firm-Year	Cash Ratio	Correlation	Domestic $\%$	# Firm-Year	Cash Ratio	Correlation	Domestic $\%$
Agriculture	65	0.173	0.915	1.000	50	0.184	0.712	0.980
Mining	1194	0.109	0.738	0.995	696	0.160	0.497	0.945
Construction	287	0.088	0.960	0.999	119	0.153	0.949	0.933
Manufacturing	5084	0.115	0.933	0.991	3688	0.348	0.743	0.927
Transportation	372	0.099	0.923	0.993	375	0.123	0.913	0.981
Communications	238	0.127	0.973	1.000	362	0.223	0.852	0.971
Technology	3027	0.147	0.880	0.990	3610	0.331	0.852	0.868
Wholesale trade	826	0.087	0.964	0.994	389	0.121	0.961	0.936
Retail trade	1342	0.105	0.957	0.999	781	0.159	0.953	0.985
Services	1297	0.139	0.922	0.996	1207	0.255	0.820	0.934
Healthcare	457	0.165	0.895	0.992	870	0.316	0.868	0.943

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are larger on average. Therefore the size effect is conflated with the globalization effect here. In contrast, separate industries and relative magnitudes of globalization were analyzed in Table 14.

On the other hand, firms with non-zero research and development expenses and firms with zero inventory have remarkably high levels of cash.²⁵ This suggests a strong precautionary motive since R&D intensive and no inventory firms may need cash to finance risky investment and have no way of using inventories to smooth cash flows. The joint dummy of non-zero R&D and zero inventory is associated with very low correlations and high cash holdings which have become even more extreme over time. In 1980-1984, firms with non-zero R&D and zero inventory or with dummy pair (1,0) have 0.809 correlation and 0.321 cash ratio while firms with dummy pair (0,1) have 0.921 correlation and 0.099 cash ratio on average. Although the latter firms are twice as large, the size effect is nowhere significant enough to generate such a great divergence. Purely grouping by firm size to achieve the same size difference would only produce less than 3% difference in correlation and less than 25% difference in cash ratio. Astoundingly in 2006-2010, firms with dummy pair (0,1) have 0.930 correlation and 0.105 cash ratio on average.

Finally, the intangible assets effect in the data seems to be reverse of what is found in Falato et al. (2013). The data appears to imply that firms with intangible assets actually have higher correlation and less cash. Though it should be noted that Falato et al. (2013) constructed a new and more accurate measure of intangible assets and the dummy variable decomposition here might be too simplistic.

 $^{^{25}\}mathrm{Gao}$ (2014) looks at the role of just-in-time inventory on the cash buildup of manufacturing firms.

Dummy		, 1	1980 - 1984		. 1	2006-2010			All	
Variable	Value	Correlation	Cash Ratio	Assets	Correlation	Cash Ratio	Assets	Correlation	Cash Ratio	Assets
Export / FI	0	0.897	0.119	183.4	0.772	0.298	238.2	0.861	0.197	190.0
Export / FI	1	0.884	0.131	177.8	0.856	0.274	317.5	0.889	0.202	236.9
R&D	0	0.904	0.107	183.1	0.872	0.168	324.6	0.907	0.123	228.2
R&D	1	0.878	0.145	179.9	0.761	0.378	230.6	0.833	0.282	184.4
Intangible	0	0.878	0.130	169.1	0.680	0.405	197.7	0.842	0.225	171.5
Intangible	1	0.923	0.108	205.4	0.849	0.247	296.7	0.899	0.175	240.8
Inventory	0	0.794	0.188	122.2	0.702	0.403	236.8	0.758	0.331	184.2
Inventory	1	0.904	0.114	188.8	0.845	0.243	284.4	0.898	0.166	213.0
(R&D, Inventory)	(0,1)	0.921	0.099	192.1	0.896	0.142	333.9	0.930	0.105	233.2
(R&D, Inventory)	(1,0)	0.809	0.321	95.1	0.593	0.580	170.2	0.665	0.540	146.8
Table 15: This tabl	le conta	vins dummy v	ariable break	downs fe	or firms with	zero versus 1	10n-zero	values for ce	rtain income	
statement and bala	nce she	et variables.	The correlatio	m colum	n refers to the	e 5 year reven	ue-opera	ating expense	s correlation,	
the cash ratio colui	nn refe	rs to the cash	n-to-assets rat	io, and i	the assets col	lumn refers to	o total a	ssets in 2010	dollars. The	
breakdown also is a	across t.	he first 5 vea:	rs. the last 5	vears, al	nd the entire	sample peric	od.			

preakdowns for firms with zero versus non-zero values for certain income	lation column refers to the 5 year revenue-operating expenses correlation,	is ratio, and the assets column refers to total assets in 2010 dollars. The	st 5 years, and the entire sample period.
Cable 15: This table contains dummy variable breake	tatement and balance sheet variables. The correlation	he cash ratio column refers to the cash-to-assets rati	reakdown also is across the first 5 years, the last 5 $_{\rm J}$