STAT AREA PART I QUALIFYING EXAM

Mathematical Statistics January 28, 2015, 9 - 11 am

NAME: _____ **ID:** _____

Instruction: Work all four problems. You may not use notes or any other assistance.

1. Let $S = [0,1]$, $X \sim \mathrm{U}(0,1)$, $X(s) = s$, and

$$\begin{aligned} X_1(s) = s + \mathrm{I}_{[0,1]}(s), \quad & X_2(s) = s + \mathrm{I}_{[0,1/2]}(s), \quad X_4(s) = s + \mathrm{I}_{[0,1/3]}(s), \\ & X_3(s) = s + \mathrm{I}_{[1/2,1]}(s), \quad X_5(s) = s + \mathrm{I}_{[1/3,2/3]}(s), \\ & \qquad\qquad\qquad\qquad\quad X_6(s) = s + \mathrm{I}_{[2/3,1]}(s), \cdots \end{aligned}$$

(a) Show that $X_n \xrightarrow{p} X$, but X_n does not converge to X almost surely.

(b) Find a subsequence of the X_i's that converges almost surely to X.

2. Suppose Y follows a multinomial distribution with probabilities

$$\pi = \left(\frac{1}{2} + \frac{\theta}{4}, \frac{1-\theta}{4}, \frac{1-\theta}{4}, \frac{\theta}{4}\right).$$

Suppose we observe $y = (n_1, n_2, n_3, n_4) = (125, 18, 20, 34)$. Find the MLE of θ using EM Algorithm.

3. Let $X_1, \cdots, X_n$ be a random sample from $\mathrm{N}(\theta, \sigma^2)$, where σ^2 is known. Assume that the prior distribution of θ is $N(\mu, b^2)$.

   (a) Find the Bayes estimator of θ under the squared error loss.

   (b) Conduct a Bayesian test for H_0: $\theta \leq \theta_0$ vs. H_1: $\theta > \theta_0$ based on the posterior distribution obtained in Part (a) using the rejection region

$$\left\{ \boldsymbol{x} : P\left(\theta \in \Theta_0^c \middle| \boldsymbol{x}\right) > \frac{1}{2} \right\}, \text{ where } \Theta_0 \text{ is the null space.}$$

4. Let $X_1, \cdots, X_n$ be iid Bernoulli(p).

(a) Find the MLE of p.

(b) Find the MSE of the MLE obtained in Part (a).

(c) Assume that the prior distribution of p is Beta$(1/2, 1/2)$ and find the Bayes estimator $\hat{p}$ of p under the squared error loss.

(d) Find the MSE of $\hat{p}$ obtained in Part (c).

(e) Compare the MSE's obtained in Part (b) and Part (d) for each of $n = 4$ and $n = 400$ and comment on it.