Saturation of Beam-Wave Interaction in an Active Medium

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Abstract. A relativistic bunch moving in the vicinity of a dielectric slab excites Cerenkov radiation. The spectrum of this radiation depends on the dielectric coefficient, the energy of the bunch and its geometric shape. If the dielectric slab is *active* and the bunch excites the proper frequency, the wake will be amplified. In the framework of a linear theory we discuss the dependence the spatial growth of the bunch on the geometric and optical parameters of the system. Further using non-linear theory, it is shown that electrons are being accelerated even when the interaction of the wave and the active medium reaches saturation.

INTRODUCTION

In all the new acceleration schemes suggested in the past two decades, the initial energy is either stored in a driving electron beam or laser pulse[(1-2)]. In recent years we have shown that there are some important advantages if the energy required for acceleration is stored in the background medium[(3-6)]. Firstly, this type of interaction mechanism virtually eliminates the constraint imposed in the other schemes on the driving electron beam or laser pulse that must carry all the necessary energy. Secondly, the longitudinal electric field is inherent in this acceleration scheme without the difficulties associated with focusing i.e., Rayleigh length in laser driven schemes. In order to illustrate the concept in the simplest possible way, consider a gas characterized by

$$\varepsilon(\omega) = \varepsilon_r + \frac{\omega_p^2}{\omega_0^2 - \omega^2 + 2j\omega\omega_1} \qquad (1)$$

where $\omega_p^2 = (N_1 - N_2)(2\omega_0\mu_{12}^2)/\hbar\varepsilon_0$, ω_0 is the resonance angular frequency, ω_1 is the resonance width and ω_p defines "plasma" frequency in terms of the microscopic parameters of the constituents; $N_1[N_2]$ is the density of atoms in which the bound electron is in the low [high] state; μ_{12} is the dipole-moment of the transition from one energy state to the other. Clearly this quantity is negative if the population is inverted ($N_2 > N_1$). In a medium described by Eq.(1) the reaction-field of the medium to the motion of a relativistic point-charge on the particle itself is

$$E_z = \frac{q}{4\pi\varepsilon_0 \varepsilon_r} \frac{2\omega_p^2}{c^2 \varepsilon_r} = \frac{q}{4\pi\varepsilon_0 \varepsilon_r} \frac{\omega_0}{c} \frac{4\mu_{12}^2 (N_1 - N_2)}{c\hbar\varepsilon_0 \varepsilon_r} \quad (2)$$

and in case of population inversion this corresponds to an accelerating field; when developing this expression, saturation was ignored. As an example consider a resonance at 10μm, a dipole-moment μ_{12}= (1.6×10^{-19}) x (2×10^{-10}), a point charge that consists of 10^7 electrons and a large population inversion N_2-N_1 = 10^{25}m$^{-3.}$ The accelerating gradient in such a case is of the order of 1.3GV/m.

There is a practical difficulty associated with this scheme. Since the gradient is proportional to the number of electrons in the bunch, the latter needs to be large in order to have a gradient that is competitive with other mechanisms. In addition, even if such a large number of electrons in a pulse that is much smaller than the resonance wavelength were available, there still is the problem of the space-charge that tends to spread its spatial distribution. For this reason it was suggested(3) to separate the bunch into two: a trigger and a witness bunch. The triggering bunch consists of a relatively small number of electrons (say 10^4-10^5) depending on the geometry of the structure and the energy of the electrons or a train of such micro-pulses that form one macro-pulse. This macro-pulse triggers the medium in the sense that it generates Cerenkov radiation. The latter is amplified by the active medium. This amplified wake has a phase velocity that is identical with the velocity of the trigger bunch. If properly located, the witness bunch that trails the trigger bunch, is accelerated by the amplified wake. In fact, the accelerated bunch should be also split into a train of micro-bunches in order to avoid de-bunching. Moreover, the witness macro-bunch may be located into the saturation region in order to minimize the energy variation from one micro-bunch to another.

MODE SELECTION

Inherently the narrow band of amplification selects among all the possible modes a small number. In the case of a wake there is a double selection process: one selection associated with the phase velocity (that musty equal the particle's velocity) and the bandwidth of amplification. A simple evaluation of a wake in space-time may be demonstrated using the background dielectric model (1) with both the material and the electromagnetic field confined by a circular waveguide of radius R. The wake generated by a point-charge is given by

$$E_z(r,z,t) = \frac{-q}{4\pi\varepsilon_0 R^2} \sum_{s=1}^{\infty} \left[\frac{2}{J_1(p_s)}\right]^2 J_0(p_s \frac{r}{R}) \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \frac{j\omega}{\varepsilon(\omega)} \frac{\left[\varepsilon(\omega) - \beta^{-2}\right] e^{j\omega(t-z/v)}}{\omega^2 \left[\varepsilon(\omega) - \beta^{-2}\right] - \omega_{c,s}^2} \right\} \quad (3)$$

where it has been assumed that the particle is on axis, $\omega_{c,s}$=cp_s/R are the cut-off angular frequencies and p_s are the zeros of $J_0(\xi)$; s=1,2,3,…. .

A few of the poles of the integrand have imaginary parts that, in turn, are responsible to the amplification process. The dependence of the imaginary part on the index of the mode is

$$\frac{\mathrm{Im}(\omega)}{\omega_0}=\frac{(\omega_p/\omega_0)^2}{4\varepsilon_c}\frac{\omega_0}{\omega_1}\begin{cases}1 & for \quad \omega_{s0}=\omega_0 \\ \left(\frac{\omega_1 R}{c}\right)^2\frac{\sqrt{\varepsilon_c}}{(\pi\Delta s)^2} & for \quad \omega_s\neq\omega_0\end{cases} \qquad (4)$$

here $\varepsilon_c=\varepsilon_r-\beta^{-2}$. In order to illustrate the "selectivity" of the process, bear in mind that for Nd:YAG, $1/\omega_1=230\mu sec$, thus the ratio $\omega_1 R/c = 4x10^{-8}$. Consequently, the "non-resonant" modes have virtually zero growth rate and only a single mode is amplified. In the next section we use this result in order to examine the saturation effect.

AMPLIFICATION AND SATURATION

So far the amplification of the Cerenkov wake was demonstrated in the framework of a *linear* theory. It is possible to formulate a 1D set of equations that determine the *non-linear* interaction of electrons and waves in an active medium. Its outcome reveals acceleration even during saturation regime i.e., when most of the energy initially stored in the medium was converted into radiation. In order to present the essence of the theory, consider the interaction of a bunch of electrons with an eigen-mode in a *passive* electromagnetic (EM) structure. The equations that describe the dynamics of a traveling wave and a beam in a passive slow-wave structure are

$$\left.\begin{aligned}&\frac{d}{d\xi}a=\alpha\left\langle e^{-j\chi_i}\right\rangle\\&\frac{d}{d\xi}\gamma_i=-\frac{1}{2}\left[a\,e^{j\chi_i}+c.c.\right]\\&\frac{d}{d\xi}\chi_i=\Omega\left(\frac{1}{\beta_i}-\frac{1}{\beta_p}\right)\end{aligned}\right\}\Rightarrow\frac{d}{d\xi}\left[\underbrace{\left\langle\gamma_i\right\rangle-1}_{Kinetic\ Energy}+\underbrace{\frac{|a|^2}{2\alpha}}_{EM\ Energy}\right]=0 \qquad (5)$$

here $\xi=z/d$ is the normalized coordinate, d is the interaction length, $a= eE_z\, d/mc^2$ is the normalized electric field that interacts with the electrons, $\Omega=\omega d/c$ is the normalized angular frequency, α is the normalized coupling coefficient between the (single) mode that propagates in the structure and the beam. The first equation is the amplitude equation that determines the change in the amplitude of the interacting wave, *a*, in terms of the particles' dynamics. The second equation is the single particle energy conservation and the third determines the change in the phase of a single particle relative to the interacting wave.

Energy conservation is inherent in this set of equations. It may be deduced by averaging the equation for a single particle energy conservation and substituting the amplitude equation. The result is illustrated in the right hand side of the expression above: the first term, represents the average kinetic energy of the particles whereas the second term represents the electromagnetic energy - both quantities are normalized with the rest mass energy of the electrons present in one period of the wave, $N_e mc^2$. In fact, $|a|^2/2\alpha$ is the normalized EM energy *per electron* in one period of the wave.

As indicated above, equations (5) determine the interaction of a wave, propagating in a passive electromagnetic structure, with a bunch of electrons. At this point, extension of the analysis to an *active* electromagnetic structure is relatively easy. In a unit volume determined by the effective cross-section of the system and one wavelength, the kinetic energy is $N_e mc^2(\langle\gamma\rangle-1)$, the electromagnetic energy is $N_e mc^2 |a|^2/2\alpha$ and it is assumed that the energy stored in excited atoms is $N_{ex} E_{ph}$ where E_{ph} is the energy of a photon stored in an atom; N_{ex} represents the number of excited atoms. Consequently, the normalized energy stored in excited atoms per electron of the micro-bunch is given by $N_{ex} E_{ph}/N_e mc^2$ and energy conservation reads

$$\frac{d}{d\xi}\left[\underbrace{\left\langle\gamma_i\right\rangle-1}_{Kinetic\ Energy}+\underbrace{\frac{|a|^2}{2\alpha}}_{EM\ Energy}+\underbrace{\frac{N_{ex}\hbar\omega}{N_e mc^2}}_{Energy\ in\ Medium}\right]=0. \qquad (6)$$

This expression reflects the fact that for acceleration purpose, the nominal number of excited atoms has to exceed the electrons number by many orders of magnitude. For example, if the input electromagnetic power is negligible and it is necessary to accelerate 10^5 electrons of 300MeV to an energy of 3GeV, then the number of excited atoms in one period of the wave must be at least 3×10^{14}; assuming a Nd:YAG slab of 6mm diameter, the required density of excited atoms is of the order of $10^{19}\mathrm{cm}^{-3}$.

In the absence of the electron-beam, according to phenomenological laser theory[(7)], the amplitude increases in space according to $da/d\xi=(\sigma n_{ex} d/2)a$ where σ is the transition cross-section and n_{ex} is the effective *density* of excited atoms; this assumption is subject to the condition that a quasi-steady-state regime has been reached. The difference between the actual density of excited atoms and the effective one is roughly the ratio between the cross-section of the active medium (A_{am}) and the effective area of the wave (A_w) i.e. $n_{ex} = N_{ex}/\lambda A_w = F_f N_{ex}/\lambda A_{am}$ where the form factor is approximately given by $F_f = A_{am}/A_w$ and $N_{ex}/\lambda A_{am}$ is the density of the population inversion in the material. Defining the normalized number of excited atoms by $N = N_{ex}E_{ph}/N_e mc^2$, the dynamics of electrons and field in an active structure is described by

$$\left.\begin{aligned}
&\frac{d}{d\xi}a=\alpha\left\langle e^{-j\chi_i}\right\rangle+\frac{1}{2}\nu N a\\
&\frac{d}{d\xi}\gamma_i=-\frac{1}{2}\left[a e^{j\chi_i}+c.c.\right]\\
&\frac{d}{d\xi}\chi_i=\Omega\left(\frac{1}{\beta_i}-\frac{1}{\beta_p}\right)\\
&\frac{d}{d\xi}N=-\nu\left(\frac{|a|^2}{2\alpha}\right)N
\end{aligned}\right\}\Rightarrow\frac{d}{d\xi}\left[\underbrace{\left\langle\gamma_i\right\rangle-1}_{Kinetic\ Energy}+\underbrace{\frac{|a|^2}{2\alpha}}_{EM\ Energy}+\underbrace{N}_{Energy\ in\ Medium}\right]=0\,; \qquad (7)$$

here $\nu=(\sigma d/A_{am}\lambda)(mc^2/\hbar\omega)N_eF_f$ is the normalized cross-section of interaction; this set of equations satisfy the energy conservation relation formulated above.

The top frame in Figure 1 illustrates the normalized population inversion, the middle one represents the energy gained by the electrons and the bottom frame shows the way the gradient develops along the interaction region. Several issues are evident: firstly, in all cases the gradient saturates, secondly, the current density of the bunch affects significantly the saturation level. Thirdly, as the electromagnetic wave is amplified by the medium, it accelerates the micro-bunch and the acceleration continues even when deep saturation is reached.

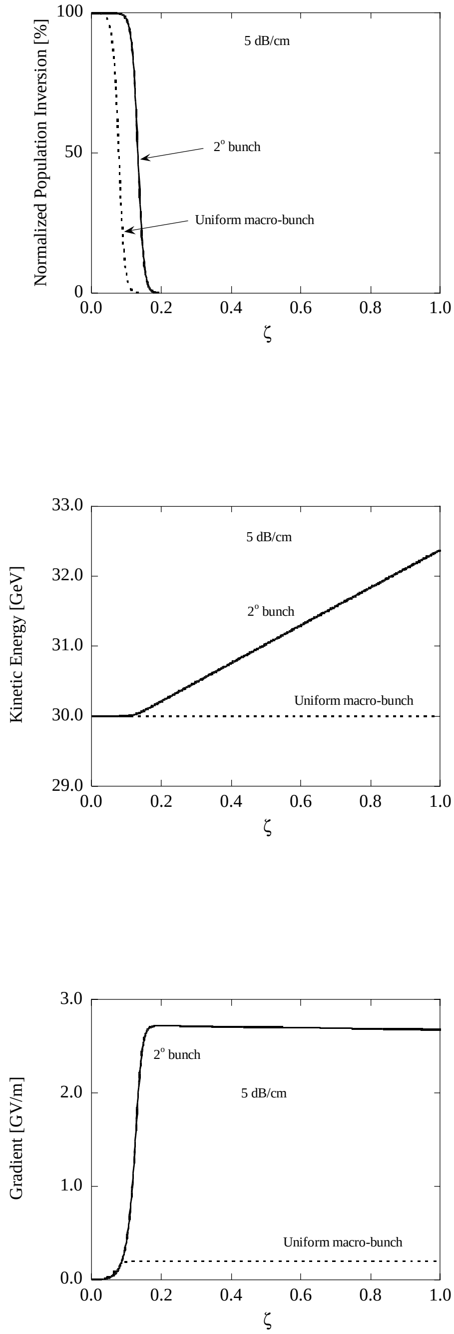

Figure 1.: Simulation results of the interaction between electrons and an electromagnetic wave in an active structure.

DISCUSSION

Electrons today are accelerated using electromagnetic energy stored in *macroscopic* cavities. The essence of the concept reviewed here is that electrons may be accelerated by energy stored in *microscopic* cavities namely, atoms or molecules. When a point-charge traverses an active medium the reaction field due to the presence of the medium tends to accelerate the particle. Moreover, the wake behind the point-charge may be amplified by the medium and further used to accelerate other electrons. Even when the saturation of the interaction between the wave and the medium is accounted for, electrons remain trapped and are still accelerated.

This work was supported by the Israel Science Foundation.

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