Methods of Laser Cooling of Electron Beams in Storage Rings

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Abstract

Methods of laser cooling of electron beams in storage rings and possible applications in physics are discussed.

1 INTRODUCTION

Different cooling methods were suggested to decrease the emittances of charged particle beams in storage rings. Except the stochastic all another methods of cooing are based on a friction of particles in external electromagnetic fields or in media when the Liouville's theorem does not work. In this paper the ordinary and enhanced methods of radiative laser cooling of electron beams in storage rings and possible applications are discussed.

2 RADIATIVE COOLING OF ELECTRON BEAMS

The ordinary three-dimensional radiative cooling of electron beams is based on using of the interaction of electron and laser beams at a straight section of a storage ring. A friction originating in the process of emission (scattering) of photons by electrons in external fields leads to a damping of theirs amplitudes of both betatron and phase oscillations. The damping is because of the friction force is parallel to the electron velocity, and therefore the momentum losses include both the transverse and longitudinal ones. Longitudinal losses are compensated by a radio frequency accelerating system of the storage ring. Meanwhile the longitudinal momentum of the electron tends to a certain equilibrium. The transverse vertical and radial momenta disappears irreversibly. Such a way the compression of phase-space density for a given ensemble of electrons takes place. The damping rates of both transverse and longitudinal oscillations can be redistributed by coupling transverse and longitudinal oscillations of electrons near betatron and synchro-betatron resonances.

In the three-dimensional radiative scheme of cooling a laser beam overlaps an electron beam, its transverse position is motionless, all electrons interact with the laser beam independent of their energy and amplitude of betatron oscillations. That is why the difference in rates of energy losses of electrons having maximum and minimum energies is small and the cooling time of the electron beam is high. E.g., when photo-electron interaction takes place in a dispersion-free straight section, the damping time of the horizontal betatron oscillation τ_x is equal to the vertical one τ_y and two times higher then phase one (τ_ϵ)

$$\tau_x = \tau_y = \frac{\tau_\epsilon}{2} = \frac{2\varepsilon}{\overline{P}}, \qquad (1)$$

where $\overline{P}$ is the average power of the radiation scattered by the electron of the energy ε.

The expression τ_ϵ in (1) is specific to the assumption that the intensity of the laser beam is constant inside the area of the laser beam occupied by the being cooled electron beam. The longitudinal-radial coupling arising in non-zero dispersion straight sections of the storage rings leads to a redistribution of the longitudinal and radial damping times when the radial gradient of the laser beam intensity is introduced.

First a one-dimensional laser cooling was applied to ion beams. It was based on the resonance Rayleigh scattering of laser photons by not fully stripped ion beams. The laser had a chirp of frequency [2] - [4]. Then a three-dimensional laser cooling of ions was suggested and developed in [5] - [8] proceeding from the analogy with the synchrotron radiation damping of amplitudes of electron oscillations in storage rings[1]. The electron version of the radiative cooling was developed in the paper [9]. In this paper Zh.Huang and R.D.Ruth have paid attention on the possibility to store in the optical high-finesse resonator laser wavepackets of picosecond duration and high intensities $I \simeq 10^{17}$ W/cm^2 to interact repetitively in a storage ring in the energy range $10 \div 10^2$ MeV with a circulating electron beam for the rapid cooling of the beam, counterbalancing of the intrabeam scattering and x-ray generation.

[1] The difference in cooling of electron and ion beams is in the dependence of the average power of scattered radiation $\overline{P}$ on the relative energy γ ($\overline{P}_e \sim \gamma^2$, $\overline{P}_{ion} \sim \gamma$) and in the nature of the Rayleigh scattering which includes an absorption and a decay. The last difference can be neglected when the length of the ion decay is smaller then the length of the straight section and the length of the period of ion betatron oscillations [5], [7].

3 THE ENHANCED LASER COOLING OF ELECTRON BEAMS

According to (1), the damping times of the three-dimensional method of cooling of transverse and longitudinal oscillations are equal to the time intervals, at which electron loose the energy equal to about the two-fold and four-fold initial energy of the electron accordingly. At the same time we know that the cooling time of an ion beam in the longitudinal plane

$$\tau_\epsilon = \frac{2\sigma_\varepsilon}{\overline{P}}. \qquad (2)$$

where σ_ε is the rms energy spread of the beam.

The cooling time (2) is less then (1) in the ratio $\varepsilon/\sigma_\varepsilon$. This is because of the selective (resonance) nature of interaction between the laser beam and ions (Rayleigh scattering). In this case moving ions interact with the laser beam only at resonance energy which is changed by scanning of the laser frequency in a frequency range $\Delta\omega_L/\omega_L = 2\sigma_\varepsilon/\varepsilon$. . They decrease energy until all of them reach the minimum energy of ions in the beam. At this position the laser beam is switched off.

The similar enhanced cooling of electrons both in the longitudinal and transverse planes separately or in turn can be done by using another selective interaction of electron and laser beams and the dispersion coupling of the radial and longitudinal directions. The idea of such cooling was presented in [10] and developed in [11]. Below we will discuss these ways of cooling.

3.1 Dispersion coupling of the transverse and longitudinal motion of electrons in storage rings

For the sake of simplicity below we will neglect the emission of the synchrotron radiation and suppose that the RF system of a storage ring is switched off, laser beam is homogeneous and has sharp edges. In this case in a smooth approximation, the movement of an electron relative to its instantaneous orbit is described by the equation

$$x_\beta = A\cos(\Omega t + \varphi). \qquad (3)$$

where $x_\beta = x - x_\eta$ is the electron deviation from the instantaneous orbit x_η, x is its radial coordinate, A and Ω the amplitude and the frequency of the electron betatron oscillations.

If the coordinate $x_{\beta\,0}$ and transverse radial velocity of the electron $\dot{x}_{\beta\,0} = -A\Omega\sin(\Omega t + \varphi)$ corresponds to the moment t_0 of change of the electron energy in a laser beam then the amplitude of betatron oscillations of the electron before interaction is $A_0 = \sqrt{x_{\beta\,0}^2 + \dot{x}_{\beta\,0}^2/\Omega^2}$. After the interaction, the position of the electron instantaneous orbit will be changed at a value δx_η, the deviation of the electron relative to the new orbit will be $x_{\beta\,0} - \delta x_\eta$, and its transverse velocity will not be changed. The new amplitude of the electron betatron oscillations will be $A_1 = \sqrt{(x_{\beta\,0} - \delta x_\eta)^2 + \dot{x}_{\beta\,0}^2/\Omega^2}$ and the change of the square of the amplitude

$$\delta(A)^2 = A_1^2 - A_0^2 = -2x_{\beta\,0}\delta x_\eta + (\delta x_\eta)^2. \qquad (4)$$

When $|\delta x_\eta| \ll |x_{\beta\,0}| < A_0$ then in the first approximation the value $\delta A = -(x_{\beta\,0}/A)\delta x_\eta$.

It follows that in order to realize enhanced cooling of an electron beam in the transverse plane we must create conditions when electrons interact with the laser beam only under deviations from the instantaneous orbit of one sign (say $x_{\beta\,0} < 0$ and the dispersion function is positive $\partial x_\eta/\partial\varepsilon > 0$ or in the opposite case $x_{\beta\,0} > 0$, $\partial x_\eta/\partial\varepsilon < 0$). In this case the rate of change of amplitudes of betatron oscillations of electrons will be proportional to the number of theirs passages through the laser beam.

We can select interaction of electrons and laser beams under deviations $x_{\beta\,0} < 0$ by the next way (see Fig.1). Let the laser beam is located at the inner side of the working region of the storage ring and the instantaneous electron orbit position $x_\eta > x_{T_1}$. In this case the interaction will take place only at deviations $x_{\beta\,0} < 0$. At that the instantaneous orbit will go in the direction of the laser beam.

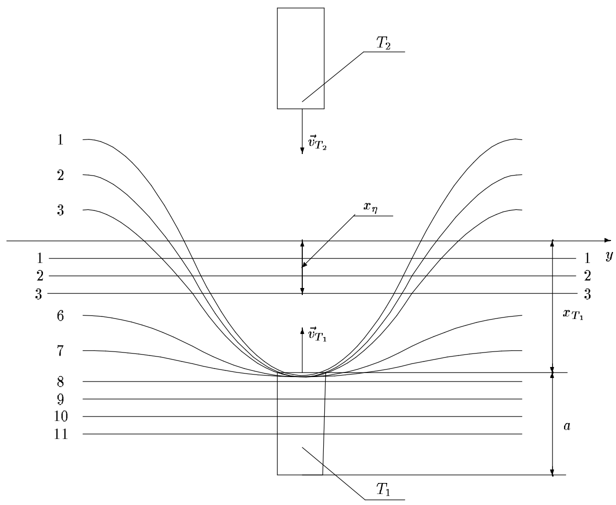

Figure 1: *The scheme of the enhanced electron cooling. The axis "y" is the equilibrium orbit of the storage ring, 1-1, 2-2 ... the location of the instantaneous electron orbit after 0,1,2 ... events of the electron energy loss, T_1, T_2 the laser beams. The transverse position of laser beams is displaced with the velocity $\vec{v}_{T_{1,2}}$ relative to the equilibrium orbit.*

In order to cool the electron beam having the spread of amplitudes of betatron oscillations and the energy spread the target must be moved in the direction of the being cooled beam.

The value $\delta x_\eta = D_x \Delta p/p$, where D_x is the local dispersion function, $p = Mc\beta\gamma$ the momentum of the electron [1]. The term $-2x_{\beta\,0}\delta x_\eta$ in (4) determines the classical processes of enhanced damping (antidamping) of radial betatron oscillations of electrons in storage rings. The scheme works when the dispersion function $D_x \neq 0$. The greater D_x the greater the rate of the transverse cooling.

To produce the enhanced cooling of an electron beam in the longitudinal plane we must create the conditions when electrons of the energy higher then minimum one will interact and electrons of minimal and lesser energy will not interact with the laser beam. E.g., the laser beam can overlap the electron beam partially in the radial direction in the straight section of the storage ring having non zero dispersion function. The degree of overlapping must be changed in time such a way that electrons of maximum energy first and then electrons of lesser energy come into interaction. When the laser beam will reach the orbit of electrons of minimum energy then it must be returned to the previous position. In this case if electrons have small amplitudes of betatron oscillations then the laser beam will interact with electrons of the energy higher then minimum energy of electrons in the beam and will not interact with electrons of minimum and lesser energy. The rate of the energy loss of electrons will not be increased but the difference in rates of the energy losses of electrons having in the beam maximum and minimum energies will be increased essentially. As a result all electrons will be gathered at the minimum energy in a short time.

3.2 Interaction of electron beams with being transversely displaced laser targets

Below we will consider two schemes of laser cooling. In these schemes the internal and external laser beams will be displaced in the transverse directions. The evolution of amplitudes of betatron oscillations and positions of instantaneous orbits in the process of the energy loss of electrons in laser beams will be analyzed.

The velocity of an electron instantaneous orbit $\dot{x}_\eta$ depends on the distance $x_{T_{1,\,2}} - x_\eta$ between the edge of the corresponding laser beam and the instantaneous orbit, and on the amplitude of electron oscillations (Fig.1). When the instantaneous orbit of an electron enters the laser beam at the depth higher than the amplitude of the electron oscillations then its velocity $\dot{x}_{\eta\, in}$ is maximum one by the value. The velocity $x_{\eta\, in}$ is given by the intensity and the length of the interaction region of the laser beam.

In the general case electrons do not interact with the laser beam every turn. That is why the velocity $\dot{x}_\eta$ can be presented in the form $\dot{x}_\eta = \dot{x}_{\eta\, in} \cdot W$, where $W < 1$ is the probability of an electron to cross the laser beam. W is the inverse ratio of a period to a part of the period of betatron oscillations of an electron which is determined by the condition that the deviation of the electron from the instantaneous orbit must be greater than the distance between the orbit and the laser beam ($|x_{T_{1\,2}} - x_\eta| \le |x_0^{'}| \le A$) and directed to the laser beam. The probability can be presented in the form $W = \varphi_{1,\,2}/\pi$, where $\varphi_1 = \pi - \arccos\xi_1$, $\varphi_2 = \arccos\xi_2$, $\xi_{1,\,2} = (x_{T_{1,\,2}} - x_\eta)/A$, labels 1, 2 correspond to laser beams.

The behavior of the amplitudes of betatron oscillations of electrons according to (4) is determined by the equation $\partial A/\partial x_\eta = -< x_{\beta\,0} > /A$, where $< x_{\beta\,0} >$ is the electron deviation from the instantaneous orbit averaged through the range of phases $2\varphi_{1,\,2}$ of betatron oscillations where the electron cross the laser beam. The value $< x_{\beta\,0} >= \pm A sinc\,\varphi_{1,\,2}$, where $sinc\varphi_{1,\,2} = sin\varphi_{1,\,2}/\varphi_{1,\,2}$, signs + and − are related to the first and second laser beams. Thus the cooling processes related to the first and second laser beams are determined by the system of equations

$$\frac{\partial A}{\partial x_\eta} = \pm sinc\varphi_{1,2}, \qquad \frac{\partial x_\eta}{\partial t} = \frac{\dot{x}_{\eta\, in}}{\pi}\varphi_{1,2}. \qquad (5)$$

From the Eqs. (5) and the expression $\partial A/\partial x_\eta = [\partial A/\partial t]/[\partial x_\eta/\partial t]$ it follows:

$$\frac{\partial A}{\partial t} = \frac{\dot{x}_{\eta\, in}}{\pi}\sin\varphi_2 = \frac{\dot{x}_{\eta\, in}}{\pi}\sqrt{1-\xi_{1,\,2}^2}. \qquad (6)$$

Let the initial instantaneous electron orbits be distributed in a region $\overline{x_\eta} \pm \sigma_{x,\varepsilon,0}$ and initial amplitudes of electron radial betatron oscillations A_0 be distributed in a region $\sigma_{x,b,0}$ relative to their instantaneous orbits, where $\overline{x_\eta}$ is the location of the middle instantaneous orbit of electrons of the beam; $\sigma_{x,\varepsilon,0}$, the mean-root square deviation of instantaneous orbits from the middle one. The value $\sigma_{x,\varepsilon,0}$ is determined by the initial energy spread $\sigma_{\varepsilon,0}$. Suppose that the initial spread of amplitudes of betatron oscillations of electrons $\sigma_{x,b,0}$ is identical for all instantaneous orbits of the beam. The velocity of the instantaneous orbit in a laser beam $\dot{x}_{\eta,in} < 0$, the transverse velocity of the first laser beam $v_{T_1} > 0$ and the transverse velocity of the second laser beam $v_{T_2} < 0$. Below we will use the relative radial velocities of displacement of laser beams $k_{1,2} = v_{T_{1,2}}/\dot{x}_{\eta,in}$. In our case $k_1 < 0$ and $k_2 > 0$.

From the definition of $\xi_{1,\,2}$ we have a relation $x_\eta = x_{T_{1,\,2}} - \xi_{1,\,2}A(\xi_{1,\,2})$. The time derivative $\partial x_\eta/\partial t = v_{T_{1,2}} - [A + \xi_{1,\,2}(\partial A/\partial \xi_{1,\,2})]\partial\xi_{1,\,2}/\partial t$, where $v_{T_{1,2}} = dx_{T_{1,2}}/dt$ is the radial velocity of displacement of the laser beam. Equating this value to the second term in (5) we will receive the time derivative

$$\frac{\partial \xi_{1,\,2}}{\partial t} = \frac{\dot{x}_{\eta\, in}}{\pi}\frac{\pi k_{1,2} - \varphi_{1,\,2}}{A(\xi_{1,\,2}) + \xi_{1,\,2}(\partial A/\partial \xi_{1,\,2})}. \qquad (7)$$

Using this equation we can transform the first value in (5) to the form

$$\pm sinc\varphi_{1,2}(\xi_{1,2}) = \frac{\partial A}{\partial \xi_{1,2}} \frac{\partial \xi_{1,2}}{\partial t} / \frac{\partial x_\eta}{\partial t} =$$

$$\frac{\pi k_{1,2} - \varphi_{1,2}}{[A + \xi_{1,2}(\partial A/\partial \xi_{1,2})]\varphi_{1,2}} \partial A/\partial \xi_{1,2}.$$

which can be transformed to

$$\frac{\partial \ln A}{\partial \xi_{1,2}} = \frac{\pm \sin \varphi_{1,2}}{\pi k_{1,2} - (\varphi_{1,2} \pm \xi_{1,2} \sin \varphi_{1,2})}.$$

The solution of this equation is

$$A_f = A(\xi_{1,2,f}) = A_0 \exp \int_{\xi_{1,2,0}}^{\xi_{1,2,f}} \frac{\pm \sin \varphi_{1,2} d\xi_{1,2,f}}{\pi k_{1,2} - (\varphi_{1,2} \pm \xi_{1,2} \sin \varphi_{1,2})}, \quad (8)$$

where the labels 0, f correspond to the initial and observation time accordingly.

The time dependence of the amplitudes and positions of the instantaneous orbits of electrons are determined by (5) through the parameter $\xi(t)$. This parameter is determined by (7) and (8). Substituting the values A and $\partial A/\partial \xi_{1,2,f}$, which are determined by (8), in (7) we can find the connection between time of observation and parameter $\xi_{1,2}$

$$t - t_0 = \frac{\pi A_0}{|\dot{x}_{\eta\, in}|} \psi(k_{1,2}, \xi_{1,2,f}), \quad (9)$$

where

$$\psi(k_{1,2}, \xi_{1,2,f}) = \int_{\xi_0}^{\xi_{1,2,f}} \frac{-[A(\xi_{1,2,f})/A_0] d\xi_{1,2,f}}{\pi k_{1,2} - (\varphi_{1,2} \pm \xi_{1,2,f} \sin \Delta\varphi_{1,2})}.$$

The equations (9) determine the time dependence of the functions $\xi_{1,2}(t_f - t_0)$. The dependence of the amplitudes $A[\xi_{1,2}(t-t_0)]$ is determined by the equation (8) through the functions $\xi_{1,2}(t-t_0)$ in a parametric form. The dependence of the position of the instantaneous orbit follow from the definition of $\xi_{1,2}$

$$x_\eta(t - t_0) = x_{T_{1,2}0} + v_{T_{1,2}}(t_f - t_0) - A[(\xi_{1,2}(t_f - t_0)] \cdot \xi(t_f - t_0). \quad (10)$$

The function $\psi(k_2, \xi_{2,f})$ for the case $k_2 > 0$ which will be considered below, according to (9), can be presented in the form

$$\psi(k_2, \xi_{2,f}) = \int_{\xi_{2,f}}^{1} dx \exp \int_x^1 \frac{\sqrt{1-t^2}/(\pi k_2 - \arccos t + t\sqrt{1-t^2})}{\pi k_2 - \arccos x + x\sqrt{1-x^2}} dt. \quad (11)$$

The instantaneous orbits of electrons having maximum initial amplitudes of betatron oscillations $A_m = \sigma_{x,b,0}$ will be deepened into the laser beam on the depth higher then their final amplitudes of betatron oscillations at a moment t_c. According to (9), this moment is $t_c = t_0 + \pi\sigma_{x,b,0}\psi(k_2, \xi_{2,c})/|x_{\eta,in}|$, where $\xi_{2c} = \xi(t = t_c) = 1$. During the time interval $t_c - t_0$ the laser beam T_2 will pass a way $l_c = |v_{T_2}|(t_c - t_0) = \pi k_2 \psi(k_2, \xi_{2,c})\sigma_{x,b,0}$. The dependence $\psi(k_2, \xi_{2,c})$ determined by (11) is presented in the Table 1.

Table 1

k_2	1.0	1.02	1.03	1.05	1.1	1.2	1.3	1.4	1.5	1.7	2.0
$\psi(k_2, \xi_{2,c})$	∞	13.80	9.90	6.52	3.71	2.10	1.51	1.18	0.980	0.735	0.538

Numerical calculations of the function $\psi(k_2, \xi_{2,f})$ for the cases $k_2 = 1.0$, $k_2 = 1.1$ and $k_2 = 1.5$ are presented in the Table 2, Table 3 and Table 4 accordingly. It can be presented in the next approximate form

$$\psi(k_2, \xi_{2,f}) \simeq C_3(k_2)\psi(\frac{1 - \xi_{2,f}}{k_2 + \xi_{2,f}}), \quad (12)$$

where $C_3(k_2) \simeq 0.492 - 0.680(k_2 - 1) + 0.484(k_2 - 1)^2 + \ldots$, $\psi[(1-\xi_{2,f})/(k_2+\xi_{2,f})]|_{k_2=1} \simeq (1-\xi_{2,f})/(1+\xi_{2,f})$.

Table 2 ($k_2 = 1.0$)

$\xi_{2,f}$	1.0	0.5	0.2	0	-0.2	-0.5	-0.8	-0.9	-1.0
$\psi(k_2, \xi_{2,f})$	0	0.182	0.341	0.492	0.716	1.393	4.388	10.187	-∞

Table 3 ($k_2 = 1.1$)

$\xi_{2,f}$	1.0	0.5	0.2	0	-0.2	-0.5	-0.8	-0.9	-1.0
$\psi(k_2, \xi_{2,f})$	0	0.163	0.300	0.423	0.595	1.033	2.076	2.759	3.710

Table 4 ($k_2 = 1.5$)

$\xi_{2,f}$	1.0	0.5	0.2	0	-0.2	-0.4	-0.6	-0.8	-1.0
$\psi(k_2, \xi_{2,f})$	0	0.116	0.202	0.273	0.359	0.466	0.602	0.772	0.980

3.3 *The enhanced transverse laser cooling of electron beams*

In the method of the enhanced transverse laser cooling of electron beams a laser beam T_1 is located at the orbit region $(x_{T_1},\ x_{T_1} - a)$, where a is the laser beam width (Fig. 1). At the initial moment the laser beam overlaps only a small part of the electron beam so that electrons with largest initial amplitudes of betatron oscillations interact with the laser beam. Then the radial laser beam position is displaced uniformly with the velocity v_{T_1} from inside of the working region of the storage ring in the direction of a being cooled electron beam or instantaneous orbits are moved in the direction of the laser beam[2].

In this case immediately after the interaction and loss of energy the position and direction of momentum of an electron remain the same, but the instantaneous orbit is displaced inward in the direction of the laser beam. The radial coordinate of the instantaneous orbit and the amplitude of betatron oscillations are decreased to the same value owing to the dispersion coupling. After every interaction the position of the instantaneous orbit approach the laser beam more and more and the amplitude of betatron oscillations is coming smaller. It will reach some small value when the instantaneous orbit will reach the edge of the laser beam. Up to this moment the instantaneous orbit was moved in the direction to the laser beam, but the electron depth of dipping in the laser beam was increased with the velocity $\sim v_{T_1}$. When the depth of dipping of the instantaneous orbit of the electron in the laser beam becomes greater then the amplitude of its betatron oscillations then the orbit will continue its movement in the laser beam with the constant velocity $\dot{x}_{\eta\, in}$.

When the laser beam will reach the instantaneous orbit corresponding to electrons having maximum energies then the laser beam must be switched off or the electron beam instantaneous orbits must be returned to the initial position for a short time. After this, electrons will have small amplitudes of betatron oscillations and increased energy spread.

The degree of the transverse cooling of the electron beam in this case is determined by the law of change of the amplitudes of electron betatron oscillations (8). It can be presented in the form

$$A_f = A_0 e^{\int_{\xi_0}^{\xi_{1,f}} \frac{\sqrt{1-\xi_1^2}\,d\xi_1}{\pi k_1 - \pi + \arccos \xi_1 - \xi_1 \sqrt{1-\xi_1^2}}}. \qquad (13)$$

The numerical calculations of the dependence of the ratio A_f/A_0 on the relative radial velocity $k_1 < 0$ of the displacement of the position of the laser beam T_1 are presented at the Fig.2 and in the Table 5 for the case $\xi_0 = -1$, $\xi_{1,f} = 1$. This dependence can be presented by the approximate expression

$$A_f \simeq A_0 \sqrt{\frac{|k_1|}{|k_1| + 1}}. \qquad (14)$$

[2] A kick, decreasing of the value of the magnetic field in bending magnets of the storage ring, a phase displacement or eddy electric fields can be used for this purpose.

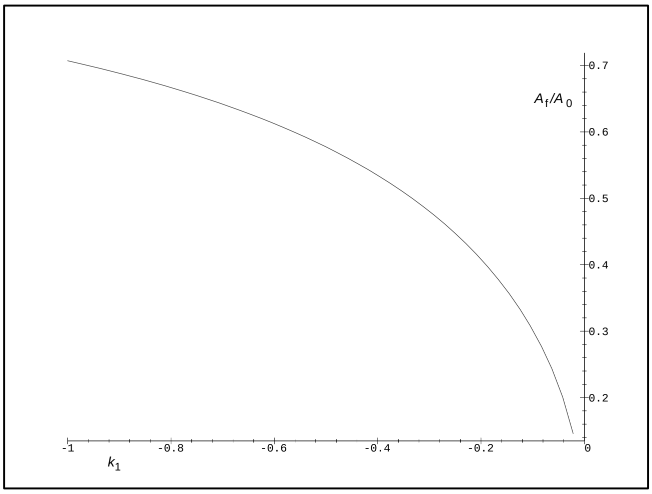

Figure 2: *The dependence of the ratio A_f/A_0 on k_1.*

Table 5

$\|k_1\|$	0	0.2	0.4	0.6	0.8	1.0
A_f/A_0	0	0.408	0.535	0.612	0.667	0.707

The time of the laser beam cooling is $\Delta t_1 \simeq \sigma_{x,0}/v_{T_1}$, where $\sigma_{x,0} = \sigma_{x,b,0} + \sigma_{x,\varepsilon,0}$ is the total initial radial dimension of the electron beam. For this time the instantaneous orbits of electrons of a beam having minimum energy and maximum amplitudes of betatron oscillations will pass the distance $\simeq |\dot{x}_{\eta\, in}|\Delta t_1$. Hence the final radial dimension of the beam determined by the final energy spread of the beam and the total radial dimension of the beam will be increased to the value

$$\sigma_{x,f} \simeq \sigma_{x,\varepsilon,f} = \sigma_{x,0} \frac{1}{|k_1|}. \qquad (15)$$

Thus, in the method of the enhanced transverse laser cooling we have a high degree enhanced cooling of electron beams in the transverse plane (14) and more high degree of heating in the longitudinal one (15). In this case it is desirable to use the straight section with low-beta and high dispersion functions as less events of the photon emission are required to cool the beam in the transverse direction. This is because the change of amplitudes of betatron oscillations is near the same as the change of positions of instantaneous orbits of the electron. Meanwhile, the spread of amplitudes of betatron oscillations is small and the step between positions of instantaneous orbits is high.

The described process of transverse cooling is based on electron interactions with laser beams. Similar interactions with external and internal targets were described in 1956 by O'Neil [12]. However, O'Neil considered the question of damping of betatron oscillations of proton beams in the transverse direction by means of motionless solid material targets. Targets in his case could not lead to three-dimensional cooling[3]. They could be used for injection and capture of only one portion of protons. For the purpose of the multi-cycle injection and storage of heavy protons O'Neil suggested the ordinary three-dimensional ionization cooling based on a thin hydrogen target jet situated in the working region of the storage ring.

3.4 *The enhanced longitudinal laser cooling of electron beams*

In the method of the enhanced longitudinal laser cooling of electron beams a laser beam T_2 is located at the orbit region (x_{T_2}, $x_{T_2} - a$) (see Fig.1). At the initial moment the laser beam overlaps only a small part of the electron beam so that electrons with largest initial amplitudes of betatron oscillations interact with the laser beam. Then the radial laser beam position is displaced uniformly with the velocity $v_{T_2} < 0$, $|v_{T_2}| > |\dot{x}_{\eta,\in}|$ from outside of the working region of the storage ring in the direction of a being cooled electron beam (or instantaneous orbits of the electron beam are moved in the direction of the laser beam). The instantaneous orbits of electrons will go in the same direction with a velocity $\dot{x}_\eta \geq \dot{x}_{\eta,in}$ after the moment of their first interaction with the laser beam. In this case, the laser beam will start to interact first with electrons having the largest amplitudes of betatron oscillations and the highest energies at some moment t_0. Then it will interact with electrons of lesser amplitudes and energies. When the laser beam will pass through the instantaneous orbit of electrons having zero amplitudes and minimum initial energies then it must be removed to the initial position.

To estimate the energy spread of the electron beam in the enhanced longitudinal method of laser cooling let us accept, for the simplicity, that the initial spread of positions of instantaneous orbits $\sigma_{x,\varepsilon,0}$ is much greater then the spread of the amplitudes of betatron oscillations $\sigma_{x,b,0}$ of the beam. In this case electrons of the beam having maximum energy and zero amplitudes of betatron oscillations will interact with the laser beam during the time $\Delta t_2^{'} \simeq \sigma_{x,\varepsilon,0}/v_{T_2}$. For this time the instantaneous orbits of electrons will pass the distance $|\dot{x}_{\eta\, in}|\Delta t_2{}' = k_2^{-1}\sigma_{\varepsilon,b,0}$. At that electrons having minimum energy and zero amplitudes of betatron oscillations will stay at rest. Hence it follows that the spread of instantaneous orbits of these electrons will be compressed to the value $\sigma_{x,\varepsilon,f} \simeq \sigma_{x,\varepsilon,0}(1 - k_2^{-1})$.

If we take into account that the behavior of the instantaneous orbit depends on the initial amplitude of electron betatron oscillations then the total radial dimension of the beam can be presented in the form

$$\sigma_{x,\varepsilon,f} \leq \frac{k_2 - 1}{k_2}\sigma_{x,0} + [\sqrt{\frac{k_2}{k_2 - 1}} - \pi(k_2 - 1)\psi(k_2, \xi_{2,c})$$

$$+0.28]\sigma_{x,b,0}, \qquad A_{T_2} > l_c, \sigma_{x,0}, \qquad (16)$$

where A_{T_2} is the amplitude of displacement of the second laser beam, the function $\psi(k_2, \xi_{2,c})$ is determined by (9) [11].

According to (16) the efficiency of the method of the longitudinal laser cooling is the higher the less the ratio of the spread of the initial amplitudes of betatron oscillations to the spread of the instantaneous orbits of the being cooled electron beam.

Notice that (5) does not take into account that the laser beam pass of a finite distance per one turn $\delta x_{T_2} = |v_{T_2}| \cdot T$, where T is the period of the electron revolution around its orbit in the storage ring. When

$$\delta x_{T_2} > \sigma_{x,b,0}, \qquad |v_{T_2}| \geq |\dot{x}_{\eta\, in}| \qquad (17)$$

then all instantaneous orbits of electrons can enter the laser beam at the distance $x_\eta - x_T > \sigma_{x,b,0}$, that is, all at once under conditions $\partial A/\partial t = 0$ ($\varphi_2 = \pi$) when they will cross the laser beam every turn. In this case there will not be any heating process in the transverse plane. It can be realized easier if we do a high-degree transverse cooling of the electron beam first, and when we install the laser beam at the straight section with low β-function and high dispersion function. Such a way we can use the considered above schemes of the transverse and longitudinal cooling in turn to realize the enhanced two-dimensional laser cooling[4].

The law of change of the amplitudes of electron betatron oscillations is determined by the equation (8), which in the method of the longitudinal laser cooling can be presented in the form

$$A_f = A_0 e^{\int_{\xi_0}^{\xi_{2,f}} \frac{-\sqrt{1-\xi_2^2}d\xi_2}{\pi k_2 - \arccos \xi_2 + \xi_2\sqrt{1-\xi_2^2}}}. \qquad (18)$$

where the parameter $\xi_{2,f}$ is determined by a moment $t_f = min\{t_{st}, t_A\}$, $t_{st} = t_0 + A_{T_2}/|v_{T_2}|$ the moment of the stop of the laser beam, A_{T_2} the amplitude of the laser beam displacement, t_A corresponds to the moment when the instantaneous orbit of an electron will reach in the laser beam the depth equal to its amplitude of betatron oscillations A, i.e. when $\xi_{2,A} = \xi_{2,f}(t_A) = -1$. After this moment the amplitude of the electron oscillations will not be changed. During the time interval $t_A - t_0$ the laser beam will pass the way $l_A \leq l_c$. When $A_{T_2} < l_A$ then, according to (18), the value A_f has to be calculated in the limits ($\xi_{2,st}, 1$), where $\xi_{2,st} > -1$ corresponds to the moment t_{st}. We will consider here the cooling of electron beams under conditions $\xi_0 = 1$, $\xi_{2,f} = -1$.

[3] Internal target could be rotated out of the medium plane only to prevent the proton beam losses.

[4] Such two-dimensional cooling will not work when the equation (17) is not valid

The ratio of a maximum amplitude of betatron oscillations of an electron to the initial one $D_{2,tr,c} = A_{f,c}/A_0$ ($A_{f,c} = A_f(\xi_{2,c} = 1)$ on the relative velocity k_2 of the second laser beam is presented in the Table 6 and at the Fig.3. According to calculations this ratio can be presented by the next approximate expression

$$A_{f,c} \simeq A_0 \sqrt{\frac{k_2}{k_2 - 1}}. \tag{19}$$

Table 6

k_2	1.0001	1.0010	1.0100	1.1000	1.5000	2.0000
$A_{f,c}/A_0$	100.005	31.64	10.04	3.32	1.73	1.414

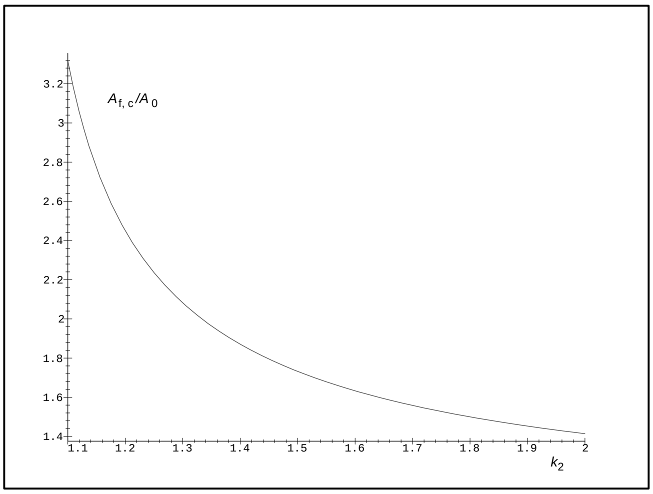

Figure 3: *The dependence of the ratio* $D_{2,tr,c} = A_{f,c}/A_0$ *on* k_2.

3.5 Damping times and emittances

The damping times of the electron beam in the transverse and longitudinal schemes of cooling are

$$\tau_x = \frac{2\sigma_{eq}}{k_1 \overline{P}}, \qquad \tau_s = \frac{2\sigma_{\varepsilon\, in}}{\overline{P}}, \tag{20}$$

where in the smooth approximation $\sigma_{eq} = \varepsilon \sigma_{x,b,0}/\alpha \overline{R}$ the energy interval corresponding to the energy spread of the electron beam whose instantaneous orbits are distributed through the interval of radii $\sigma_{x,b,0}$; $k_1 \sim 0.1 \div 0.2$; α, the momentum compaction function.

The transverse emittances of beams are proportional to theirs damping times. It means that the emittance of a beam in the plane "i" in the two-dimensional method of cooling $\epsilon_i^{(2)}$ is equal to the emittance corresponding to a three-dimensional one $\epsilon_i^{(3)}$ multiplied by the ratio of their damping times

$$\epsilon_i^{(2)} = \epsilon_i^{(3)} \frac{\tau_i^{(2)}}{\tau_i^{(3)}}. \tag{21}$$

3.6 Discussion

The dynamics of instantaneous orbits and amplitudes of betatron oscillations of electrons depends on the depth of deepening of their instantaneous orbits in the laser beam. Moreover, the being displaced laser beam begins to interact with electrons of the beam located at different instantaneous orbits at different moments of time and for different durations of time. These features of interaction of moving laser beams can be used for the enhanced three-dimensional cooling.

In the method of transverse laser cooling, according to (13), (15) there is a significant decrease of amplitudes of betatron oscillations and, at the same time, a greater increase of the spread of instantaneous orbits. If the degree of transverse cooling (14) is defined by the coefficient of compression $C_{1,tr} = A_0/A_f = \sqrt{(1 + |k_1|)/|k_1|}$ then, according to (15), the increase of the spread of the instantaneous orbits of the beam (decompression) will be $D_{1,tr} \simeq C_{1,tr}^2$ times.

In the method of longitudinal laser cooling there is a significant decrease of the spread of instantaneous orbits of electrons defined by the compression coefficient $C_{2,l} = \sigma_{x,\varepsilon,0}/\sigma_{x,\varepsilon,f}$ and, according to (16), (19), lesser value of increase of amplitudes of betatron oscillations. If the condition (17) is not fulfilled then the degree of the transverse heating can be about $D_{2,tr} \simeq \sqrt{C_{2,l}}$. When the condition (17) is fulfilled then heating process can be neglected at all. The successive application of two methods of the two-dimensional cooling considered above in tern will lead to cooling of the electron beam in both degrees of freedom only in the case when the condition (17) is fulfilled.

In the longitudinal method of laser cooling, contrary to the transverse one, the degree of longitudinal cooling can be much greater then the degree of heating in the transverse plane. That is why we can use the emittance exchange between longitudinal and transverse phase spaces (e.g., using the synchro-betatron resonance) and such a way to have enhanced two-dimensional cooling of the electron beam based on the longitudinal laser cooling only.

When the synchrotron radiation of electron beams in guiding magnetic fields of lattices of storage rings is high then we can do an additional enhanced laser cooling of such beams in the radio frequency buckets. Such cooling in the longitudinal plane can be produced by using of a being displaced target T_2 (see section 3.4). Cooling in the transverse plane can be produced by a coupling of transverse and longitudinal dimensions. This is another problem which will be considered in a separate paper.

Notice that in the case of the three dimensional laser cooling of electron beams considered in [9] the spread of amplitudes of betatron oscillations will be small in a straight section with high dispersion and low β-function. At the same time the energy spread and the spread of instantaneous orbits of electrons will be high. That is why we can locate the laser beam T_2 at this section and produce an additional longitudinal cooling (see sect.3.4) one, two or more times. Then we can use the beam with low both transverse and longitudinal emittances in the storage ring for emission of spontaneous or stimulated radiation or extract it for linear colliders.

4 CONCLUSION

In this paper we have presented different methods of laser cooling of electron beams in transverse and longitudinal directions[5]. We hope that the development and adoption of these methods will lead to new generations of light sources of spontaneous incoherent and stimulated radiation in optical to X-ray and γ-ray regions. Using of circular polarized laser beams for cooling can lead to a longitudinal polarization of stored $e^{\pm}$ beams in storage rings [13, 14]. Hard circular polarized photons produced in the process of the Backward Compton Scattering of laser photons by electrons in storage rings can be used for production in material targets of longitudinally polarized positron beams for linear colliders [15].

5 REFERENCES

[1] A.A.Kolomensky and A.N.Lebedev, Theory of Cyclic Accelerators. North Holland Publ., C^o, 1966; M.Sands, "The physics of electron storage rings, an Introduction", SLAC Report 121, Nov. 1970 (unpublished); H.Bruk, Accelerateurs Circulaires de Particules (Press Universitaires de France, 1966); H.Wiedemann, Particle Accelerator Physics I & II (Springer-Verlag, New York, 1993.

[2] P.J.Channel, J.Appl.Phys., v. 52(6), p.3791 (1981); L.D.Selvo, R.Bonifacio, W.Barletta, Optics Communications, v.116, (1995), p.374.

[3] S.Shröder, R.Clein, N.Boos et al., Phys. Rev. Lett., v. 64, No 24, p.2901 (1990).

[4] J.S.Hangst, M.Kristensen, J.S.Nielsen et al., Phys. Rev. Lett., v. 67. 1238 (1991); J.S.Hangst, K.Berg-Sorensen, P.S.Jessen et al. Proc. IEEE Part. Accel. Conf., San Francisco, May 6-9, NY, 1991, v.3, p.1764.

[5] E.G.Bessonov, a) Preprint FIAN No 6, 1994; b) Proc. of the Internat. Linear Accel. Conf. LINAC94, Tsukuba, KEK, August 21-26, 1994, Vol.2, pp.786-788; c) Journal of Russian Laser Research, 15, No 5, (1994), p.403; Proc. of the Internat. Conf. EPAC94, London, June 27 - July 1, 1995, Vol. 1, p.888 - 890

[5] The using of solid targets instead of laser beams is possible for enhanced muon cooling as well [11].

[6] E.G.Bessonov, Proc. of the 15th Int. Free-Electron Laser Conference FEL94, Nucl. Instr. Meth. v.A358, (1995), pp. 204-207.

[7] E.G.Bessonov and Kwang-Je Kim, Preprint LBL-37458 UC-414, June 1955; Phys. Rev. Lett., 1996, vol.76, No 3, p.431.

[8] E.G.Bessonov, K.-J.Kim, Proc. of the 1995 Part. Accel. Conf. and Int. Conf. on High-Energy Accelerators, p.2895; Proc. 5th European Particle Accelerator Conference, Sitges, Barcelona, 10-14 June 1996, v.2, p. 1196.

[9] Zh. Huang, R.D.Ruth, Phys. Rev. Lett., v.80, No 5, 1998, p. 976.

[10] E.G.Bessonov, K.-J Kim, F.Willeke, Physics/ 9812043.

[11] E.G.Bessonov, physics/0001067.

[12] O'Neil G., Phys. Rev., 102, 1418 (1956); A.Shoch, Nucl. Instr. Meth, v.11, p.40 (1961).

[13] Yu.A.Bashmakov, E.G.Bessonov, Ya.A.Vazdik, Sov. Phys. Tech. Lett., v.1, p.239 (1975).

[14] J.E.Clendenin, SLAC-PUB-8465, 20 July 2000.

[15] E.G.Bessonov, Proc. 15th Int. Accelerator Conf. on high Energy Accelerators, v.1, p.138, 1992, Hamburg, Germany.