The impact of radiative collective effects on the quality of a high-peak-current ultra relativistic electron beam

Gianluca Geloni*, Vladimir Goloviznin, Jan Botman and Marnix van der Wiel

Technische Universiteit Eindhoven

P.O. Box 513, 5600 MB Eindhoven, The Netherlands

Abstract

Very short, high-charge bunches will be used in the next generation light sources. For example, bunch compression chicanes will be often required in order to produce very high-peak-current beams to be used in X-ray SASE-FELs. However, the production and utilization of such bunches may face considerable difficulties due to radiative collective effects: radiative interactions between electrons in the bunch can spoil the required high quality. These questions are currently a matter of active investigation. In the present contribution we address the energy spread induced by the coherent synchrotron radiation in a bunch of relativistic electrons following a curved trajectory in vacuum without shielding. Our considerations include an innovative feature; namely, we assume a privileged direction of motion for the electrons by considering the transverse velocity to be small. This results in a consistent use of a small-angle approximation, which eventually makes the computation of the collective effects more flexible and intrinsically more efficient. Another conceptual advantage of our approach is that we do not switch to a non-Cartesian reference frame, as it is usually done. First computational results will be reported and compared with results obtained by other authors.

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I. INTRODUCTION

Very short, high-charge bunches of electrons will be used in the next generation light sources. Bunch compression chicanes are expected to be often used in order to provide very high peak-current beams for X-ray SASE-FELs. However, their production and utilization may prove difficult due to radiative collective effects.

Consider a short, relativistic electron beam moving along a circular trajectory. Due to coherent addition of electromagnetic waves from individual particles, up to photon wavelengths comparable to the bunch size the intensity of radiation emitted per unit frequency interval increases quadratically with the number of electrons [1].

This number is typically 10^8-10^{10}, which explains the high magnitude of the effect. Similar effects are observed when an electron bunch passes bending magnets, magnetic chicanes, and other beam-optics elements. In all such cases, in contrast to the above steady- state circular motion, also transient collective phenomena take place. Their study has been a matter of active theoretical [2], [3], [4], numerical [5–7] and experimental [8] research in the past few years. Measurements and computations are in reasonable agreement.

In the present paper we address the problem of radiative collective interactions within a short electron bunch following its trajectory in vacuum without shielding. The novel feature of our consideration is that we consistently apply the small-angle approximation, a natural technique for ultrarelativistic particles. By this, the efforts necessary for the treatment of an arbitrary trajectory is considerably reduced; on the other hand, the class of allowed trajectories is somewhat restricted.

From a conceptual point of view, we expect it will account more easily for the effects of finite transverse extent of the bunch, because it does not make use of polar coordinates like other techniques do [3], [9], thus getting rid of any extra term arising from the Jacobian of the transformation. Eventually, this route is expected to lead to an efficient computational tool for the design of magnetic systems for high-peak-current electron bunches. A more detailed presentation of our approach is given in [10].

II. THE SMALL-ANGLE APPROXIMATION

We will consider the bunch as a 'rigid', 1D, charged object with a given linear charge density distribution, together with [2], [4], [11], [12]. We define a cartesian reference frame (x, y, z) as shown in Fig. 1, where the z-axis coincides with the direction of the initial velocity.

We will assume that, before and after the magnets ($z < 0$ or $z > \overline{z}$), the bunch moves along a rectilinear path with constant velocity, while inside the magnetic system ($0 < z < \overline{z}$) it follows a path subject, in the spirit of the small-angle approximation, to the only constraint that the angle θ formed by the velocity vector with the z-axis is always small, i.e. $\theta \ll 1$. Note that θ can still be small or large as compared to the other small parameter of the problem, γ^{-1}, where $\gamma \gg 1$ is the usual Lorentz factor.

It is quite natural to neglect differences in transverse velocities of the electrons, that is $l_b(dv_{x,y}/dz) \ll v_{x,y}$, where l_b is the longitudinal extent of the bunch and $v_{x,y}(z)$ are the components of the transverse velocity.

Moreover we will consider the bunch moving on a fixed trajectory by assuming zero initial energy spread and no change of particles energy during the passage of the bunch through the magnetic system. The back influence of radiative effects on the motion of particles is therefore neglected: the consistency of this assumption must, of course, be verified a posteriori.

Let us define the local particle velocity $\mathbf{v}$ and a unit vector $\hat{\mathbf{n}}$ connecting two points lying on the same trajectory. One has to distinguish explicitly between their longitudinal and transverse components, assuming the latter to be small, according to the small-angle approximation. Keeping first and second order terms one gets the following well known expressions:

$$n_z \simeq 1 - \frac{1}{2}\boldsymbol{n}_\perp{}^2 , \quad v_z \simeq c\left(1 - \frac{1}{2\gamma^2}\right) - \frac{\boldsymbol{v}_\perp{}^2}{2c} . \qquad (1)$$

After the bunch trajectory is known, the problem of radiative collective effects within the bunch reduces to properly accounting for signal retardation in pairwise interactions between individual electrons. Let us consider a test particle inside the bunch. Its present velocity and its present position in the laboratory frame of reference will be denoted as $\boldsymbol{v_0}(t)$ and $\boldsymbol{r_0}(t)$, respectively. We are interested in its interaction with some other bunch particle −the source particle− whose present position will be denoted as $\boldsymbol{r}(t)$. Causality defines the well-known retardation condition between the two particles:

$$|\boldsymbol{r_0}(t) - \boldsymbol{r}(t')| = c\,(t - t') \ , \tag{2}$$

where $\boldsymbol{r}(t')$ denotes the retarded position of the source particle (t' being the so called retarded time), and $(t - t')$ is the time delay associated with signal propagation.

The small-angle aproximation considerably simplifies the treatment of the above retardation condition. Firstly, using Eq.(1) one gets, for the transverse ($\boldsymbol{\rho} = x\ \hat{\boldsymbol{x}} + y\ \hat{\boldsymbol{y}}$) and longitudinal coordinates of a particle

$$\boldsymbol{\rho}(t) = \int_0^t \boldsymbol{v}_\perp(\tau) d\tau \ , \tag{3}$$

$$z(t) \simeq z(0) + \int_0^t \left[c\left(1 - \frac{1}{2\gamma^2}\right) - \frac{{\boldsymbol{v}_\perp}^2(\tau)}{2c} \right] d\tau \ . \tag{4}$$

The transverse velocity, in its turn, is easily found once the configuration of external fields is defined, which makes this approach rather convenient.

Secondly, the positions of the test and of the source particle are related through

$$\boldsymbol{r_0}(t) = \boldsymbol{r}(t + \delta) \ , \tag{5}$$

because all particles in the bunch are assumed to follow the same trajectory. In the small-angle approximation the time difference δ is easily translated into the difference between z-coordinates of both particles:

$$\Delta z = z_0 - z \simeq c\delta \ . \tag{6}$$

Note that for $\delta > 0$ the position of the source particle is *behind* that of the test particle: as has been argued in [2], interactions with particles that are *ahead* of the test particle contain only trivial Coulomb repulsion, which has to be subtracted from final expressions in order to get a non-singular result(see the discussion of the Coulomb singularity in Section III). For this reason, in the following we will always assume $\Delta z > 0$.

Thirdly, it is convenient to switch from time-retardation to a retardation condition expressed in z, which is possible since, in the small-angle approximation, t and z are uniquely mapped onto each other. Upon this the retardation condition in the small-angle approximation can be represented as:

$$\frac{(z_0 - z')}{\gamma^2} + \int_{z'}^{z_0} d\zeta {\boldsymbol{\beta}_\perp}^2(\zeta) - \frac{1}{(z_0 - z')} \left(\int_{z'}^{z_0} d\zeta \boldsymbol{\beta}_\perp(\zeta) \right)^2 \simeq 2\Delta z \ . \tag{7}$$

III. TWO PARTICLE SYSTEM AND COULOMB SINGULARITY

If we multiply the retarded electric field $\boldsymbol{E}$ generated by a source particle at an observation point $\boldsymbol{r_0}(t)$ by the velocity of the test particle $\boldsymbol{v_0}$, and by the electron charge e, we get the change of the energy of the test particle due to its interacton with the source particle:

$$\left(\frac{dE}{dt}\right) = \frac{e^2}{4\pi\varepsilon_0} \left[\frac{c}{\gamma^2} \frac{\hat{\boldsymbol{n}} \cdot \boldsymbol{\beta_0} - \boldsymbol{\beta} \cdot \boldsymbol{\beta_0}}{R^2 (1 - \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta})^3} + \frac{(\hat{\boldsymbol{n}} \cdot \dot{\boldsymbol{\beta}})(\hat{\boldsymbol{n}} \cdot \boldsymbol{\beta_0} - \boldsymbol{\beta} \cdot \boldsymbol{\beta_0}) - (\boldsymbol{\beta_0} \cdot \dot{\boldsymbol{\beta}})(1 - \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta})}{R (1 - \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta})^3} \right] . \tag{8}$$

where $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$ are, respectively, the dimensionless velocity and its time derivative at the retarded time t', R is the distance between the retarded position of the source particle and the observation point, and $\hat{n}$ is a unit vector along the line connecting those two points.

The first term on the right side of Eq. (8) is proportional to R^{-2} and is called Coulomb term: as has been argued in [2], it is singular in the limit $R \to 0$ (that is, $\Delta z \to 0$) due to the infinitely small transverse size of the bunch taht we use in our model. Following [2], we will cure the situation by subtracting from Eq.(8) its purely Coulomb counterpart corresponding to rectilinear motion of the same two particles with constant velocity:

$$\left(\frac{d\widehat{E}}{dt}\right) = \left(\frac{dE}{dt}\right) - \frac{e^2\beta c}{4\pi\varepsilon_0\gamma^2(\Delta z)^2} \ . \tag{9}$$

The resulting expression appears to be regular in the limit $\Delta z \to 0$. This regularized formula will be used in all following calculations.

In the small-angle approximation, one has to expand the above expressions up to second order terms in the transverse velocity.

Performing this expansion and putting, with the same accuracy, $R \simeq (z_0 - z')$, one gets

$$\left(\frac{d\widehat{E}}{dt}\right) \simeq \frac{e^2}{4\pi\varepsilon_0}\frac{2\gamma^2}{1+\gamma^2(\boldsymbol{n}_\perp - \boldsymbol{\beta}_\perp(z'))^2}\left\{[C]+[R]\right\} \ , \tag{10}$$

where $\boldsymbol{n}_\perp$ is given by

$$\boldsymbol{n}_\perp = \frac{1}{(z_0 - z')}\int_{z'}^{z_0} d\zeta \boldsymbol{\beta}_\perp(\zeta) \tag{11}$$

and $[C]$ and $[R]$ stand for the Coulomb and the Radiative part, respectively:

$$\begin{aligned}
[C] \equiv{}& \frac{2c}{(z_0 - z')^2} \\
\times\Bigg\{&\frac{1-\gamma^2\left(\boldsymbol{\beta}_\perp(z_0)-\boldsymbol{n}_\perp\right)^2+\gamma^2\left[\boldsymbol{\beta}_\perp(z_0)-\boldsymbol{\beta}_\perp(z')\right]^2}{\left[1+\gamma^2\left(\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')\right)^2\right]^2} \\
&-\frac{1+\gamma^2\left[\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')\right]^2}{\left[1-\gamma^2\boldsymbol{n}_\perp{}^2+\gamma^2\left(z_0-z'\right)^{-1}\int_{z'}^{z_0}\boldsymbol{\beta}_\perp{}^2(\zeta)d\zeta\right]^2}\Bigg\} \ ,
\end{aligned} \tag{12}$$

$$\begin{aligned}
[R] \equiv{}& 2\gamma^2\frac{\dot{\boldsymbol{\beta}}_\perp}{(z_0-z')\left\{1+\gamma^2\left[\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')\right]^2\right\}^2} \\
\times\Bigg\{&\left[\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z')\right]\Big[1+\gamma^2\left(\boldsymbol{\beta}_\perp(z_0)-\boldsymbol{\beta}_\perp(z')\right)^2 \\
&-\gamma^2\left(\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z_0)\right)^2\Big]-\left[\boldsymbol{\beta}_\perp(z_0)-\boldsymbol{\beta}_\perp(z')\right] \\
&\times\left[1+\gamma^2(\boldsymbol{n}_\perp-\boldsymbol{\beta}_\perp(z'))^2\right]\Bigg\} \ .
\end{aligned} \tag{13}$$

A rather straightforward calculation shows that the obtained expression is, indeed, regular in the limit $\Delta z \to 0$ or, equivalently, $(z_0 - z') \to 0$. Namely, it is sufficient to consider the case of constant transverse acceleration $\dot{\boldsymbol{\beta}}_\perp = Const$. Without loss of generality, let us put $\dot{\beta}_x = \alpha$, $\dot{\beta}_y = 0$. By shifting the origin and denoting $z_0 - z' \equiv \tau$, one has $\beta_y(z_0) = 0$, $\beta_x(z_0) = \alpha\tau$. Upon this, the Coulomb part becomes

$$[C] \simeq \frac{\gamma^2\alpha^2}{6} \frac{\left(1 - \frac{1}{3}\gamma^2\alpha^2\tau^2\right)}{\left(1 + \frac{1}{4}\gamma^2\alpha^2\tau^2\right)^2 \left(1 + \frac{1}{12}\gamma^2\alpha^2\tau^2\right)^2} \ . \tag{14}$$

which clearly has no pole as $\tau \to 0$. Similarly, one can check the absence of singularity in the second term on the right side of Eq. (8) that, being proportional to R^{-1}, is called radiative term.

IV. ENERGY LOSS FOR A TEST PARTICLE IN A 1D BUNCH

Next we evaluate the energy change for a test particle interacting with the whole bunch. Under our assumptions it is logical to express the bunch density, that we will call λ, in terms of the longitudinal distance from the test particle. The corresponding variable, Δz, has been already introduced in Eq.(6).

Clearly, in the evaluation of the energy change, it is more convenient to perform integration over the retarded position z' rather than over the distance between particles Δz, since this eliminates the necessity of solving Eq.(7) against z'. After several calculations, the energy change can finally be written as

$$\left(\frac{dE}{dt}\right)_B (z_0) = \int_{z_0}^{-\infty} \left(\frac{d\widehat{E}}{dt}\right)(z_0, z')\lambda(\Delta z)\frac{d(\Delta z)}{dz'}dz' \ , \tag{15}$$

where $'B'$ stands for Bunch and $\lambda(\Delta z)$ is assumed to behave in such a way that the integral converges. Moreover note that, in Eq. (15), $\lambda(\Delta z)$ is to be considered as a shorthand for $\lambda\left(\Delta z(z, z')\right)$ and that

$$\frac{d(\Delta z)}{dz'} = -\left(1 - \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta}(z')\right) \ . \tag{16}$$

If we want to obtain the energy loss during the entire trajectory we have to integrate over z_0 (or, equivalently, over t) which, finally, gives

$$\Delta E \simeq \frac{e^2}{4\pi\epsilon_0 c} \int_{-\infty}^{+\infty} dz_0 \int_{-\infty}^{z_0} dz' \{[C] + [R]\} \lambda(\Delta z) \ . \tag{17}$$

where $[C]$ and $[R]$ are defined by Eqs. (12) and (13), and Δz by Eq. (7).

All we need to know in order to evaluate the energy change is now the transverse velocity of the bunch as function of the propagation distance. This latter is fully defined by the (pre-designed) configuration of external magnetic fields.

A useful particular case of the above equation is the one of a rectangular current profile: $\lambda(\Delta z)$ is assumed to be constant, $\lambda(\Delta z) = \lambda_0$, over the whole length of the bunch l_b. If the test particle is situated at a distance s_0 from the head of the bunch, then the expression for the energy loss becomes

$$\Delta E(s_0) \simeq \frac{e^2 \lambda_0}{4\pi\epsilon_0 c} \int_{-\infty}^{+\infty} dz_0 \int_{z'_*(l_b - s_0)}^{z_0} dz' \{[C] + [R]\} \ . \tag{18}$$

where $z'_*(l_b - s_0)$ stands for the solution of Eq. (7) corresponding to $\Delta z = l_b - s_0$, and s_0 is understood positive for particles that lie behind the head of the bunch.

We have performed a comparison of the above expressions with some earlier results obtained without the use of the small-angle approximation in [2], where the problem of a 'rigid', one-dimensional (1D) electron bunch entering a circular path from a straight path in vacuum has been carefully studied.

The authors of [2] introduce normalized expressions for the bunch length ($\hat{l}_b = l_b \gamma^3 / A$, where A is the radius of curvature of the bend) and for the angular dimension of the magnet ($\hat{\phi}_m = \gamma \phi_m$), and they consider the bunch and the magnet long (or short) if the corresponding normalized expression are greater or smaller than unity. Upon this they discuss several limiting cases.

In one of those limiting cases, the comparison is particularly simple: if the bunch is short and the bending magnet is, in the normalized sense, much longer than the bunch then, with [2], we can assume all the retarded positions of the sources to lie within the bending magnet, and the situation becomes stationary.

For a rectangular bunch containing N particles, one finds upon a calculation similar to that in Eq. (14)

$$\left(\frac{dE}{dt}\right)_B = -\frac{1}{4\pi\varepsilon_0}\frac{4Ne^2\gamma c}{Al_b}\frac{\gamma u_s\left(8+\gamma^2 u_s^2\right)}{\left(4+u_s^2\gamma^2\right)\left(12+\gamma^2 u_s^2\right)}\ , \tag{19}$$

where

$$u_s \simeq \frac{2\gamma^2(l_b - s_0)}{A}\ . \tag{20}$$

One can easily check that Eqs. (19) and (20) coincides with those found in [2].

In general, the expressions are rather complicated, and the corresponding comparison can only be done numerically. A computer code has been developed and benchmarked against several limiting cases given in [2]. The results are presented in table 1. Cases 1 and 2 deal with a short bunch and a magnet longer than the bunch: here the crucial factor is the energy of the beam. The difference by a factor of 2 in the Lorentz factor is responsible for the increase by a factor of 16 in the energy change. In cases 3 and 4 the magnet is *long* and the bunch is *much longer* (again in the normalized sense) than the magnet; these two cases have been computed, respectively, with low- and high-energy bunches.

In all cases we observe a good agreement between our numerical computations and the corresponding analytical estimates (a relatively large discrepancy of order 10% in cases 3 and 4 is presumably a result of the logarithmic accuracy of the analytical expressions in [2]). It is also worth mentioning that in all four cases the total energy change is small as compared to the initial particle energy; specifically, the largest relative energy change of about 4% is found in case 2. This confirms consistency of the computational scheme, as has been discussed in Section I.

As one more test, we have calculated the instantaneous power radiated by a particle located at the head of a bunch that enters into bending magnet. This case demonstrates pronounced transient collective effects. To be specific, we considered a 1 mm-long, 40 MeV bunch with rectangular electron density distribution entering a circular trajectory with a radius $A = 1$ m from a straight path. The dependence of the radiated power on the angle of deflection θ is shown in Fig. 2.

The observed dependence is in agreement with well-known results [3], [4].

V. CONCLUSIONS AND SPECULATIONS

The problem of radiative collective effects within an ultra relativistic electron bunch has been addressed in a new analytical approach. The systematic use of the small-angle approximation results in a new expression for the energy exchange between a test particle and the bunch, and all we need to know to evaluate it is the transverse velocity of the bunch as a function of the propagation distance, which is directly determined by the external field configuration.

Analytical and numerical comparison of the obtained formulas with earlier results by other authors has been performed and a good agreement has been demonstrated. The technique is applicable to an arbitrary bunch trajectory subject with the only restriction of a small deviation from the initial direction. A conceptual advantage of the new route is that, due to the choice of geometry, we do not switch to a polar frame of reference, thus getting rid of any extra terms arising from the Jacobian of the transformation. We expect that, in future studies, this will allow to account more easily for the finite transverse size of the bunch.

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* e-mail address: g.a.geloni@tue.nl

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FIGURES

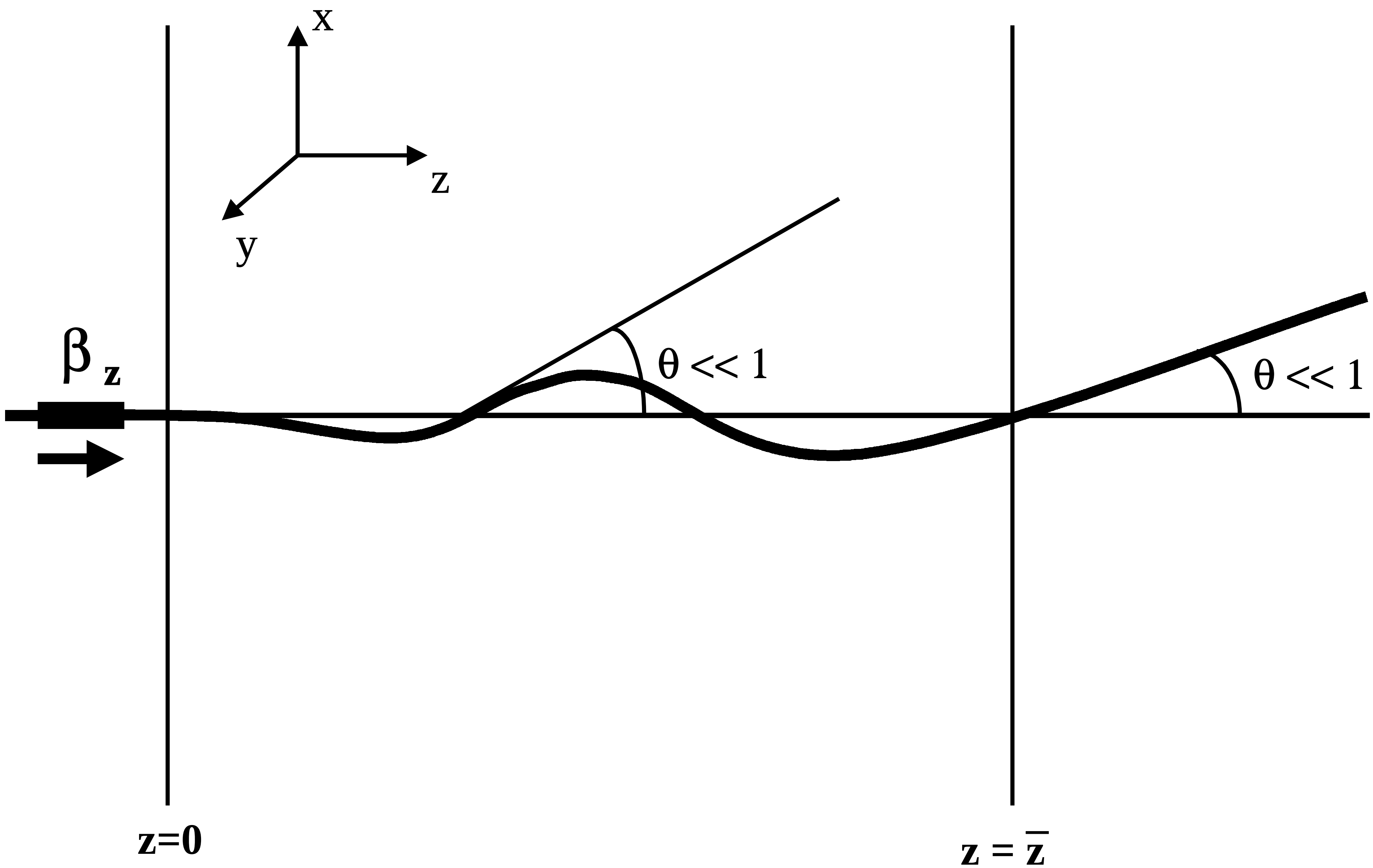
FIG. 1. Schematic of a particle trajectory in the small angle approximation

FIG. 2. Normalized transient power loss for a bunch with rectangular density distribution going into a bend

TABLES

TABLE I. Energy change, in Joules, for an electron located at the head of a bunch with rectangular density distribution. A comparison is made between an evaluation with completely analytical formulas found by other authors and with our simuation. B is the magnetic field, in Tesla, l_b is the bunch length in meters, γ is the Lorentz factor, $\overline{z}$ is the length of the interaction zone, in meters and N is the number of particles considered in the bunch

	$B(T)$	$l_b(m)$	γ	$\overline{z}(m)$	$N/10^9$	$Analytical Results(J)$	$Simulation Results(J)$
1	0.043	$1.0 \cdot 10^{-6}$	25	$1.2 \cdot 10^{-2}$	6.0	$8.7 \cdot 10^{-15}$	$8.3 \cdot 10^{-15}$
2	0.085	$1.0 \cdot 10^{-7}$	50	$8.0 \cdot 10^{-3}$	10.0	$1.54 \cdot 10^{-13}$	$1.50 \cdot 10^{-13}$
3	0.17	0.45	50	$9.9 \cdot 10^{-2}$	10.0	$3.7 \cdot 10^{-17}$	$3.4 \cdot 10^{-17}$
4	0.85	0.2	500	0.02	10.0	$8.4 \cdot 10^{-17}$	$9.3 \cdot 10^{-17}$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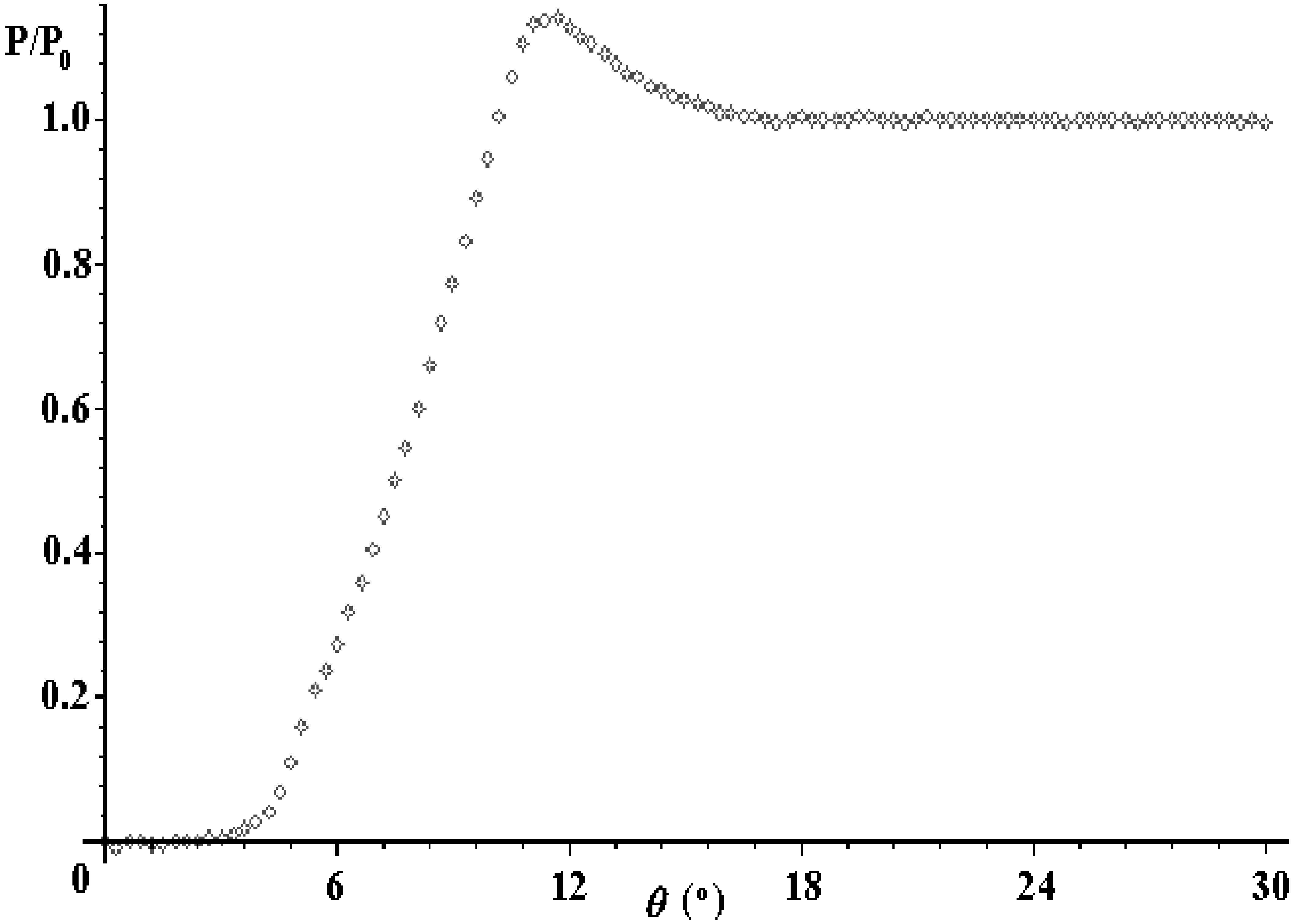
P/P0
1.0
0.8
0.6
0.4
0.2
0
6
12
θ (°)
18
24
30