# FREE ELECTRON LASER THEORY USING TWO TIMES GREEN FUNCTION FORMALISM

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In this paper, we present a quatum theory for free electron laser obtained by firstly using the Two time's Green Function method developed by Matsubara for solid physics theory. The dispersion relation for the laser photon obtained is limited to the case of low intensity of the laser due to the decoupling the correlation function in low order. For the analysis of the self-amplified emission (SASE), the high intensity laser radiation which strongly affect the trajectory of the free electron is involved, the use of the classical approximation for laser can formulate the laser radiation with multiple frequency. To get the quantum effects in the high intensity laser, use of the perturbation theory, and the expansion methods of state function using the coherent, squeeze and super-radiant states have discussed.

## 1. Introduction

The classical formalism for an electro-magnetic field has been used to formulate a theory for the free electron laser Because the electron beam used for the free electron laser was not very well refined in the past, its formulation by using quantum mechanical formalism was not required. However due to the recent refinements of the electron particle beam used for the free electron laser, it is necessary to formulate a theory and to it give a good foundation by taking the quantum effect into account the Twenty years ago, I derived a quantum theory based on the two times Green function formalism derived by Matsubara for solid physics theory[1]. The dispersion relation is compared with the classical theory obtained by Kwan[2] .

When a relativistic electron moving in a electro-magnetic field and the magnetic field B , The Hamiltonian for such a system may be written as

$$H = \sum_i ( c\, \alpha_i.(p_i - -eA/c) + \beta_i mc^2 ) + H_r + H_{ee} + H_s \qquad (1.1)$$

where A is the sum of the vector potential $A_s$ of the B-field and the vector potential $A_r$ of the electromagnetic wave; $A = A_s + A_r$, m and c are the mass of the electrons and the speed of light, respectively, while $\alpha$ and $\beta$ are the Dirac matrix operators, the suffix stands for the ith particle. $H_r$ is the Hamiltonian for the electromagnetic wave. $H_{ee}$ is the Hamiltonian for the interaction between electrons , and $H_s$ is the Hamiltonian for the Static magnetic field. The last Hamiltonian, which is the constant of motion, is not incorporated in the dynamic of the system; it can thus be neglected in the following derivation.

The Hamiltonian $H_{ee}$ and $H_r$ are given by

$$H_{ee} = \sum_{i>j} e^2 / r^{ij}, \qquad (1.2)$$

$$H_r = \sum_{q\lambda} h\, \omega_q\, b^+_{q,\lambda}\, b_{q,\lambda} \qquad (1.3)$$

where $b^+_{q,\lambda}$ and $b_{q,\lambda}$ are the photon creation and annihilation operator, respectively, for photon momentum h q and polarization state $\lambda$. . The photon energy is given by $h\, \omega_q$ = hcq. The unperturbed system is described by an electron moving in the B-field and the free radiation. Hamiltonian for the unperturbed system is

$$H_0 = H_e + H_r \qquad (1.4)$$

where

$$H_e = \sum_i ( c\, \alpha_i (p_i - -eA_{si} /c) + \beta_i\, mc^2 ) \qquad (1.5)$$

The interaction Hamiltonian is expressed as

$$H_{int} = - \sum_i e\, \alpha_i A_{ri} + \_H_{ee} \qquad (1.6)$$

For simplicity we consider the B-field to be periodic with spacing d in the direction, and having no spacial dependence in the transverse (x-y) plane . Hence it will be described by

$$B_{0\perp} = B_0 (e_x \cos K_0 z + e_y \sin K_0 z) \qquad (1.7)$$

where $K_0 = 2\pi / d$.

We note that He is invariant under all transformations in the transverse plane and discrete translations in the z direction corresponding ti the periodicity of the field. This results in the Bloch type solution

$$\psi_k (r) = e^{ikr} u_k (z) \qquad (1.8)$$

where $u_k(z)$ has the same periodicity as the static magnetic field. It is important to note that the function is a four component spinor

$$u_k (z) = \sum_{K=+-K0} C_{Kk}, e^{ik.z} \qquad (1.9)$$

where C is a free electron spinor with an energy eigenvalue

$$E_k^2 = h^2 c^2 k^2 + m^2 c^4 \qquad (1.10)$$

To derive the dispersion relation of the free electron laser, the Hamiltonian of the relativistic electron moving in a electron-magnetic field and he magnetic field B may be written as The second quantization formalism for the Hamiltonian for the unpertubed system can be obtained eq,(2.4) , as follows ; as

$$H_0 = \sum_{k\sigma} T_{k\sigma} a^+_{k\sigma} a_{k\sigma} + \sum k_{q\lambda} h\omega_{q\lambda} b^+_{q,\lambda} b_{q,\lambda} \qquad (1.11)$$

where $a^+_{k\sigma}$ is the electron creation operator for the electron momentum hk and spin $\sigma$, ak$\sigma$ is the corresponding annihilation opertor, and $T_k$, is its energy . The interaction Hamiltonian can be expressed by

$$H_{int} = \sum_{k2k2q;\sigma1\sigma2\lambda} M_{k1k2q,\sigma1,\sigma2\lambda} a^+_{k3,\sigma2} (t) a_{k4,\sigma2}(t) (b_{q,\lambda} + b^+_{-q,\lambda})$$

$$+ 1/2 \sum_{k1k2k3k4,\sigma1,\sigma2} V_{k1k3k4k2,\sigma1,\sigma2\sigma2\sigma1}(k)\, a^+_{k1,\sigma1}(t) a_{k4,\sigma2} (t) a_{k2,\sigma1}(t) \qquad (1.12)$$

The first term is due to the electron creation-photon interaction with

$$M_{k1k2q,\sigma1,\sigma2\lambda} = -e \sqrt{[4 \pi c^2 h / 2 \omega_{q,\lambda} v]}\, (2\pi)^2 (\delta(k_{1x} - k_{2x} + q_x)\, \delta(k_{1y} - k_{2y} + q_y)$$

$$N \sum_{K=-K0}^{K0} (\delta(k_{1z} - k_{2z} + q_z, K)\, F(^{k1k2}, _{\sigma1\sigma2}) \qquad (1.13)$$

In equation (1.25), N the number of spacing, is derived from the

## 2. The Green function of the photons

The propagation of the photon in this system can be obtained by solving for Green's function of the photon.

The one photon retarded Green function can be written as

$$G_{q,\lambda}(t-t') =\langle\langle A_{q,\lambda}(t); A_{q,\lambda}^{+}(t')\rangle\rangle = -i\,\theta(t-t')\langle[A_{q,\lambda}(t); A_{q,\lambda}^{+}(t')]\rangle \quad (2.1)$$

And where <> denote the statistical average.

Where [C,D] = CD –DC,

$$\theta(t) =1 \text{ for } t>0, \text{ and } =0 \text{ for } t<0, \quad (2.2)$$

$$A_{q,\lambda} = b_{q,\lambda} + b^{+}_{q,\lambda} \quad (2.3)$$

and where <,> denotes the statistical average

To obtain the equation of motion for Green's function , we consider the equations of the motion for the creation and annihilation operators of the photons and electrons from the Hamiltonian defined in eqs. (1.11) and (1.12):

$$ih\, \partial A_{q,\lambda}(t)/\partial t = [A_{q,\lambda}(t), H] = h\omega_{q,\lambda}(b_{q,\lambda}(t)-b^{+}_{q,\lambda}(t)) = h\omega_{q,\lambda}B_{q,\lambda}(t) \quad (2.4)$$

where $H = H_0 + H_{int}$ (2.5)

$$ih\, \partial B_{q,\lambda}(t)/\partial t = [B_{q,\lambda}(t), H] = h\omega_{q,\lambda}A_{q,\lambda}(t) + \sum_{k3q1,\sigma3\lambda1} M_{k2k3q1,\sigma2,\sigma3\lambda1}a$$

$$a^{+}_{q,\lambda}(t))a_{q,\lambda}(t) \quad (2.6)$$

$$ih\, \partial a_{k1,\sigma1}(t)/\partial t = [a_{k1,\sigma1}(t), H] = Ta_{k1,\sigma1a\, k1,\sigma1}(t)- +\sum_{k2q2;\sigma1,2\lambda} M_{k1k2q,\sigma1,\sigma2\lambda}\, a$$

$$a_{q,\lambda}(t)A_{q,\lambda}(t) + \sum_{k2k3k4,\sigma2} V_{k1k3k4k2,\sigma1,\sigma2\sigma2\sigma1}(t)\, a^{+}_{k3,\sigma2}(t)a_{k4,\sigma2}(t)a_{k2,\sigma1}(t) \quad (2.7)$$

$ih\,\partial a_{k1,\sigma1}(t)/\partial t = [a_{k1,\sigma1}(t), H] = Ta_{k1,\sigma1}a_{k1,\sigma1}(t) - +_{+} \sum_{\substack{k1k2 \\ \sigma1\sigma2}} M_{\substack{k1k2q\sigma \\ \sigma1\sigma2\lambda}}$

$\sum_{k2q2;\sigma1,2\lambda} M_{k1k2q,\sigma1,\sigma2\lambda}\, a$

$a_{q,\lambda}(t)A_{q,\lambda}(t) + \sum_{k2k3k4,\sigma2} V_{k1k3k4k2,\sigma1,\sigma2\sigma2\sigma1}(t)\, a^{+}_{k3,\sigma2}(t)a_{k4,\sigma2}(t)a_{k2,\sigma1}(t)$ (2.7)

$ih\,\partial a^{+}_{k2,\sigma2}(t)/\partial t = [a^{+}_{k2,\sigma2}(t), H] = -Ta^{+}_{k2,\sigma2}(t) + \sum_{k1q;\sigma1,\lambda1} M_{k1k2q,\sigma1,\sigma2\lambda}\, a^{+}_{kqk1\sigma1}(t)A_{q,\lambda}(t)$

$+\sum_{k1k3k4,\sigma2} V_{k1k3k4k2,\sigma1,\sigma2\sigma2\sigma1}(k)\, a^{+}_{k3,\sigma2}(t)a^{+}_{k2,\sigma1}(t)a_{k4,\sigma2}(t)$ (2.8)

$-h2\,(\partial^{2}(t)/\partial t^{2} + \omega_{q,\lambda}^{2})\, G(t-t') = 2\,\omega_{q,\lambda}h^{2}\, k\delta(t-t') + \omega_{q,\lambda}h \sum_{k1k2\sigma1;\sigma2} M_{k1k2q,\sigma1,\sigma2\lambda}$
$G_{k1\,\sigma1k2,\sigma1;q\lambda}(t-t')$, (2.9)

where $G_{k1\,\sigma1k2,\sigma1;q\lambda}(t-t') = \langle\langle\, , a^{+}_{k1,\sigma1}(t)a_{k2,\sigma2}(t); a^{+}_{k3,\sigma2}(t)\, A_{q,\lambda}{+}(t')\rangle\rangle$ (2.10)

To solve eq.(2.9), the equation of motion for the Green's function defiend in eq.(2.10) is obtained by differentiating it with respect to time t as sollows;

$i\hbar\, \partial G_{k1\sigma1k2\sigma;q\lambda}(t-t')/\partial t = (-T_{k1\sigma1} + T_{2\sigma k2})\, G_{k1\sigma1k2\sigma;q\lambda}(t-t')$

$+\sum_{k3q1,\sigma3\lambda1} M_{k2k3q1,\sigma2,\sigma3\lambda}1 << a+_{k1\sigma1}(t)a+_{k2\sigma2}(t)\, A_{q1},_{\lambda}(t); A_{q},_{\lambda}{}^{+}(t') >>$

$-\sum_{k3q1,\sigma3\lambda1} M_{k3k1q1,\sigma3,\sigma13\lambda1} << a+_{k3\sigma3}(t)a+_{k2\sigma2}(t)\, _{Aq1,\lambda}(t); _{Aq,\lambda}+(t') >>$

$+\sum_{k3,k4,k5,\sigma3,k} V_{k5k3k4k1\sigma1\sigma3,\sigma3,\sigma1}(k) << a+_{k5\sigma1}(t)a+_{k3\sigma3}(t)\, a_{k4\sigma3}(t)a_{k2\sigma2}(t); A_{q,\lambda}+(t') >>$

$-\sum_{k3,k4,k5,\sigma3,k} V_{k2k3k4k1\sigma2\sigma3,\sigma3,\sigma2}(k) << a+_{k1\sigma1}(t)a+_{k3\sigma3}(t)a_{k4\sigma3}(t)a_{k5\sigma2}(t); A_{q,\lambda}+(t') >>$

(2.11)

By successive differentiation of the Green's function with respect to t, hierarchy of equations is obtained. In order to close the hierarchy equation, we approximate the higher order Green's function by expressing it in terms of the low order Green's function. The de0coupling of the higher order Green's function is carried out as follows:

$<< a+_{k1\sigma1}(t)a+_{k3\sigma3}(t)\, A_{q1,\lambda}(t); _{Aq,\lambda}+(t') >> \cong$

$\delta_{k1,k3}\delta_{\sigma1,\sigma3} n_{k1,\sigma1} \delta_{q1;q}\, \delta_{\lambda1;\lambda} << A_{q1,\lambda}(t); A_{q,\lambda}+(t') >>$ (2.12)

where $n_{k1,\sigma1} = < a+_{k1\sigma1}(t)a+_{k3\sigma3}(t) >$ is the density of electrons with momentum h $k_1$ and spin $\sigma_1$.

$<< a+_{k5\sigma1}(t)a+_{k3\sigma3}(t)a_{k4\sigma3}(t)a_{k2\sigma2}(t); Aq,\lambda+(t') >> \sim$

$\delta_{k5,k2}\delta_{\sigma1,\sigma2} n_{k2,\sigma1} << a+_{k3\sigma3}(t)a+_{k4\sigma3}(t); A_{q,\lambda}+(t') >>$

$-\delta_{k5,k4}\delta_{\sigma1,\sigma3}\, n_{k4,\sigma1} << a+_{k3\sigma3}(t)a+_{k2\sigma2}(t); A_{q,\lambda}+(t') >>$

$-\delta_{k3,k2}\delta_{\sigma3,\sigma2} n_{k1,\sigma1} << a+_{k3\sigma3}(t)a+_{k2\sigma2}(t); A_{q,\lambda}+(t') >>$ (2.13)

The first term on the r. h.s. of eq.(2.13) contribute to the direct Coulomb interaction; the second and third terms contribute to the exchange Coulomb interaction.

The fourier components of the equation of motions for the Green's function (2.13) $G_{k1\sigma1k2\sigma;q\lambda}(\omega)$ is thus obtained as

$(h\ \omega+T_{k1\sigma1} - T_{k2\sigma2})\ G_{k1\sigma1k2\sigma;q\lambda}(\omega) =$

$\sum_k 4\pi e^2/k^2[n_{k2\sigma2}\ H\ (k_1, k_2,k, \sigma_1) - n_{k1\sigma1}\ H\ (k_1, k_2,k, \sigma_2)\ ],\ x\sum_{k3,k4} H^*\ (k_3, k_4,-k, \sigma_3)$

$G_{k3\sigma3k4\sigma3;q\lambda}(\omega) + (n_{k2\sigma2} - n_{k1\sigma1})\ M_{k2k1q;\sigma2\sigma1\lambda\ Gq\lambda}\ (\omega)$. (2.14)

Solving eq.(2.14) and substituting it into eq. (2.9), we obtain the equation of motion for photon Green's function:

$[[\ h^2\ (\omega^2-\omega_{q,\lambda}^2) - h\omega_{q,\lambda}\{\ \sum_{k3,k4\ \sigma1\sigma2} M_{k1k2q;\sigma1\sigma2\lambda}\ (h\ \omega+T_{k1\sigma1} - T_{k2\sigma2})^{-1}$

$\sum_k [H\ (k_1, k_2,k, \sigma_1)\ n_{k2\sigma2} - H\ (k_1, k_2,k, \sigma_2) n_{k1\sigma1}\ ],\ x$

$(4\pi e^2/k^2))[\ 1 + 4\pi e^2/k^2 \sum_{k3k4\ \sigma3}\ [n_{k4\sigma3} - n_{k3\sigma1}]\ (h\ \omega+T_{k1\sigma1} - T_{k2\sigma2})^{-1}$

$|\ H\ (k_1, k_2,k, \sigma_2)|^2\ ]^{-1}/\sum_{k3,k4\sigma3} H^*\ (k_3, k_4,-k, \sigma_3)\ M_{k4k3q,\ \sigma3\sigma3\lambda xx}(n_{k2\sigma2} - n_{k1\sigma1})$

$(h\ \omega+T_{k1\sigma1} - T_{k2\sigma2})-1 + \sum_{k1,k2q;\sigma1\sigma2\lambda}|\ M_{k4k3q,\ \sigma3\sigma3\lambda x}||\ 2(n_{k2\sigma2} - n_{k1\sigma1})\ (h\ \omega+T_{k1\sigma1} - T_{k2\sigma2})-1]]$

$G_{\lambda q}\ (\omega) = h^2\omega_{q\lambda}/\pi$ (2.15)

The term [ ] on the r.h.s. of eq. (2.15) account for the Coulomb shielding factor for the photon-electron interaction . In the case of a weak B –field, the primal process ( K=0) is predominant in the Coulomb interaction, and H (k1, k2,k, σ2) can be approximated by the δ function, δ(k1-k2+k).

$\omega^2-q^2c^2 - \omega^2 p/2\gamma\ [1/4(\ hq^2\ /m\gamma)^2\ /(\ \omega-qV)^2 - 1/4(\ hq^2\ /m\gamma)^2\ ]$

$= \omega^2_p\ \omega^2_p\ /4\gamma^3 \sum_{k=+-k0}(((q+K)/K)^2[(\omega-(q+K).Vb)^2 - 1/4(\ h(q+K)/m\gamma)^2 + \omega^2_p\ /\gamma]^{-1}$

((4.17)

$[\omega^2-q^2c^2 - \omega^2_p/\gamma] =$

$[1/2\omega^2_p\ \omega^2_{ce}\ /\gamma^5(q+K_0)^2/K_0)^2\ [(\omega-(q+K_0\ ).V_b)^2 - \omega^2_p\ /\gamma^3\ (\ 1+3(q+K_0)^2)\lambda D_2\ ])]^{-1}$ (4.18)

Although I formulated the quantum mechanical theory for the free electron laser, it was limited to a low intensity laser because of the approximation used to decouple the higher-order correlated function of Eq.(2.12) in I. Hence, it could not be applied for the large intensity amplification of the laser, such as the self amplification laser (SALE), which produces a high-intensity laser from the small noise signal or input signal.

To formulate a theory for a high intensity laser, the higher correlated function should not be decoupled as in eq. (2.12 ). Further, the equation of the motion of the high-order correlated Green function should be obtained by differentiating it the same way as in Eq. (2. ). This differentiation creates a correlated function of a much higher order. By successively differentiating them, a correlated function which is of a higher order than the previous one can be derived. Since the higher correlated function becomes smaller as the order increases, by decoupling the higher order's functions as products of the lower correlation ones, similar to eq. (3, ) succesive simultaneous equations can be closed. However, it is difficult to solve the simultaneous equations analytically; therefore a numerical method using a computer might be required, although it poses the problem how to integrate these continuous functions.

To obtain the closed form of the correlation function, the use of the classical description for the EM field greatly simplifies the formula. When the laser intensity is high, and the number of photons associated with this laser intensity becomes large, their photons are not correlated, such that their phases are randomly distributed, and the EM field can be treated as classical one. Although the photons are not correlated each other, the high -intensity laser affects to the trajectory of electrons, the laser produced from the emission from the electrons is very much affected by this high-intensity laser. In analyzing electron plasma under such a high -intensity laser field, I derived the Green's function of electrons by treating the EM wave as the classical one. In this formalism without a lengthy integration of the simultaneous equation, as used in the above, the intense laser field with multiple of the frequency are simply calculated from the formula of the electron's correlated function,. However, the highly correlated photon is not taken into account this classical formula, and the delicacy of the coherence due to quantum effects is also totally discarded.

When the EM is treated as classical field A(t), the wave function of the electron can be expressed as

$$\psi^{(i)}(r,t) = c^{(i)} \exp (ik^{(i)} r - i /(2m^{(i)}) \int_0^t | k^{(i)} + q^{(i)} A(t')|^2 dt$$

The total Hamiltonian H is expressed as

$$H = \sum_s \{ \sum_s (1/(2m)\,(k^{(i)} + q^{(i)} A(t))^2 - EF^{(i)}) + 1/2 \sum_{s,t} V^{(i,i)} (r_s - r_t)) \}$$

$$+ \sum_{s,t}(i \neq j) \sum_{s,t} V (r_s - r_t)) \quad (1.3)$$

where Vs are the Coulomb interaction Hamiltonians.

In this Hamitonian, Hamitonian of electron is expressed by $1/(2m)\,(k^{(i)} + q^{(i)} A(t))^2$ which includes the classical vector field A(t)

The density operator of the i-th particle is expressed in the second quantization formalism as

$$\rho^{(i)}(r,t) = \psi^{*(i)}(r,t)\, \psi^{(i)}(r,t) = \sum_{k1,k2} a+_{k1}{}^{(i)} \exp(ik^{(i)}{}_1 r)\, a_{k2}{}^{(i)} \exp(ik^{(i)}{}_2 r)$$

The Fourier transformation of the density correlation function operator $<\rho(i)(r,t),\rho(i)(r,t)>$ can be expressed by

$$\iint dr\, dr' <\rho^{(i)}(r,t), \rho^{(i)}(r,t)> \exp(ik^{(i)}(r-r'))$$

$$= \sum_{k1,k2\sigma1\sigma2} <a+_{k1\sigma1}{}^{(i)}(t)\, a_{k1\sigma1}{}^{(i)}(t) a+_{k2+k\sigma2}{}^{(j)}(t') a_{k1\sigma 2}{}^{(j)}(t')>$$

Fourier Transformation of Green's functions

$$G_{k1\sigma1}{}^{(i,j)}(k, t-t') = \sum_{k2,\sigma2} <<a+_{k1\sigma1}{}^{(i)}(t)\, a_{k1\sigma1}{}^{(i)}(t) a+_{k2+k\sigma2}{}^{(j)})(t') a_{k2\,\sigma2}{}^{(j)}(t')>>$$

and the vector potential A(t) is sum of many modes as

$$A(t) = \sum_s [A_{xs} \cos(\omega_s t + \theta_s) + A_{ys} \sin(\omega_s t + \theta_s)] \quad (1.2)$$

then, Fourier Transformation of Green's functions $G_{k1\sigma1}{}^{(i,j)}(k, E)$ can be obtained as

$$G_{\sigma1}{}^{(i,j)}(k, E+\sum_s k_s\omega_s) = \sum_{k1} G_{k1\sigma1}{}^{(i,j)}(k, E+\sum_s k_s\omega_s)$$

$$= \{ \Pi_s \sum_{Ls} J_{L-Ns}(z_s{}^{(i)}{}_s) \sum_{Ms} J_{L-Ms}(z_s{}^{(i)}{}_s) \exp[-i(\theta_s - \Delta_s)(N_s - M_s)] \}$$

$$\{ L_{\sigma1}{}^{(i,)}(k, E+\sum_s k_s\omega_s)\, \delta_{ij} + (4\pi/k^2)\, q^{(i)} L^{(i)}(k, E+\sum_s k_s\omega_s) \sum_{j' \neq i}(q^{(j')} G^{(i,j)}(k, E+\sum_s k_s\omega_s)$$

$$+- (4\pi/k^2)\, q^{(i)2} L_{\sigma1}{}^{(i)ex}(k, E+\sum_s k_s\omega_s) G_{\sigma1}{}^{(i,j)}(k, E+\sum_s M_s\omega_s) \}$$

where

$$L_{k1\sigma1}{}^{(i)}(k, E) = (n_{k1\sigma1}{}^{(i)} - n_{k2\,\sigma2}{}^{(i)}) / (E + T_{k1\sigma1}{}^{(i)} - T_{k2\,\sigma2}{}^{(i)})$$

$$L_{\sigma1}{}^{(i)}(k, E) = \sum_{k1} L_{k1\sigma1}{}^{(i)}(k, E)$$

$$L^{(i)}(k,E) = \sum_{\sigma1} L_{\sigma1}{}^{(i)}(k, E)$$

$$L_{\sigma 1}^{(i)ex}(k, E) = [1/k^2 \sum_{k1} 1]^{-1} \sum_{k1k2} 1/(k_2-k_1)^2 \; L_{k2\sigma 1}^{(i)}(k, E) \quad (1.30)$$

**Perturbation Theory**

One way to save partially the quantum effect on the laser is to use the perturbation method by treating a vector potential $A_{em}$ associated with EM field as sum of a classical field $A_c$ and the $A_p$. By treating $A_p$ as the operator-expressed creation and annihilation of the photons, the quantum effects on the high-intensity laser can be analyzed as in the first paper, where the vector potential A was composed of the wiggler field $A_w$ and the photon fields. By adding the classical vector potential Ac to the wiggler vector potential, $A_w$ , we can deal with high intensity laser in a similar way as adopted the first paper.

**3. Coherent , Squeeze, and Super-Radiant Theories**

Another way to deal with this problem is to use the coherent state description which gives the sound foundation of the classical formula.

The coherent state is defined as

$$|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0 \text{ to } \inf} \alpha^n/\sqrt{n!}\,|n\rangle \quad (3.1)$$

Here $|n\rangle = 1/\sqrt{n!}\,(a+)^n|\varphi_0\rangle$, $|\varphi_0\rangle = |\text{vacume state}\rangle$ (3.2)
is the eigen state of th number operator N=a+ a containing light quanta of ( k, $\eta$).
For our consideration it is useful to split a+ and a into a sum of Hermitian opertors i.e.

$$a = (u+ip)/\sqrt{(2h)} \quad a+ = (u-ip)/\sqrt{(2h)} \quad (3.3)$$

Coherent state $|\alpha\rangle$ is the eigen state of the non-Hermitian annihilation operator a with the complex eigen value $\alpha = (u+ip)/\sqrt{(2h)}$

If this complex eigen value $\alpha$ , which labels the coherent states runs over the whole complex plane, the coherent states becomes over-complete for the Hilbert space. Such an over-complete sets of the coherent states can not be used for our consideration because they are linearly dependent. However, Bargmann al. and Perelomov proved that subset of the over complete sub-state form a complete set.

This subset is given by

$$\{ |\alpha\rangle : \alpha = \sqrt{\pi}(l + im) ; l = 0,+-1, +-2,; m= = 0,+-1, +-2,\ldots\ldots \} \quad (3.4)$$

This fact was originally stated by von Neumann, without proof. Therefore these states are called von Neumann lattice coherent state(VNLCS).

Using the VNLCS state, Toyoda et al [4] provide the sound foundation of the classical formalism for the high intensity photon field. However, in their formulation, classical approximation is applied to the radiation field, assuming that the electron state is not subjected to such an approximation. To use the coherent state for electron, the formalism developed by Ohnuki et al [5] should be applied which describe the coherent state of fermion particle

$$|(\xi)_n| > \cong \exp(-\sum_{j=1}^{n}\xi_j a_j^{+})| 0> \qquad (3.5)$$

where ξs are Grassmann number instead of the complex number α for boson field .

This discretization introduces the uncertainty principle of quantum formulation, the delicacy of the quantum effect on the theory such as the super radiant state and the squeeze states which is prominent in the quantum theory of the laser can be studied as discussed as the paper on .

—The one of advantage of using these coherent state description is a many modes created by the EM field can be formulated without difficulty. Further more another transition associated with the super-radiant and between squeezed state can be formulated by including it in the eigen values α,

The squeezed states which might be created by the free electron laser is described by the u or p in the Eq.(3.3)

As mentioned in the state can be expanded by including the these squeeze or super – radiant state, we can formulate the free electron theory which has effect of the these states.

This coherent state is over complete state and the some dicretalization is needed to reduces the over-completeness to the conventional completeness. Here the uncertainty of the quantum description is comes in.

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