LASER POLARIZATION OF POSITRON BEAM.

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1 Introduction.

As it was pointed out in works [1, 2], for a number of physical studies which are planned to be made with the next generation colliders, it is necessary to use polarized beams of both electrons and positrons. The problem of producing and acceleration of polarized electrons may be considered to be solved [3], but the existing approaches to create polarized positron beams [4–8] do not ensure the parameters required (see [9]).

This work proposes a new approach to produce polarized positron beams, which is based on the process of multiple Compton backscattering of unpolarized ultrarelativistic positrons in the field of intense circularly polarized laser wave.

Under sufficiently high intensity of a laser flash a positron undergoes $N \gg 1$ successive collisions with laser photons and at each collision a positron loses a part of its initial energy. A positron beam acquires (on the average) a partial polarization along the direction coinciding with the photon momentum.

As shown in work [10] the deflection of a relativistic positron (electron) after a single act of Compton back-scattering (CBS) is characterized by the angle $\theta_0 \sim \frac{\omega_0}{mc^2} \ll \gamma_0^{-1}$, if $\gamma_0 \leq 10^5$, here $\omega_0$ is the laser photon energy ($\sim 1 \text{ eV}$), $\gamma_0$ is the Lorentz-factor of an initial particle. Therefore, one can assume as a first approximation that the divergence of positron beam does not change, and, further it is possible to consider the characteristics of a positron beam, as a whole, after its passage through a laser flash.

The quantization axis ($z$-axis) is chosen to be parallel to the initial positron momentum (i.e., antiparallel to the laser photon momentum). It is clear that for obtaining a nonzero polarization of the final beam, it is necessary that after a multiple process of CBS the "occupancies"of spin states should differ:

$$N_\uparrow \neq N_\downarrow,$$

if for the initial unpolarized beam

$$N_\uparrow = N_\downarrow.$$
In a single act of CBS the spin flip processes are described by \( w_{\uparrow \downarrow} \) and \( w_{\downarrow \uparrow} \) probabilites. In case when \( w_{\uparrow \downarrow} = w_{\downarrow \uparrow} \), the process of laser polarization is impossible, while at \( w_{\uparrow \downarrow} \neq w_{\downarrow \uparrow} \) in the process of multiple Compton scattering the positron beam can achieve the polarization degree equal to:

\[
\xi_{\max} = \frac{w_{\uparrow \downarrow} - w_{\downarrow \uparrow}}{w_{\uparrow \downarrow} + w_{\downarrow \uparrow}}.
\]

Here \( \xi_{\max} \) is the mostly achievable value of the \( z \)-component of the final beam polarization.

In this work the expressions for \( \omega_{\uparrow \downarrow} \) and \( \omega_{\downarrow \uparrow} \), were obtained and it was shown that \( \omega_{\uparrow \downarrow} \neq \omega_{\downarrow \uparrow} \); the requirements to the parameters of the laser flash to obtain the significant polarization were analyzed; the process of laser polarization of particles in the storage ring during a short-time period was considered.

## 2 Compton-effect cross-section on longitudinally polarized electron (positron) with spin flip.

The process of Compton backscattering (CBS) will be considered in a system where an electron (positron) is at rest (rest frame - RF), and a circularly polarized laser photon moves against the axis \( z \). We will be also interested in the component of the recoil electron spin along the axis \( z \), since, because of the azimuthal symmetry of the process after averaging over the assembly of the initial beam particles, the transverse projections of the spin are set to zero.

The authors of classical work [11] calculated the cross-section of Compton effect for an electron at rest, considering the correlations among polarizations of all four particles taking part in the reaction.

After summing up over polarizations of a scattered photon according to [11], one can get

\[
\frac{d\sigma}{d\Omega} = 2r_0^2 \left( \frac{k}{k_0} \right)^2 \left\{ \Phi_0 + P_c \xi_{0z} \Phi_1 + P_c \xi_z \Phi_2 + \xi_{0z} \xi_z \Phi_3 \right\}.
\]

Here \( r_0 \) - a classical radius of an electron; \( k_0 \) and \( k \) - are the energies of the initial and scattered photons; \( P_c \) - is the degree of a circular polarization of a laser photon; \( \xi_{0z}, \xi_z \) - are the spin components of initial and scattered electrons; the functions of \( \Phi_i \) will be determined later. A similar cross-section for a positron may be obtained after a simple changing the signs of the terms proportional to \( P_c \).

It should be pointed out that in (1) the term which is proportional to \( P_c \xi_{0z} \xi_z \) is absent.

For electrons with the initial energy \( \gamma_0 \leq 10^4 \) and the energy of the laser photon \( \omega_0 \sim 1 \) eV in the laboratory frame (LF) the initial photon energy in the RF system is obtained after the Lorentz transformation (the system \( \hbar = m = c = 1 \) is used)

\[
k_0 = (1 + \beta_0) \gamma_0 \omega_0 = 2 \gamma_0 \omega_0 << 1,
\]

and the energy of a scattering photon may be obtained from the conservation laws:

\[
k = \frac{k_0}{1 + k_0(1 - \cos \theta)} \approx k_0 [1 - k_0(1 - \cos \theta)].
\]
Here $\theta$ is the angle of a scattered photon in the RF.

In equation (3) the terms of order of $k_0^3$ and higher are neglected. This approximation will be also used further. Let us write down the functions $\Phi_i(k_0, \cos \theta)$:

$$\Phi_0 = \frac{1}{8} \left[ 1 + \cos^2 \theta + k_0^2 (1 - \cos^2 \theta) \right],$$

$$\Phi_1 = -\frac{1}{8} \cos \theta (1 - \cos \theta) [2k_0 - k_0^2 (1 - \cos \theta)],$$

$$\Phi_2 = -\frac{1}{8} (1 - \cos \theta) [2k_0 \cos \theta - k_0^2 (1 + \cos \theta - 2 \cos^2 \theta)],$$

$$\Phi_3 = \frac{1}{8} \left[ 1 + \cos^2 \theta - k_0^2 \left( \frac{1}{2} \cos \theta - 2 \cos^2 \theta + \cos^3 \theta \right) \right].$$

From (1) and (4) after simple integration one has:

$$\sigma = \pi r_0^2 \left[ \frac{4}{3} (1 - 2k_0 + \frac{26}{3} k_0^2) + P_c \xi_{0z} \frac{1}{3} \left( 2k_0 - 10k_0^2 \right) + P_c \xi_z \frac{1}{3} \left( 2k_0 - 8k_0^2 \right) + \right.\xi_{0z} \xi_z \frac{4}{3} \left( 1 - 2k_0 + \frac{22}{5} k_0^2 \right) \right] = \sigma_0 + P_c \xi_{0z} \sigma_1 + P_c \xi_z \sigma_2 + \xi_{0z} \xi_z \sigma_3. \tag{5}$$

Section (5) is invariant and may be used both in the RF, and the LF. The probability of the Compton effect with a spin flip is proportional to cross-section (5) when $\xi_{0z}$ and $\xi_z$ have opposite signs:

$$w_{\uparrow} = w(\xi_{0z} = +1, \xi_z = -1) = \text{const} \{ \sigma_0 + P_c \sigma_1 - P_c \sigma_2 - \sigma_3 \}$$

$$w_{\downarrow} = w(\xi_{0z} = -1, \xi_z = +1) = \text{const} \{ \sigma_0 - P_c \sigma_1 + P_c \sigma_2 - \sigma_3 \}$$

As it follows from (5) $\sigma_1 \neq \sigma_2$, therefore, the polarization of scattering electrons in CBS (laser polarization) is, in principle, possible.

Cross-section (5) can be derived from the invariant expressions with taking into account the polarization of all particles [12, 13]. Thus, the authors in [13] obtained the CBS section depending on the longitudinal polarization of a scattered electron (i.e., on the projection of a scattered electron spin on its momentum) in the invariant form. Following from the above work, let us write down the cross-section where the component of a scattered electron polarization is maintained along the same axis $z$ as that of the initial electron:

$$\frac{d\sigma}{dy} = \pi r_0^2 \frac{1}{x} \left\{ F_0 + (s_z G_2 + c_z G_3) \xi_z \right\} \tag{6}$$

In (6) averaging over the azimuthal angle was performed. The invariants $x, y$ are found in a standard way:

$$x = 2pk, \quad y = 1 - \frac{pk'}{pk} \tag{7}$$

where $p, k, k'$ are the four-vectors of the initial electron and photon and the scattered photon, respectively. The functions $F_0, G_2, G_3$ are determined as:

$$F_0 = \frac{1}{1 - y} + 1 - y - s^2 + \frac{2 - y}{1 - y} c \xi_{0z} P_c$$
\begin{equation}
G_2 = yscP_c + (1 + c^2 - yc^2) \ s \xi_{0z},
\end{equation}
\begin{equation}
G_3 = \left( \frac{y}{1-y} + yc^2 \right) P_c + \left[ \frac{1}{1-y} + (1-y)c^2 \right] \ c \xi_{0z},
\end{equation}
where \( s = 2\sqrt{r(1-2)}, \ c = 1 - 2r, \ r = \frac{y}{(1-y)x}. \)

The coefficients \( s_z, \ c_z \) are none other than the coefficients of the rotation matrices from the basis used in [12] to the basis where one of the axes coincides with the axis \( z \).

Substituting (1) into (6) let us write down the section in the form as in (1):
\begin{equation}
d\sigma \ = \ \pi r_0^4 x \left( \frac{1}{1-y} + 1 - y - s^2 + \xi_{0z} \ c \ \frac{2y - y}{1-y} + \right.
\end{equation}
\begin{equation}
+ P_c \xi_{0z} \left[ s_z ysc + c_z \left( \frac{y}{1-y} + yc^2 \right) \right] + \xi_{0z} \xi_z \left[ s_z s(1 + c^2 - yc^2) + \right.
\end{equation}
\begin{equation}
+c_z c \left( \frac{1}{1-y} + (1-y)c^2 \right) \right] = \pi r_0^4 x \left\{ \Phi_0 + \Phi_1 P_c \xi_{0z} + \Phi_2 P_c \xi_z + \Phi_3 \xi_{0z} \xi_z \right\}
\end{equation}
One can easily see that, if \( s_z = s, \ c_z = c, \) (which corresponds to the longitudinal polarization of a scattered electron), \( \Phi_1 = \Phi_2 \). However, generally speaking, \( s_z \neq s, \ c_z \neq c \), which can be easily seen in the RF. Let us calculate section (9) in the RF and compare with the results obtained earlier (Eqs. (4), (5)). Coefficients \( s_z, c_z \) are defined as
\begin{equation}
s_z = -\frac{k_0}{k_0} \bar{n}_2', \ c_z = -\frac{k_0}{k_0} \bar{n}_3',
\end{equation}
where the unit vectors \( \bar{n}_2', \bar{n}_3' \) determine the basis used in [12]. In the RF by neglecting the terms \( \sim k_0^3 \) and higher, one has from (10)
\begin{equation}
s_z = \frac{\sin \theta(1 + k_0)}{1 + k_0(1 - \cos \theta)} \left[ 1 - k_0 \frac{1 - \cos \theta}{2} \right],
\end{equation}
\begin{equation}
c_z = \frac{\cos \theta - k_0(1 - \cos \theta) - k_0^2(1 - \cos \theta) \cos \theta}{1 + k_0(1 - \cos \theta)},
\end{equation}
while in the same system (see [13])
\begin{equation}
s = \sin \theta, \ c = \cos \theta.\)
\end{equation}
Passing from the invariant variables to those used earlier
\begin{equation}
x = 2k_0, \ y = 1 - \frac{k}{k_0} = \frac{k_0(1 - \cos \theta)}{1 + k_0(1 - \cos \theta)}
\end{equation}
and substituting (11), (12), (13) into cross-section (9), one can show that the section obtained completely coincides with (5), in the same approximation as above.
3 Process of Multiple Compton Backscattering. Main characteristics.

The number of hard photons $N$ produced in colliding of one electron with photons of the laser flash is a random value as well as the other characteristics of the process of multiple CBS (total radiation losses, the electron multiple scattering angle, etc.).

For simplicity, further we will consider only mathematical expectations of these random values, omitting, as a rule the averaging symbol $\bar{N} = <N>$. In addition, we will deal with collisions of single positron bunches with photons. The $N$ value (according to work [10] it is the conversion coefficient) can be found by using the luminosity of the process:

$$ N = \frac{L\sigma}{N_{e^+}}. $$

Here $N_{e^+}$ is the number of positrons in a bunch; $\sigma$ is the cross-section of CBS process; $L$ is the luminosity:

$$ L = 2eN_0N_{e^+}\int \int dV \int dt f_{ph}(x, y, z + ct) f_e(x, y, z - \beta ct). \tag{14} $$

In (14) $N_0$ is the number of photons in a laser flash; $f_{ph}, f_e$ are the normalized distributions of photons and positrons in bunches.

For the analytical estimations we will consider monodirected positron and photon beams with the Gauss distribution in the transverse and longitudinal directions:

$$ f_e = \frac{2}{(2\pi)^{3/2}\sigma_e^2 l_e} \exp \left\{ -\frac{r^2}{2\sigma_e^2} - \frac{(z - \beta ct)^2}{2 l_e^2} \right\}, \tag{15} $$

$$ f_{ph} = \frac{2}{(2\pi)^{3/2}\sigma_{ph}^2 l_{ph}} \exp \left\{ -\frac{r^2}{2\sigma_{ph}^2} - \frac{(z + ct)^2}{2 l_{ph}^2} \right\}, $$

$$ r^2 = x^2 + y^2. $$

In this case (the so-called case of short-length bunches [10]) the luminosity is calculated analytically

$$ L = N_{e^+} N_0 \frac{2}{\pi(\sigma_e^2 + \sigma_{ph}^2)} \tag{16} $$

and it does not depend on the bunch lengths $2l_e, 2l_{ph}$ (in other words, on the time of interaction). For $k_0 \ll 1$ we have:

$$ N = 2N_0 \frac{\sigma}{\pi(\sigma_e^2 + \sigma_{ph}^2)} \approx \frac{2A}{\omega_0} \cdot \frac{2\sigma_0}{\pi(\sigma_e^2 + \sigma_{ph}^2)}, \tag{17} $$

where $A$ is the laser flash energy, $\omega_0$ is the laser photon energy. The cross-section summed up over the spin of a scattered positron and averaged over the spin of initial ($\sigma_1 \ll \sigma_0$) is substituted into (17). If the beam after the "damping-ring" is used as an initial one, in this case $\sigma_e^2 \ll \sigma_{ph}^2$ and, therefore,

$$ N = \frac{A}{\omega_0} \frac{16}{3} \left(\frac{r_0}{\sigma_{ph}}\right)^2 = 4 \frac{A}{\omega_0} \frac{\sigma_\tau}{\lambda Z_n} = 2n_\tau \sigma_\tau l_{ph}. \tag{18} $$

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Here $\lambda$ is the wavelength of the laser photon; $Z_n$ is Rayleigh length; $n_z$ is the photon concentration in the laser flash; $\sigma_{\tau} = \frac{8}{3} \pi r_0^2$; $2l_{ph}$ is the length of the laser pulse.

Let us evaluate the average positron energy after the process of multiple CBS. The average energy after the first collision act is found from the condition

$$< \gamma_1 > = \gamma_0 - < \omega_1 >,$$

where $< \omega_1 >$ is the average energy of an emitted photon in the LF, which depends on the average characteristics of a photon in the RF:

$$< \omega_1 > = \gamma_0 (< k > - \beta_0 < k || >),$$

$$< k > = \int k \left( \frac{k}{k_0} \right)^2 \Phi_0 d\Omega / \int \left( \frac{k}{k_0} \right)^2 \Phi_0 d\Omega = k_0 (1 - k_0),$$

$$< k || > = \int k \cos \theta \left( \frac{k}{k_0} \right)^2 \Phi_0 d\Omega / \int \left( \frac{k}{k_0} \right)^2 \Phi_0 d\Omega \approx \frac{6}{5} k_0^2.$$

Thus,

$$< \gamma_1 > = \gamma_1 \approx \gamma_0 (1 - k_0) = \gamma_0 (1 - 2 \gamma_0 \omega_0).$$

Before the second scattering act a laser photon in the RF will have a smaller energy

$$< k_1 > = k_1 = 2 \gamma_1 \omega_0 = k_0 (1 - k_0).$$

In a similar way one can obtain the relation (further, for the sake of simplificatoton the averaging symbol is neglected again)

$$\gamma_{i+1} = \gamma_i (1 - 2 \omega_0 \gamma_i).$$

(20)

From (20) we get the equation in the finite differences

$$\Delta \gamma_i = \gamma_{i+1} - \gamma_i = 2 \omega_0 \gamma_i^2.$$

If the number of scattering acts $k \gg 1$, then, passing from the finite difference equation to the differential one and solving the latter, we have:

$$\gamma_k = \frac{\gamma_0}{1 + 2 \gamma_0 \omega_0 k} = \frac{\gamma_0}{1 + 2 \mu}.$$

(21)

By making use of (21) one can estimate the total radiation losses by each positron after $N$ collisions

$$\Delta \gamma = \gamma_0 - \gamma_N = \gamma_0 \frac{2 \mu}{1 + 2 \mu} = 2 \gamma_0^2 \omega_0 \frac{N}{1 + 2 \gamma_0 \omega_0 N}.$$

(22)

V. Telnov [14] proposed to use the process of a multiple CBS for cooling the head-on electron (positron) beam. Let us estimate the laser flash parameters and the number of hard photons to decrease an electron initial energy by one order of magnitude:

$$\gamma_N = \frac{\gamma_0}{1 + 2 \gamma_0 \omega_0 N} = \frac{\gamma_0}{10}.$$
Hence, we have the following relations for the collision number
\[ N_{10} = \frac{9}{2\gamma_0 \omega_0}, \]
and laser energy:
\[ A = \frac{9 \lambda Z_R}{8 \gamma_0 \sigma_r}. \]  
(23)

It follows from the rough consideration (similar to the V. Telnov approach [14]) that monochromaticity of a positron beam improves with the growth of \( N \):
\[ \frac{\Delta \gamma_N}{\gamma_N} \approx \sqrt{\frac{\gamma_0 \omega_0}{1 + 2\gamma_0 \omega_0 N}} \approx \frac{1}{2N}, \quad N \gg 1, \quad \mu \gg 1. \]

4 Single-pass laser polarization of positron beam.

If the initial positron is unpolarized and the laser photons possess a 100 % right-hand circular polarization \( (P_c = +1) \), then, after the first scattering act the recoil positrons, on the whole, become partly polarized. The value of the polarization component along the \( z \) axis can be found from cross-section (5)
\[ \xi_1 = \frac{\sigma_2}{\sigma_0} \approx \frac{k_0}{2} (1 - 2k_0) = \gamma_0 \omega_0 (1 - 4\gamma_0 \omega_0). \]  
(24)

Henceforth the index \( z \) will be neglected. Taking into account that before the second scattering act the photon energy in the RF equal to \( k_1 = k_0 (1 - k_0) \) from (5) one can find the positron polarization after the second scattering act:
\[ \xi_2 = \frac{\sigma_2 + \xi_1 \sigma_3}{\sigma_0 + \xi_1 \sigma_1} \approx \frac{\frac{2}{3} k_1 (1 - 4k_1) + \xi_1 \frac{4}{3} (1 - 2k_1)}{\frac{4}{3} (1 - 2k_1) + \xi_1 \frac{2k_1}{3} (1 - 5k_1)} \approx \]
\[ \approx k_0 (1 - \frac{7}{2} k_0) = 2\gamma_0 \omega_0 (1 - 7\gamma_0 \omega_0). \]  
(25)

Comparing (25) and (24) one can see that \( |\xi_2| > |\xi_1| \). By applying the method similar to the one used to derive (21) the polarization value after \( N \) collisions can be found
\[ \xi_N = \frac{\gamma_0 \omega_0 N}{1 + \gamma_0 \omega_0 N} = \frac{\mu}{1 + \mu}. \]  
(26)

However, it should be pointed out that the immediate use of the derived formula in order to evaluate the final polarization of the positron beam is illegitimate. The problem is that, as an unpolarized positron beam is passing through a laser flash, the positrons which have not interacted with the laser photons become polarized but in the opposite direction [15]. Thus, the positrons in this part the beam will have a nonzero polarization before the first collision act, and that is why the use of formulas (25,26) in this case gives a wrong results.

To obtain a correct result it is necessary to perform a Monte Carlo simulation of CBS process. But the equilibrium polarization value can be found from the elementary balance equations. In passing polarized positrons through a laser flash having the photon
concentration $n_z$ hard photons are emitted in the process with a spin flip ($\xi_z = -\xi_{0z}$),
and without one ($\xi_z = +\xi_{0z}$). The change of the number of positrons having the spins
oriented along the positive $N^\uparrow (\xi_z = +1)$ and negative direction of the quantization axis
$N^\downarrow (\xi_z = -1)$ is described by the equation resulting from cross-section (9):

$$\frac{dN^\uparrow}{dz} = 2n_z[-(\sigma_0 + \sigma_1 - \sigma_2 - \sigma_3)N^\uparrow + (\sigma_0 - \sigma_1 + \sigma_2 - \sigma_3)N^\downarrow],$$

$$\frac{dN^\downarrow}{dz} = 2n_z[-(\sigma_0 + \sigma_1 - \sigma_2 - \sigma_3)N^\downarrow + (\sigma_0 - \sigma_1 + \sigma_2 - \sigma_3)N^\uparrow].$$

(27)

By adding and subtracting Eqs. (27) we obtain standard balance equations [16]:

$$\frac{d(N^\uparrow + N^\downarrow)}{dz} = 0$$

$$\frac{d(N^\uparrow - N^\downarrow)}{dz} = 2n_z\left[-2(\sigma_0 - \sigma_3)(N^\uparrow - N^\downarrow) - 2(\sigma_1 - \sigma_2)(N^\uparrow + N^\downarrow)\right].$$

(28)

Balance equations (28) have the following solution:

$$\xi_z(z) = \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} = -\frac{\sigma_1 - \sigma_2}{\sigma_0 - \sigma_3}\{1 - \exp[4n_z(\sigma_0 - \sigma_3)z]\}. \quad (29)$$

Here $z$ is the thickness of the "laser"target. It follows from (29) that the maximum
polarization degree (at $z \to +\infty$):

$$\xi_{z,\text{max}} = -\frac{\sigma_1 - \sigma_2}{\sigma_0 - \sigma_3} = -\frac{5}{8}.$$ 

(30)

The characteristic length of the laser bunch $Z_{pol}$, after passing which the positron beam
acquires the polarization degree $(1 - e^{-1})\xi_{z,\text{max}} \approx 0.63 \xi_{\text{max}}$, is found from the following
relation:

$$Z_{pol} = \frac{1}{4n_z(\sigma_0 - \sigma_3)}. \quad (31)$$

From (18) and (31) we can find another characteristic of the laser polarization process
- the average number of $N_{pol}$ photons emitted by each positron after passing the length
$Z_{pol}$:

$$N_{pol} = \frac{8\pi}{3}m_c^2Z_{pol} = \frac{2\pi r_0^2}{3} \cdot \frac{1}{\sigma_0 - \sigma_3} = \frac{5}{8k_0^2} = \frac{5}{32\gamma_0^2\omega_0^2}. \quad (32)$$

Formula (32) is valid for $k_0 = 2\gamma_0\omega_0 \ll 1$, since it is in this approximation that the
expressions for $\sigma_0$ and $\sigma_3$ (see (9)) were obtained. The value $N_{pol}$ does not depend on
the specific laser parameters and is a more suitable value, for example, in the case of a
multiple passage of positrons through the laser bunch in the storage ring [9].

As it follows from (32) and (18) the characteristic number of hard photons necessary
for cooling and polarization processes depends on the energy of a laser photon in the RF
such as $\sim 1/k_0$ and $\sim 1/k_0^2$. 

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But for the above case $\gamma_0\omega_0 = 0.05$ and, consequently,
\[ N_{\text{pol}} = \frac{5}{32(0.05)^2} = 62 \sim N_{10}. \]

Having rewritten formula (29) in which one should pass from the laser bunch length to the number of collision acts (i.e., to the number of emitted photons (see (18)):
\[ \exp(-\frac{z}{Z_{\text{pol}}}) = \exp(-\frac{n_z\gamma_\tau z}{N_{\text{pol}}}) = \exp(-\frac{N}{N_{\text{pol}}}), \]
one gets the expression for the highest attainable polarization of the positron beam
\[ \xi_z = \xi_{z\text{max}} \left[ 1 - \exp\left( -\frac{N}{N_{\text{pol}}} \right) \right], \quad (33) \]
For the example considered $\xi_z = 0.41$.

5 Laser polarization in storage ring.

It was shown in the previous part that by applying a sufficiently powerful laser one can, in principle, attain a polarized positron beam during a single pass through a laser flash. However, the laser parameters to create the significant beam polarization with the time structure necessary for linear colliders are beyond the present-day technology. An obvious way to increase luminosity of the process is to use a high Q-factor optical resonator matched with the storage ring where a positron beam is circulating.

In work [17] the authors discussed the possibility of applying a laser with circularly polarized radiation in order to reduce the time of electrons self-polarization in the storage ring, and they showed that the CBS process can be used for this purpose.

In his recent work [9] J. Clendenin proposed using a resonator and a damping ring for simultaneous laser cooling of a positron beam and its laser polarization. It is clear that the requirements to the laser power are, in this case, considerably decreased (see the estimations the work cited).

J. Clendenin considered a damping ring with a circumference of 297 m for positrons with $E = 1.98$ GeV, which is designed for synchrotron cooling radiation. For laser cooling of 100 MeV electrons the authors of [18] proposed a small-sized ring which 2 m in diameter. It is of special interest to estimate the feasibility of this ring for laser polarisation.

For a laser with the wavelength $\lambda \sim 1$ mm ($\omega_0 = 1.25$ eV) the parameter $k_0 = 2\gamma_0\omega_0 \approx 10^{-3}$, therefore, $N_{\text{pol}} = 6.1 \cdot 10^5$.

During a single pass through the laser pulse having the energy $\sim 1$ J and $\lambda_z = 1$ mm each positron experiences, on the average, $N = 2A/\omega_0 \sigma_\tau \lambda z_R = 0.82$ collisions with laser photons, and the average energy of each emitted photon $<\omega_1> = 2\gamma_0^2\omega_0$.

Consequently, the energy losses by each positron $\Delta \gamma = 4\sigma_\tau \gamma_0^2 A/\lambda z_R$, which coincides with formula (3) of work [18].

Thus, the necessary number of turns in the ring to attain $N_{\text{pol}}$ is:
\[ n = \frac{N_{\text{pol}}}{N} = 7.4 \cdot 10^5, \]
which corresponds to the time $\tau_{pol} = \pi d/c = 15.5 \cdot 10^{-3}$ sec. It seems that a serious problem for such a ring is to maintain power in the resonator during the time mentioned.

It takes $\sim 10^3$ laser injections into the resonator during the time $\tau_{pol}$ if the cavity Q-factor $\sim 10^3$.

6 Conclusion

All the results were obtained with neglect of the contribution of the nonlinear processes. It is clear that for the laser flash parameters discussed in Parts 3 and 4 a linear approximation will be valid, if the length of the laser pulse is rather high: $l_{ph} >> Z_a$. To maintain a transverse size of the laser beam at a suitable level along the full length $l_{ph}$, it is necessary to apply a channeled laser beam, e.g., in a plasma channel [19] or in a capillary dielectric tube [20].

For a substantially nonlinear process, when during a single collision act $n > 1$ laser photons are absorbed, the average energy of an emitted photon will considerably exceed the one for a linear process, which will result in decrease in the value $N_{pol}$. In other side it should be noted the value of polarization degree $\xi_{max} = 5/8$ was obtained in work [21] for moving electron in a long weak helical undulator (see, also, [17]).

Authors of cited paper considered also the case of strong helical undulator where electron polarization may be much higher ($\xi_{max} = 0.92$). It is known the trajectory of electron in a helical undulator is similar to one in a field of the intense circularly-polarized wave. Having in mind the analogy between radiative processes in the helical undulator and circularly-polarized wave one may expect that positron polarization during nonlinear multiple CBS process may exceed the value of (30) and approach to 92%, typical value for self-polarization mechanism due to synchrotron radiation.

It is evidently the polarization of positron through nonlinear multiple CBS process demands the detailed investigations.

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Список литературы


