

MASSES OF NEUTRON STARS AND BLACK HOLES IN X-RAY BINARIES.

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The determination of masses for compact stars in X-ray binaries is useful for at least four reasons [1]. The values of the masses provide (1) a unique test of the combined predictions of theories of dense (hadronic or quark) matter and of general relativity, (2) information about the final stages of stellar evolution, (3) answers to questions about the nature of the X-ray source itself, (Cyg X-1 is a good example since the mass inferred for the X-ray star exceeds the theoretical limits for the masses of neutron stars and of white dwarfs, and is hence commonly regarded as a black hole), and (4) the means to understand specific phenomena, such as the spin-up rate of the pulsar period and various other processes that depend on mass transfer.

The mass determinations are achieved through a combination of Newton's law of gravity and Kepler's laws of planetary motion, and involve the determination of the so-called "mass function" [1]

$$f(M_1, M_2, i) = \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2} = \frac{Pv_1^3}{2\pi^2}. \quad (1)$$

Above, M_1 and M_2 are masses of objects in circular orbit about their center of mass (elliptic orbits require separate treatment), the angle i is the inclination of the orbital plane to the line of sight, P is the orbital period, and v_1 is the projection of the orbital velocity of M_1 along the line of sight. The mass function f depends only on the observable quantities P and v_1 . Because $M_1/M_2 > 0$ and $\sin i \leq 1$, it follows that $M_2 > f(M_1, M_2, i)$: the measured mass function f is a lower limit to the mass M_2 of the compact object.

Although straightforward numerical computations can yield the value of M_2 , analytical solutions are invaluable in obtaining additional insights. Towards this goal, we cast Eq. (1) in the form

$$M_2^3 - \bar{f}(M_1 + M_2)^2 = 0, \quad (2)$$

where $\bar{f} = f/\sin^3 i$. Regarding \bar{f} and M_1 as independently known quantities, we found the physically relevant real root of the above cubic equation to be

$$\begin{aligned} M_2 = & \left[\frac{\bar{f}M_1^2}{2} \left(1 + \frac{2\bar{f}}{3M_1} \right) + \frac{\bar{f}^3}{27} + \frac{\bar{f}M_1^2}{2} \left(1 + \frac{4\bar{f}}{27M_1} \right)^{1/2} \right]^{1/3} \\ & + \left[\frac{\bar{f}M_1^2}{2} \left(1 + \frac{2\bar{f}}{3M_1} \right) + \frac{\bar{f}^3}{27} - \frac{\bar{f}M_1^2}{2} \left(1 + \frac{4\bar{f}}{27M_1} \right)^{1/2} \right]^{1/3} + \frac{\bar{f}}{3}. \end{aligned} \quad (3)$$

We verified that the results from Eq. (3) matched exactly those obtained from numerical computations.

The analytical solution in Eq. (3) allows us to examine some interesting limiting situations:

(1) In the case $M_1/\bar{f} \ll 1$, we find that

$$M_2 \cong \bar{f} \left[1 + 2 \left(\frac{M_1}{\bar{f}} \right) - 3 \left(\frac{M_1}{\bar{f}} \right)^2 + 10 \left(\frac{M_1}{\bar{f}} \right)^3 - 42 \left(\frac{M_1}{\bar{f}} \right)^4 + 198 \left(\frac{M_1}{\bar{f}} \right)^5 \dots \right], \quad \text{and} \quad (4)$$

(2) For $\bar{f}/M_1 \ll 1$, we obtain the result

$$M_2 \cong \frac{\bar{f}}{3} + M_1 \left[\left(\frac{\bar{f}}{M_1} \right)^{1/3} \left\{ 1 + \frac{10}{81} \left(\frac{\bar{f}}{M_1} \right) - \frac{22}{6561} \left(\frac{\bar{f}}{M_1} \right)^2 + \frac{374}{1594323} \left(\frac{\bar{f}}{M_1} \right)^3 - \dots \right\} \right. \\ \left. + \frac{2}{3} \left(\frac{\bar{f}}{M_1} \right)^{2/3} \left\{ 1 + \frac{7}{162} \left(\frac{\bar{f}}{M_1} \right) - \frac{13}{6561} \left(\frac{\bar{f}}{M_1} \right)^2 + \frac{247}{1594323} \left(\frac{\bar{f}}{M_1} \right)^3 - \dots \right\} \right]. \quad (5)$$

The first few terms in each of the cases considered above can be obtained easily by the use of binomial expansions. The remaining terms, although straightforward, require tedious algebra. These terms were obtained using the symbolic algebraic program MAPLE.

It is worthwhile to note that the convergence of the series in Eq. (4) is less rapid than that in Eq. (5) principally because the coefficients in the alternating series of Eq. (4) grow rapidly in magnitude in contrast to those in Eq. (5) which decrease rapidly in magnitude.

These analytical solutions and their utility in the analysis discussed below are among the principal new results of this project.

We also examined the uncertainty in the inferred value of M_2 caused by specified uncertainties in the observable values of M_1 , f , and i . Following the procedure of error analysis outlined in Ref. [3], the variance (square of the standard deviation) in M_2 may be obtained from

$$\sigma_{M_2}^2 = \left(\frac{\partial M_2}{\partial M_1} \right)^2 \sigma_{M_1}^2 + \left(\frac{\partial M_2}{\partial f} \right)^2 \sigma_f^2 + \left(\frac{\partial M_2}{\partial i} \right)^2 \sigma_i^2 \\ + 2 \left(\frac{\partial M_2}{\partial M_1} \right) \left(\frac{\partial M_2}{\partial f} \right) \sigma_{M_1 f}^2 + 2 \left(\frac{\partial M_2}{\partial M_1} \right) \left(\frac{\partial M_2}{\partial i} \right) \sigma_{M_1 i}^2 + 2 \left(\frac{\partial M_2}{\partial f} \right) \left(\frac{\partial M_2}{\partial i} \right) \sigma_{f i}^2. \quad (6)$$

The first three terms in this equation are averages of squares of deviation weighted by the squares of the appropriate partial derivatives, and may be considered to be the averages of the squares of the deviations in M_2 produced by the uncertainties in M_1 , f , and i , respectively. The remaining covariant terms represent the average of the cross terms involving products of deviations in the possible pairs weighted by the product of the partial derivatives. Only in the case of entirely uncorrelated fluctuations in the measured quantities do these terms vanish. In the following, we turn to evaluate the necessary derivatives for an estimation of the error in determining M_2 .

Explicitly,

$$\begin{aligned} \frac{\partial M_2}{\partial M_1} &= \frac{2M_2}{(3M_1 + M_2)} \quad \text{or} \quad \frac{\partial \ln M_2}{\partial \ln M_1} = \frac{2}{3+q}, \\ \frac{\partial M_2}{\partial f} &= \frac{M_2}{f} \left(\frac{M_1 + M_2}{3M_1 + M_2} \right) \quad \text{or} \quad \frac{\partial \ln M_2}{\partial \ln f} = \frac{1+q}{3+q}, \\ \frac{\partial M_2}{\partial i} &= -3M_2 \left(\frac{M_1 + M_2}{3M_1 + M_2} \right) \cot i \quad \text{or} \quad \frac{\partial \ln M_2}{\partial \ln i} = -3 \left(\frac{1+q}{3+q} \right) i \cot i. \end{aligned} \quad (7)$$

We are performing an in-depth investigation of this issue in order to reliably estimate the errors involved in inferring the mass M_2 .

As a part of this project, we analyzed three photographs of the visual binary Krüger 60 taken in 1908, 1915, and 1920 [2]. Given the parallax of 0.254 seconds of arc and that a field star was 30 seconds of arc from the binary system in 1908, we performed measurements using the photographs that enabled us to determine the period $P \simeq 45$ years, the average angular separation $s \simeq 2.38$ seconds of arc, the distance from earth $d \simeq 8.14 \times 10^5$ AU (1 AU = 1.5×10^8 km), the linear separation between the binary stars $a = 9.36$ AU, and the combined mass $M_1 + M_2 \simeq 0.4M_\odot$ ($1M_\odot \simeq 2 \times 10^{33}$ g).

To date 14 binary systems are thought to host black holes. Optical and X-ray observations have yielded that $0.14 < f/M_\odot < 6.86$, $30^\circ < i < 78^\circ$, and $0.15 < M_{\text{opt}}/M_\odot < 20$ (we have used the central values in quoting these ranges) [4]. The inferred values of the masses of compact objects lie in the range $4 < M_{\text{BH}}/M_\odot < 12$. We are analyzing archival data of X-ray binaries purported to harbor black holes.

We are eagerly awaiting the opportunity to make our own observations of a new binary system containing a black hole. The frustration caused by not having a local "Telescope Facility" to explore space was unbearable.

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References

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