

The Quantum X-ray Compton Free Electron Laser

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The quantum operation mode in the X-ray Compton free-electron laser (FEL) is presented. This FEL mode emerges when the quantum emission exceeds the electron beam energy spread and FEL homogeneous linewidth, determined by the optical undulator length. Such a FEL becomes a two-level quantum oscillator with a completely inverted active medium. In the nonlinear interaction regime, inversion can be completely removed in a single pass, in analogy with π -pulse formation in coherently amplifying medium. Estimates show the quantum FEL provides a promising compact high-power high-brightness source of coherent x-ray radiation. The design parameters for the quantum FEL are proposed for a compact high brightness coherent X-ray radiation source

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1. Introduction.

Since the demonstration of the first FEL, lasing in these devices has been achieved over a broad frequency range. It has been found that most of the experimental results can be explained by both the quantum and classical theories [1,2]. It was accepted that the complete agreement between the results of the classical and quantum approaches signifies that an FEL is essentially a classical device, more akin to conventional devices of classical electronics, rather than an intrinsically quantum oscillator.

In this paper, we intend to show that quantum effects become dominant at definite conditions in the x-ray Compton FEL. The quantum nature of laser-particle interaction in free-electron lasers becomes crucial when the laser quanta exceeds the electron beam energy spread and FEL homogeneous linewidth, determined by the number of undulator periods. Such a FEL becomes a two-level quantum oscillator with a completely inverted active medium. In the nonlinear interaction regime, inversion can be completely removed in a single pass, in analogy with π -pulse formation in coherently amplifying medium. Possible test experimental schemes based on a high peak power laser and electron accelerators are discussed.

2. Classical and quantum FEL operation modes

The quantum nature in any laser - matter interaction process is revealed when the state of material system changes significantly as a result of photon absorption or emission event. Or, in other words, when the characteristic energy scale of material (atomic, molecular, electron, etc.) system is of the same order of magnitude as the radiation quantum. In free-electron lasers, such a characteristic energy scale is determined by the condition of an effective interaction of electron beam with the slow ponderomotive potential wave, which is formed by the undulator and signal fields.

In the X-ray Compton FEL, the high-intensity laser pulse is scattered by counter propagating relativistic electron beam [3]. The dependencies of the emission and absorption probabilities for high-frequency photons in FEL on the beam energy, i.e., laser absorption and emission lines, are thus of resonant nature. The centers of absorption (a) and emission (e) lines are at energies $\varepsilon_{a,e} = \varepsilon_0 \pm \eta\omega / 2$, where the energy $\varepsilon_0 = mc^2(1 - \omega/kc)^{-2}$ corresponds to the synchronism with the ponderomotive wave, of which frequency $\omega = \omega_s - \omega_i$ and wave number $k = k_s + k_i$, where $\omega_{s,i}$ and $k_{s,i}$ are frequencies and wave numbers of pumping (i) and signal (s) waves respectively [1].

The quantum properties of an FEL are thus determined by the ratio of the separation $\eta\omega$ between the absorption and emission lines and their effective width $\Delta\varepsilon/\varepsilon_0 = \max\{1/4N, \Delta\gamma/\gamma, <\Delta\Theta^2 >/2\}$, which is caused by the finite length of interaction region $L = N\lambda_i$ (homogeneous broadening, $\sim 1/4N$), and inhomogeneous broadening due to the beam energy spread $\Delta\gamma/\gamma$ and divergence $<\Delta\Theta >$, caused by beam emittance. It is convenient to introduce the quantum parameter η [4]

$$\eta = \frac{\eta\omega}{\Delta\varepsilon} \quad (1)$$

In the classical FEL operation regime $\eta \ll 1$, the emission and absorption profiles practically overlap (see Fig.1a). The state of electron cannot change significantly as the probabilities of a direct Compton scattering and an induced process for each electron are practically the same. The gain coefficient formula does not contain the Plank constant and agrees with the formula derived from classical theory [3.5].

In the opposite limit $\eta \gg 1$, which is the case of our particular interest, the emission and absorption lines are completely separated, see Fig.1b. The state of electron changes dramatically within the scale of $\eta\omega$, if the electron energy is near the emission maximum ε_e , it cannot absorb a quantum and vice versa. In this regime, the quantum effects become dominant. The FEL dynamics reduces to the quantum transitions between these two energy levels, ε_e and ε_a [4]. An adequate description of lasing in this regime thus requires the quantum theory and can not be done within the classical electrodynamics.

For the case of the initial laser quantum $\eta\omega_i \sim 1$ eV and Rayleigh length $L \sim 1$ cm, the FEL linewidth is determined mainly by the beam energy spread, and the Compton FEL will operate in the quantum regime at $\gamma > 1.25 \cdot 10^5 \Delta\gamma/\gamma$, that corresponds to the beam electron energy of ~ 60 MeV for the energy spread of $\sim 0.1\%$. The emitted quanta are thus in the X-ray region, $\eta\omega_s \approx 4\gamma^2\eta\omega_i \geq 40$ keV.

3. Electrons as the quantum anharmonic oscillators.

To describe correctly two-level X-ray FEL dynamics, the Klein - Gordon equation should be analyzed in association with the wave equation for the X-ray signal amplitude [5]. Taking into account the momentum and energy conservation laws

$$p_n = p_0 + nk, \quad \varepsilon(p_n) = (p_n^2 c^2 + m^2 c^4)^{1/2}, \quad (2)$$

where p_0 is the initial momentum of electrons, one can represent the electron wave function in the form

$$\Psi = \sum_n \left(\frac{mc^2}{2\Omega\varepsilon(p_n)} \right)^{1/2} a_n(z, t) \exp\left\{ \frac{i}{\eta} [p_n z - \varepsilon(p_n)t] \right\}, \quad (3)$$

which leads to a set of equations for amplitudes of the energy levels given by Eq. (2) [5]

$$\begin{aligned} i \dot{a}_n &= g_n a_{n-1} \exp(-i\Delta_n t) + g_{n+1} a_{n+1} \exp(i\Delta_{n+1} t) \\ g_n &= (e^2 / \eta) A_i A_s (\varepsilon_{n-1} \varepsilon_n)^{-1/2} \end{aligned} \quad (4)$$

Here, $A_{i,s}$ are the vector potential amplitudes of the pumping (i) and X-ray signal (s) coherent waves. In fact, Eqs. (2)-(4) describe an anharmonic oscillator, which is characterized by the detuning from the resonance, increasing with the number of level

$$\Delta_n = (\varepsilon_n - \varepsilon_{n-1}) / \eta - \omega \approx 4 \frac{c}{\lambda_i} \frac{\varepsilon(p_0) - \varepsilon_e + n\eta\omega}{\varepsilon_0} \quad (5)$$

At the interaction length,

$$L > \frac{1}{32\pi} \frac{mc^2}{\eta\omega_i} \frac{\lambda_i}{\gamma_i} \sim 0.497 / \gamma \text{ cm}, \quad (6)$$

all the exponents in Eq. (4) with $n \neq 0$ oscillate sufficiently fast so that the excitation of correspondent states is negligible. As a result, emission of coherent x-ray photons is related to the stimulated transitions between only two energy levels $n = 0, -1$ [4].

4. Two-level FEL coherent nonlinear dynamics (X-ray π -pulse generation)

In the quantum regime the Compton FEL becomes a two-level quantum oscillator in a completely inverted active medium [4]. Such a quantum system is known very well in the conventional quantum electronics, and it is natural to apply a correspondent well-elaborated technique to our analysis. In this technique, Eqs.(4) in the two-level limit forms the Maxwell - Bloch system of equations in association with the wave equation for the X-ray signal amplitudes, the solution to which is usually seeking in terms of pulse square [6]

$$\chi(z) = \frac{\mu}{c\eta} \int_0^z E_s(z) dz, \quad \mu = e^2 E_i / m\omega_i^{1/2} \omega_s^{3/2} \quad (7)$$

Here μ is an effective dipole moment of the two-level system. The energy state amplitudes for initially completely inverted system then are

$$a_0 = \cos(\chi(z)/2), \quad a_{-1} = -i \sin(\chi(z)/2), \quad (8)$$

and the wave equation becomes the pendulum equation

$$\frac{\partial^2 \chi}{\partial z^2} = \alpha^2 \sin \chi, \quad \alpha = \left(\frac{2\pi n_e e^4 E_i^2}{\eta \omega_s^2 \omega_l m^2 c^2} \right)^{1/2} \quad (9)$$

It is easy to see that α , in fact, is the linear gain coefficient. Note that the gain coefficient formula contains the Plank constant in explicit form. It means that the regime under consideration has essentially quantum nature and qualitatively differs from all the conventional FEL operation regimes.

The Eq. (9) is nonlinear. In association with “material” equations (8), it describes the stimulated emission of coherent X-ray quanta in the collective nonlinear coherent interaction of the laser with the electron beam. The coherence emerges due to all the electrons in the same quantum state during the interaction.

The Eqs. (7)-(9) reveal a well-known in the nonlinear optics effect of π -pulse generation: when the total square of the pulse reaches the value $\chi(L) = \pi$, the system becomes in the lower energy state, and the complete removal of inversion emerges. For the intensity of the x-ray signal, we have from Eq. (9)

$$I_s = dn^{-2} \left(\frac{\alpha L}{\sqrt{\kappa}}, \kappa \right) I_s(0) \quad \kappa = 1 / (1 + I_s(0) / I_b) \quad (10)$$

which is the periodical function of the interaction length L . The maximum values are

$$I_s(L_{opt}) = I_s(0) + I_b \quad (11)$$

Here $I_b = n_e \eta \omega_s c$ is the maximum X-ray intensity which can be emitted by a bunch of two-level electrons. The physical interpretation of Eq. (11) is clear – as a result of the collective coherent laser-beam interaction, every “two-level” electron emits one X-ray quanta.

Note, the maximum output intensity (11) does not depend on the pump laser intensity, but just on the density of electron beam, which is quite different from the conventional FEL classical regimes.

In Fig.2, the dependence of signal intensity on the longitudinal coordinate is shown at three different pump laser intensities. The calculations are made for the electron beam of the $n_e = 10^{15} \text{ cm}^{-3}$ and energy $\gamma = 100$, and for 10 initial x-ray photons. One can see that the most of X-ray quanta are emitted in the nonlinear regime when the collective interaction becomes dominant.

The optimum interaction length, corresponding to the X-ray π -pulse is determined by the half-period of the Jakobi dn -function

$$L_{opt} = K(\kappa)\sqrt{\kappa}\alpha^{-1} \quad (12)$$

where $K(x)$ is the complete elliptic integral. In Fig 3, the dependence of optimum interaction length on the pump laser intensity is shown for the above parameters and three different electron beam densities ($0.5 \cdot 10^{15} \text{ cm}^{-3}$, $1 \cdot 10^{15} \text{ cm}^{-3}$, $5 \cdot 10^{15} \text{ cm}^{-3}$).

Discussions

In conclusion, let us discuss some estimates. The beam energy spread of 0.1% lead to the lower limit of beam energy of ~ 50 MeV. The correspondent energy of x-ray quanta are in the region ~ 40 keV (for initial laser wavelength of $\sim 1 \mu\text{m}$). For the bunch of $\sim 10^{10}$ electrons (1.6 nC) the energy of emitted coherent X-ray radiation is $\sim 64 \mu\text{J}$, or in terms of intensity $I_{s,\text{max}} \sim 1.9 \cdot 10^{11} \text{ W/cm}^2$ at the electron density $n_e = 10^{15} \text{ cm}^{-3}$. The interaction length at pump intensity of $\sim 10^{20} \text{ W/cm}^2$ is ~ 2 cm (see Fig.3) which allows to focus the electron beam and reach sufficiently high beam densities. Supposing the beam diameter of $\sim 10 \mu\text{m}$, the divergence of X-ray pulse is $\sim 2.5 \mu\text{rad}$ that leads to the high brightness of the quantum x-ray FEL. Really, if we define the brightness as the photon flux in the spherical angle determined by divergence, we have $\sim 1.6 \cdot 10^{32} \text{ mm}^{-2} \text{ mrad}^{-2} \text{ sec}^{-1}$ at the pulse duration of 100 fsec, which is comparable to SLAC/LCLS X-ray sources.

The above parameters, however, may not be unique and available with the current technologies. Using the high-power table-top femtosecond lasers, the intensities $> 10^{21} \text{ W/cm}^2$ are now realized. The race-track microtrons can provide electron bunches of the charge of 1-5 nC and energy in the region 50-150 MeV at sufficiently low energy spread ($< 0.1\%$) and emittance. Thus, the realization of the Compton free-electron laser in the quantum operation regime can provide us a compact source of high-brightness coherent X-ray radiation.

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Figure 1. The dependencies of x-ray quantum absorption and emission probabilities W on the electron energy \mathcal{E} (i. e., FEL absorption and emission lines reciprocal position) in the two limiting cases: a) the classical FEL operation regime, $\eta \ll 1$; b) the quantum FEL operation regime, $\eta \gg 1$.

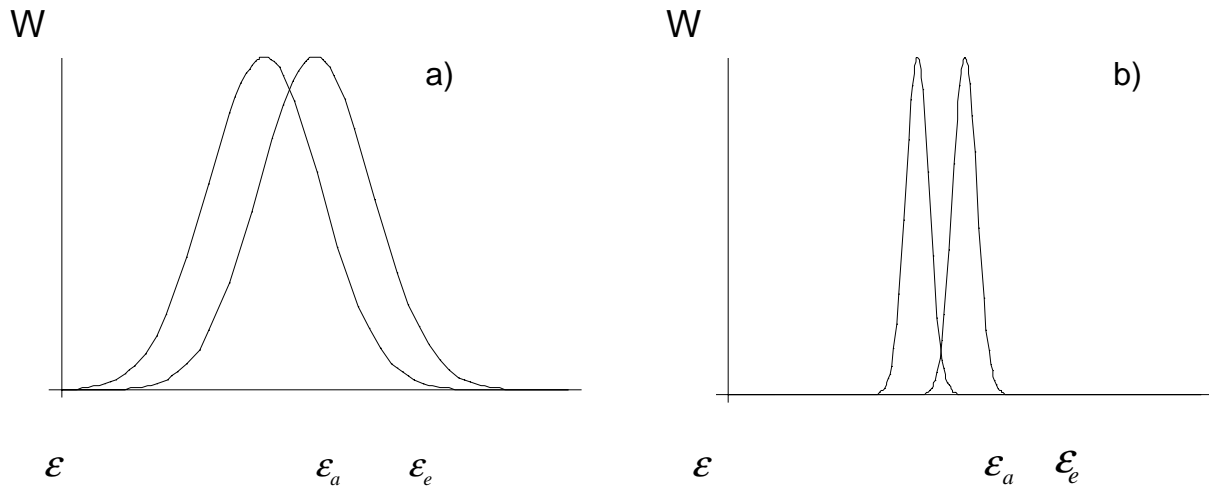


Figure 2. The dependence of the normalized x-ray signal intensity $I_s(z)/I_b$ on the longitudinal coordinate z at the pump laser intensities (a) $3 \cdot 10^{19} \text{ W/cm}^2$, (b) 10^{20} W/cm^2 , (c) $3 \cdot 10^{20} \text{ W/cm}^2$ for the beam energy $\gamma = 100$, the beam density $n_e = 10^{15} \text{ cm}^{-3}$ and the number of initial X-ray photons - 10.

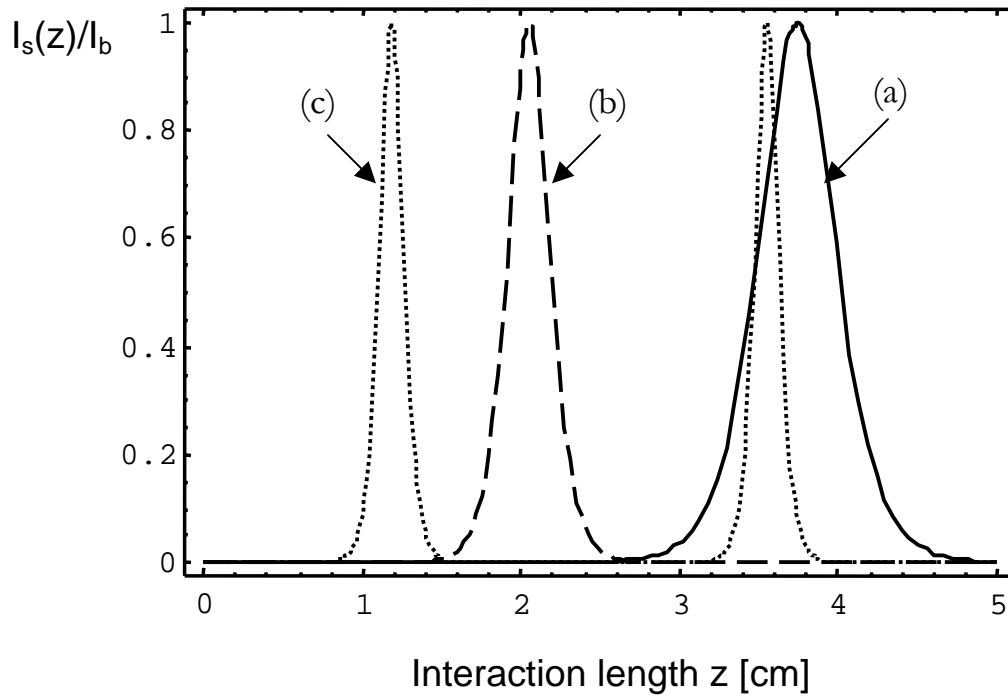


Fig. 3. The dependence of the optimum interaction length L_{opt} on laser intensity (in units 10^{19} W/cm^2) at the electron beam densities (a) $0.5 \cdot 10^{15} \text{ cm}^{-3}$, (b) $1 \cdot 10^{15} \text{ cm}^{-3}$, (c) $5 \cdot 10^{15} \text{ cm}^{-3}$ for the beam energy $\gamma = 100$, and the number of initial X-ray photons - 10.

