Stress assignment and subsequentiality

While computational studies of stress patterns as phonotactics have yielded restrictive characterizations of stress (Rogers et al. 2013) with provably correct learning procedures (Heinz 2009), an outstanding question is the nature of stress assignment as a *function* which assigns stress to an underlying bare string of syllables. This paper fills this gap by locating stress patterns with respect to the *subsequential* class of functions (Mohri 1997), which are argued to be important for phonology in that the vast majority of phonological functions fall within the subsequential boundary (Heinz & Lai 2013, Chandlee 2014), with the notable exception of tone (Jardine 2016). The main result is that—while all Quantity Insensitive (QI) stress systems are subsequential functions—not all Quantity Sensitive (QS) stress systems are. Counter-intuitively, so-called *default-to-opposite* QS patterns are subsequential, but *default-to-same* QS patterns are provably not. We conclude that Jardine (2016)'s hypothesis that tone has access to non-subsequential functions should be extended to all suprasegmentals.

We describe subsequential functions with a *quantifier-free* (QF; Chandlee and Lindell, to appear) logic that is extended with *least-fixed point* operators (Libkin 2004). This allows for functions that refer to local information in the output by defining them recursively (Koser et al., to appear), but are sure to describe subsequential functions (Chandlee and Jardine, 2019). *Logical transductions* (Courcelle 1994) are defined with a predicate logic written with *terms* that range over positions in the input string and *atomic predicates* that evaluate the labels at those positions. For example, $\sigma(x)$ is true when x is evaluated at a position that is an unstressed syllable. Variables like x are terms; to refer to order in the string we also use a *successor function* s, whose application to a term is also a term. Thus $\sigma(s(x))$ is true when the position immediately following x is an unstressed syllable. By convention, we consider strings in which the successor of the final position is itself. An equality predicate $t_1 \approx t_2$ evaluated to true when terms t_1 and t_2 are evaluated as the same position in the string.

To illustrate, we can define shorthand predicates for the last and penultimate syllables as follows:

(1) $last(x) \overset{d}{=} s(x) \approx (x) \quad penult(x) \overset{d}{=} (last(s(x)) \wedge \neg last(x)) \vee only(x)$

We define $first(x)$ and $peninitial(x)$ in the same way, and $only(x) \overset{d}{=} first(x) \wedge last(x)$.

We can then define transductions by defining output stress in terms of the input. Thus, a non-iterative initial stress assignment function would be defined as in (2):

(2) $\acute{\sigma}(x) \overset{d}{=} first(x) \qquad \sigma\sigma\sigma\sigma \mapsto \acute{\sigma}\sigma\sigma\sigma$

$\sigma(x) \overset{d}{=} \neg first(x)$

The transduction states that an element is labeled as stressed in the output if its predecessor and itself are the same element. All non-iterative QI stress patterns—initial, peninitial, final, penultimate, antepenultimate, etc. (Hyman 1977, Gordon 2002)—can be defined in this QF way; that is, with a first-order logic that has no quantifiers.

In order to refer to the output, we use a QFLFP logic, which allows us to define QF predicates recursively. Instead of the full least-fixed point formalism, we employ *implicit definitions* of predicates (Rogers 1997), where the definition of a predicate can recursively refer to itself. A definition $\acute{\sigma}(x) \overset{d}{=} \acute{\sigma}(p(x))$, for example, states that a syllable is stressed in the output if its predecessor is also stressed *in the output*. Recursive predicates in a definition can refer to the successor function *or* a *predecessor function* p, but never both, as this exceeds the subsequential boundary (Chandlee and Jardine 2019).

For example, iterative QI patterns can be defined in QFLFP and are thus subsequential. In Pintupi (Hansen & Hansen 1969), primary stress is assigned to the first syllable, and secondary stress is assigned to every alternating syllable thereafter except the final syllable.

(3) $\acute{\sigma}(x) \overset{d}{=} first(x)$ (a)

$$\grave{\sigma}(x) \overset{d}{=} \underbrace{(first(p(p(x))) \wedge \neg peninitial(x))}_{\text{(b)}} \vee \underbrace{\grave{\sigma}(p(p(x)))}_{\text{(c)}} \wedge \underbrace{\neg(last(x) \wedge \neg only(x))}_{\text{(d)}}$$

1. $\sigma\sigma\sigma\sigma\sigma\sigma\sigma \mapsto \acute{\sigma}\sigma\grave{\sigma}\sigma\sigma\sigma\sigma$ 2. $\acute{\sigma}\sigma\grave{\sigma}\sigma\sigma\sigma\sigma \mapsto \acute{\sigma}\sigma\grave{\sigma}\sigma\grave{\sigma}\sigma\sigma$

The formulae are interpreted as follows. Evaluation of $\acute{\sigma}(x)$ puts primary stress on the first syllable, as seen in (3a). The first pass of evaluation of $\grave{\sigma}(x)$ puts secondary stress on the third syllable via disjunct (3b)—its predecessor's predecessor is the first element in the string, and it is not peninitial. The second pass puts secondary stress on the fifth syllable—the element two positions to the left bears secondary stress in the output, satisfying disjunct (3c). The conjunct in (3d) prevents stressing of the final syllable while still allowing a monosyllabic form to bear stress. No other element satisfies the definition, and so the transduction converges on the correct surface form for Pintupi. A QFLFP transduction can be written for all iterative QI patterns in Gordon (2002), thus all are subsequential.

However, not all QS patterns are subsequential. In default-to-opposite patterns, a heavy syllable nearest an edge is stressed, or a light syllable at the opposite edge is stressed if no heavy syllables are present. The leftmost-heavy or right (LHOR) pattern of Kwakw'ala (Hayes 1995) is an example. A light syllable is only stressed when in the final position and not preceded by a heavy at any point. For a heavy syllable, it is exactly the same—it is only stressed when no heavies precede it, making it the leftmost heavy. Thus, the search of the string proceeds in the *same direction* via the predecessor function when checking if either a light or heavy syllable should be labeled as stressed. This is shown in (4a). This can be defined in QFLFP with a predecessor function only, and is thus subsequential.

However, default-to-same patterns are *not* subsequential. In default-to-same patterns, a heavy syllable nearest an edge is stressed, or a light syllable at the same edge is stressed if no heavy syllables are present. Take the leftmost-heavy or left (LHOL) pattern of Lushootseed (Hayes 1995). For heavy syllables, the generalization is the same as above. A light syllable, however, can only be stressed when no heavy syllables *follow* it. This is contrasted with LHOR in (4b).

(4) a. LHOR: $\overleftarrow{\text{LL}}\boxed{\acute{\text{H}}}$LHL $\overleftarrow{\text{LLLLLL}}\boxed{\acute{\text{L}}}$ b. LHOL: $\overleftarrow{\text{LL}}\boxed{\acute{\text{H}}}$LHL $\boxed{\acute{\text{L}}}\overrightarrow{\text{LLLLLL}}$

This means that the computation of the LHOL function must be sensitive to information that may lie unboundedly far in both directions, meaning it is not subsequential (see, e.g., Jardine 2016). In QFLFP terms, the function must be defined with *both* the successor function *and* the predecessor function. This result is counter-intuitive—the "opposite-sided" nature of the pattern might lead one to assume it is more complex, whereas a computational study reveals that the opposite is true.

In sum, QI stress assignment is subsequential, but the same is not true of QS stress. In particular, default-to-same patterns are provably non-subsequential—a counter-intuitive result given that default-to-opposite stress *is* subsequential. A question that arises is *why* some stress functions should be non-subsequential. Jardine (2016) proposes that tonal processes can access non-subsequential functions. We take the results of this paper as evidence that this should be extended to all suprasegmental processes. This may be representational—if tone and stress are defined over multi-tiered representations, then the computational power needed to parse these representations may be higher than is needed for simpler segmental phenomena. Future work will further articulate this hypothesis. Another avenue of research is to investigate the evidence for the non-subsequentiality of stress by studying the veracity of the LHOR cases, and to study stress typologies for other cases of non-subsequential stress.

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