

Industrial Espionage with a Noisy Intelligence (18/09/2011)

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1. Introduction.

We analyze the role of the industrial espionage in a simple model of two firms. The first firm is an incumbent monopolist (M), and the second firm is a potential entrant (E). The monopolist who wishes to deter E from entering the market. For this purpose M considers an investment for capacity expansion.

E will stay out if she knows that the capacity of M is expanded. E will enter if she knows that M did not expand his capacity. Hence, E enters only if she believes that with high probability M does not expand his capacity.

The entrant E operates an Intelligence System (IS) which set to detect M's action of whether or not to expand capacity. The IS can send one out of two signals. The signal i which indicates that M invests in new capacity and the signal ni which indicates the opposite. The precision of the IS is α , $\frac{1}{2} \leq \alpha \leq 1$. Namely, the signal sent by the IS is correct (it sends the signal i when M invests and sends the signal ni otherwise) with probability α . If $\alpha = 1$ the IS is a perfect device and E can perfectly detect the action of M. The case $\alpha = \frac{1}{2}$ is equivalent to not using any intelligence system. Based on the signal received, E decides whether or not (or with what probability) to enter the market.

There are four possible outcomes: (NI, NE), (NI, E), (I, NE), (I, E), where I stands for "invest" and NI stands for "not invest". The interpretation of E and NE is similar. The best outcome for M is the status quo outcome (NI, NE). His second best outcome is (I, NE). M prefers the outcome (I, NE) on (I, E), and if the investment cost not too high, then M prefers the outcome (I, E) on (NI, E). As for E, her best outcome is (NI, E) and it is better for her than either (I, NE) or (NI, NE) (in both cases E obtains zero). The worst outcome for E is (I, E).

We first analyze the case where the value of α is common knowledge to both firms. Let us summarize the results we obtain. If the investment cost is high (M prefers not to invest even if he knows that E enters) the results is straightforward: It is a dominant strategy for M not to invest and hence E will enter the market. This result holds for any precision of the IS.

Suppose next that the investment cost is relatively small so that M prefers to invest if he knows that E enters. Suppose that E obtains the signal ni . If the IS is not too accurate (α falls below a certain threshold) then with probability 1 E enters (believing that M did not invest with high probability). However, if the IS is significantly accurate then E observing the signal ni "hesitates" and enters with a probability lower than 1 and stays out with a significant probability. There is no surprise if the signal is i . In this case, if the IS is less accurate E hesitates and enters with a probability less than 1 (taking into account the possible mistake of the IS). If the IS is significantly accurate E when observing the signal i stays out with probability 1.

Regarding the benefits of the two players, M benefits from the IS more than its owner, E, if the IS is relatively accurate. Whenever α exceeds the threshold, E obtains zero and she is indifferent between spying on M or not. If the cost of building an IS does not depend on α , the optimal accuracy of the IS for E is the threshold value (a value in between $\frac{1}{2}$ and 1). While M is best off with a perfect IS (namely $\alpha = 1$). The implication is that M should subsidize E for building a perfect IS, even though this means that E will be able to perfectly monitor M's action.

The intuition is the following: If the precision of the IS is relatively high, M knows that if he does not expand his capacity, E will detect it with high probability and she is likely to enter the market. Hence, M expands capacity with high probability.

Consequently, the signal ni is not likely to occur. If E observes this signal, she should not trust its accuracy and should "treat it with care". This is the reason that E when observing ni , with positive probability does not enter. If the precision of the IS is less accurate, M expands capacity with smaller probability, knowing that there is a good chance that his action will not be detected. The signal ni is now more likely to be sent.

In this case this signal is more reliable and when E observes it, she enters the market with probability 1.

In equilibrium the unconditional probability that E enters the market decreases the higher is the precision of the IS. Hence, M benefits from a better precision of the IS.

In the second part of the paper we analyze the asymmetric case where the precision, α , of the IS is the private information of its owner, E. The incumbent knows the distribution of α but not the actual realization of α . In particular it covers the case

where E does not use an IS ($\alpha = \frac{1}{2}$) but M believes with positive probability that E

does operate an IS of a precision $\alpha > \frac{1}{2}$. We find that the equilibrium strategies are

qualitatively similar to the common knowledge case, but M's payoff behaves

differently. If the expected value of α is below the threshold, the ex-post payoff of M is

constant for low values of α and it is decreasing for higher values of α . In this case,

contrary to the complete information case, M is best off when E does not spy on him

(or if the precision of the IS is low). If the expected value of α is above the threshold,

the ex-post payoff of M is constant for low values of α but then onward it can increase

or decrease. If the reward of M from expanding capacity when E stays out, is relatively

large, then the ex-post payoff of M is increasing in α and M is best off when E

operates a perfect IS. If the reward is relatively small, then the ex-post payoff of M is

decreasing in α and M is best off when E does not spy on him. Similarly to the

complete information case, the ex-post expected payoff of E is nondecreasing. It is

increasing for large values of α , and for low values of α it is constant.

Let us briefly review the literature on economic espionage.

A closely related paper is Solan and Yariv (2004) (hereafter SY). The set-up in SY is similar to ours and treats general games of espionage. Still the results are different in nature. The SY model considers a general class of costly and noisy intelligence systems.

Contrary to our model, the IS is the strategic choice of the player who spies on his

opponent. We on the other hand, assume that the owner of the IS owns it before the

game starts, and the quality of his IS is either common knowledge or it is his private

information. This is the case if a firm has already a spying technology before it

encounters a new rival (e.g., a firm that has the ability to plant a Trojan Horse in the computer system of her rivals) . The SY paper provides a sort of a “folk theorem” . Namely, for every espionage game it provides a characterization of the set of distributions over the entries of the payoff matrix that for some set of information devices and some cost structure can arise from espionage equilibria. Contrary to our model, the SY paper provides for every IS in a given set of devices and for every cost structure neither the equilibrium strategies and payoffs nor their qualitative properties. Another related paper with similar structure as SY is Provan (2008). This paper is more computational-based approach for two-person-zero-sum games and it uses linear programming solutions.

Matsui (1989) considers a two person repeated game where every player has a small probability of perfectly detecting the other player action and revise his strategy accordingly. The papers shows that if the probability is sufficiently small any subgame perfect equilibrium payoff is Pareto efficient.

Whitney and Gaisford (1999) study the Cournot competition between two companies (Airbus and Boeing as an example) where one company (or both) spy on the other company in attempt to learn its technology and as a result to be able to lower its own marginal cost. The intelligence system can result with either a pure success or a pure failure and it does not generate any noise. The outcome of the IS is assumed to be common knowledge. Similar to our model, the spying company benefits from the IS but in contrast to our model the other company is worse off. If the two companies compete in prices the results change considerably. Both firms are worse off if anyone of them is spying on the other.

In a recent paper, Billand, Bravard, Chakrabarti and Sarangi (2009) (BBCS) study espionage in a Cournot model of several firms with differentiated goods. In BBCS each firm can spy on every other firm and perfectly detects the quality of their products. Spying is costly, and depends on this cost BBCS characterize all the possible espionage configurations. Their model is of symmetric information where firms observe the full espionage activity before choosing their quantity levels. For some parameters of the model they obtain a result similar to us where firms benefit from being spied upon.

In contrast our paper deals with a noisy IS which may generate false signals. We analyze both the case where the precision of the IS is commonly known but also for the asymmetric case where the precision of the IS is private information of its owner. Our paper is similar in spirit to Biran and Tauman (2009) (BT). The context of the BT paper is different and deals with the role of intelligence in nuclear deterrence. The preferences of the players are different and so are the results. For further discussion on the literature see SY.

2. The basic model.

There are two firms, M and E. The Incumbent Firm, M, is a monopolist and E is a potential entrant. In an attempt to deter E from entering M considers whether to invest or not to invest in new capacity. E has an Intelligence System (IS) that suppose to monitor the action of M. The IS sends one of two signals i or ni . The signal i indicates that M invests and the signal ni indicates that M does not invest. The IS sends the right signal with probability α and the wrong signal with probability $1-\alpha$. It is assumed w.l.o.g that $\frac{1}{2} \leq \alpha \leq 1$. If $\alpha = \frac{1}{2}$ then the IS is of no relevance and if $\alpha = 1$ then the IS is perfect.

The following tree describes this situation:

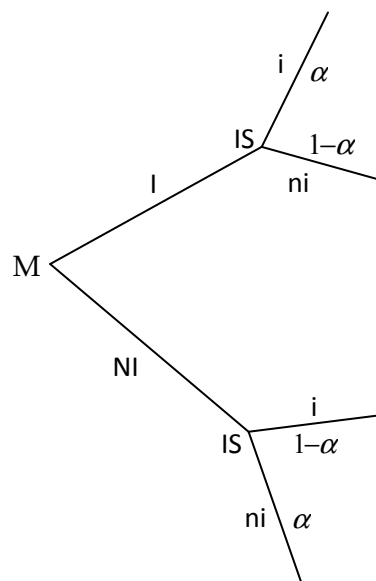


Figure 1

Based on the signal received E decides whether or not to enter the market. We first focus on the case where α is common knowledge. In particular, M knows that E spies on him with an IS of precision α .

The following table describes the payoffs of the two firms based on their possible actions:

		E	
		E	NE
M	I	$b, -1$	$c, 0$
	NI	$a, 1$	$1, 0$

Figure 2

It is assumed that $\max(a, b) < c < 1$ and $\min(a, b) \geq 0$.

We consider two cases: (i) $a > b$ and (ii) $b > a$. The first case is when the investment cost is high and M prefers not to invest even if he knows that E enters. In this case NI is a strictly dominant strategy for M and E enters the market irrespective the signal received. The more difficult case is where the investment cost is not too high and M prefers to invest if he knows that E enters.

So from now on we assume that

$$0 \leq a < b < c < 1 \tag{A1}$$

Without loss of generality assume $a = 0$. Then, we replace the table in Figure 2 by

		E	
		E	NE
M	I	$b, -1$	$c, 0$
	NI	$0, 1$	$1, 0$

Figure 3

3. The extreme cases.

Suppose that (A1) holds. Let us analyze the two extreme cases where $\alpha = \frac{1}{2}$ and where $\alpha = 1$.

Let us start with the case $\alpha = \frac{1}{2}$. This is basically the case where E does not operate an IS on M, and the strategic game between M and E is described in Figure 2. This game has a unique Nash equilibrium in which: M invests with probability $\frac{1}{2}$ and E enters the market with probability $\frac{1-c}{1-c+b}$. The payoff of E is zero (as E is indifferent between entering and not entering). The probability that E enters is decreasing in both b and c . Namely, the higher is the payoff of M from expanding his capacity the lower is the probability that E enters.

The second case is the case where $\alpha = 1$. This is the case where M's action is completely detected by E. In this case, E chooses her action based on the action taken by M. This game can be described by the following tree:

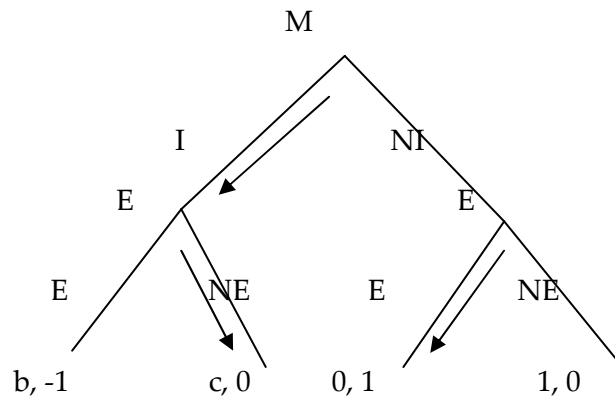


Figure 4

This game has a unique Nash equilibrium outcome: M expands his capacity and E does not enter the market. This outcome yields a higher payoff for M than the equilibrium outcome for $\alpha = \frac{1}{2}$. However the payoff of E is zero in both two cases. Hence, only M benefits from a perfect IS. The entrant, who operates the IS to spy on M and who is able

to perfectly detect M's action before taking her action, does not benefit at all from using it. This result follows by the assumption that is α commonly known.

4. The general case $\frac{1}{2} < \alpha < 1$.

The entrant has four pure strategies. A pure strategy of E is a pair (x, y) where both x and y are in $\{E, NE\}$, x is the action of E if she observes the signal ni and y is her action if she observes the signal i . The following figure describes the game, G_α , between M and E in strategic form (see Figure 1 and Figure 3):

E M	(E, E)	(E, NE)	(NE, E)	(NE, NE)
I	$b, -1$	$\alpha c + (1 - \alpha)b, -1 + \alpha$	$\alpha b + (1 - \alpha)c, -\alpha$	$c, 0$
NI	$0, 1$	$1 - \alpha, \alpha$	$\alpha, 1 - \alpha$	$1, 0$

Figure 5

For instance, the strategy (E, NE) of E is to enter the market if the signal is ni and not to enter if the signal is i . The strategy (E, E) is to enter the market irrespective of the signal.

Note that the strategy (NE, E) of E is strictly dominated by her strategy (E, NE) , since $\alpha > \frac{1}{2}$. Therefore the strategy (NE, E) can be removed, and the resulting game is:

		E	(E, E)	(E, NE)	(NE, NE)
		M			
p	I		$b, -1$	$b + (c - b)\alpha, -1 + \alpha$	$c, 0$
1-p	NI		$0, 1$	$1 - \alpha, \alpha$	$1, 0$

Figure 6

$$\text{Let } \bar{\alpha} = \max \left[\frac{1}{2}, \frac{1-b}{1-b+c} \right].$$

We first analyze the case where $1 - c \leq b$. Namely (see Figure 3), the cost of making a mistake for M is larger when E enters than when E does not enter. In this case $\bar{\alpha} = \frac{1}{2}$.

Proposition 1. Suppose that $1 - c \leq b$. Then the game has a unique Nash equilibrium.

(1) The Entrant does not enter the market if the signal is i and randomizes between entering and not entering if the signal is ni . The Incumbent randomizes between expanding and not expanding its capacity. (2) The probability that the Incumbent expands his capacity is increasing in α . The probability that the Entrant enters the market is decreasing in α . (3) The expected payoff of the Incumbent increases in α . The expected payoff of the Entrant is zero.

Proof. See Appendix.

The possible mistake of M if E enters is not to expand his capacity. Since $b \geq 1 - c$ the cost of this mistake for M is relatively high. Thus M invests in capacity expansion with relatively high probability. Actually, this probability is shown to be α and since $\alpha > \frac{1}{2}$ then $\alpha > 1 - \alpha$. As a result E expects to observe the signal i with higher probability than the signal ni . Hence, E stays out for sure if she observes the signal i and E somewhat hesitates if she observes the less expected signal ni . In the latter case E mixes her two pure actions. The higher is the precision α of the IS the higher is the probability that M invests and the higher is the probability that E stays out. Hence, the expected payoff of M increases with α and the expected payoff of E is zero since she

mixes her two actions and therefore she is indifferent between entering and staying out.

We conclude that for $1 - c \leq b$ even if the IS is cost free E has no incentive to use IS since her payoff is zero irrespective of the quality α .

We next deal with the case where $1 - c > b$. In this case $\bar{\alpha} > \frac{1}{2}$ and the equilibrium outcome depends on whether $\alpha < \bar{\alpha}$ or $\alpha > \bar{\alpha}$.

Proposition 2. Suppose that $1 - c > b$ and $\alpha \neq \bar{\alpha}$. The game has a unique Nash equilibrium. In equilibrium (1) If E observes the signal ni she enters the market with probability 1 if $\alpha < \bar{\alpha}$ and she randomizes her two actions if $\alpha > \bar{\alpha}$. If E observes the signal i she randomizes her two actions if $\alpha < \bar{\alpha}$ and stays out with probability 1 if $\alpha > \bar{\alpha}$. (2) The probability that E enters the market is decreasing in α for all $\alpha \in (\frac{1}{2}, 1)$. (3) The probability that M expands capacity is decreasing in α for $\frac{1}{2} < \alpha < \bar{\alpha}$ and it is increasing in α for $\bar{\alpha} < \alpha < 1$. (4) The expected payoff of M is increasing in α , for all α , and the expected payoff of E is increasing in α for $\frac{1}{2} < \alpha < \bar{\alpha}$ and it is zero for all α , $\bar{\alpha} < \alpha < 1$.

Suppose next that $\alpha = \bar{\alpha}$. Then the game has a multiplicity of equilibrium points. If E observes the signal ni she enters the market with probability 1 and stays out with probability 1 if she observes the signal i .

Proof. See Appendix.

The proposition asserts that if the cost of building an IS is positive, the best IS for E is one with a precision smaller than $\bar{\alpha}$. If the cost is zero (or constant) for all α , E is better off with an IS of a precision as close as possible to $\bar{\alpha}$, from below (see Figure 7). The incumbent firm is best off with a perfect IS, even though that means perfect monitoring of his actions. Let Π_E be the equilibrium expected payoff of E when the cost of building any IS is zero.

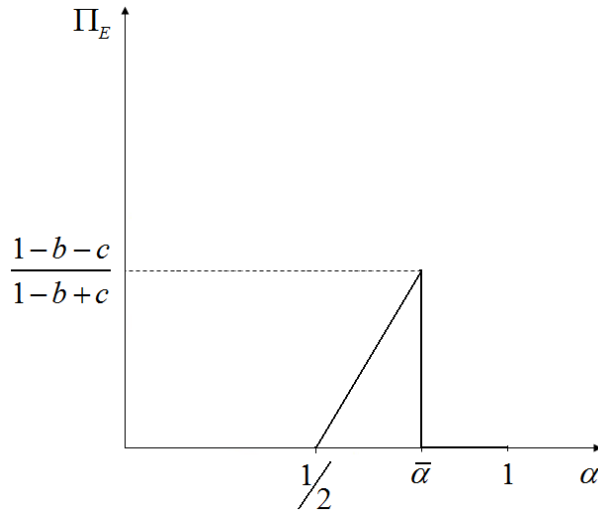


Figure 7

Let us provide intuition for these results. If the precision α of the IS is sufficiently large ($\alpha > \bar{\alpha}$), M who knows α knows that his action will be correctly detected with high probability. He therefore expects that if he does not expand his capacity, E is likely to enter. Hence, M expands his capacity with high probability which can be shown to be α . Consequently, the signal ni is not likely to occur. If E observes this signal, she should not trust its accuracy and she does not enter with positive probability. On the other hand, E expects the signal i and when she observes it, she trusts its accuracy and does not enter with probability 1.

If the precision of the IS is not too accurate ($\alpha < \bar{\alpha}$), M assigns significant probability that his action will not be accurately detected. Therefore, in an attempt to conceal his action, M mixes his two strategies I and NI, both with significant probabilities ($1-\alpha$ and α respectively). Thus both signals i and ni are likely to occur. However the signal ni is more likely than the signal i since the probability that M expands capacity is smaller than the opposite (as $\alpha > 1-\alpha$). As a result E enters with probability 1 if the signal is ni and randomizes her action if the signal is i .

5. The strategic choice of α .

Let Γ_0 be defined like the game G_α (see section 4) except that α is a strategic variable of E and $\gamma(\alpha)$ is the cost of building an IS of quality α . We assume that M observes

the choice α of E before he chooses whether to invest or not. The game Γ_1 , defined later on, deals with the case where M does not observe E's choice of α .

The game Γ_0 is a four-stage game in which E chooses first the precision α of the IS. In the second stage M observes α and chooses whether to invest or not to invest. In the third stage the IS sends a signal "i" or "ni", and in the last stage, based on the signal observed, E chooses whether or not to enter. We analyze the subgame perfect equilibrium (s.g.p.e.) of Γ_0 .

The general case:

Suppose that the cost of building an IS is $\gamma(\alpha)$. Assume that $\gamma(1/2) = 0$, $\gamma'(\alpha) > 0$ and $\gamma''(\alpha) > 0$.

Case 1: $b + c < 1$. In this case $\bar{\alpha} = \frac{1-b}{1-b+c} > \frac{1}{2}$. By Proposition 2, for every α the equilibrium expected payoff of E is given by

$$\Pi_E(\alpha) = \begin{cases} 2\alpha - 1 - \gamma(\alpha), & \frac{1}{2} \leq \alpha < \bar{\alpha} \\ -\gamma(\alpha), & \bar{\alpha} < \alpha \leq 1 \\ \in \left[-\gamma(\bar{\alpha}), \frac{1-b-c}{1-b+c} - \gamma(\bar{\alpha}) \right], & \alpha = \bar{\alpha} \end{cases} \quad (1)$$

The next proposition characterizes the subgame perfect equilibrium choice of α .

Proposition 3. Let $b + c < 1$. Consider a subgame perfect equilibrium of the game Γ_0

with the cost function satisfying $\gamma(1/2) = 0$, $\gamma'(\alpha) > 0$ and $\gamma''(\alpha) > 0$ for all

$\alpha \in [1/2, 1]$. Let α^* be the equilibrium choice of E. Then, $\alpha^* = (\gamma')^{-1}(2)$ and (i)

$\alpha^* = 1/2$ and E does not operate an IS on M iff $\gamma'(1/2) \geq 2$, (ii) $\alpha^* = \bar{\alpha}$ iff $\gamma'(\bar{\alpha}) \leq 2$,

(iii) $1/2 < \alpha^* < \bar{\alpha}$ iff $\gamma'(1/2) < 2$ and $\gamma'(\bar{\alpha}) > 2$. In the last two cases E obtains a positive net payoff and in the first case his payoff is zero.

Proof:

First recall that the equilibrium strategy of M is

$$\sigma_M(\alpha) = \begin{cases} 1-\alpha, & \frac{1}{2} \leq \alpha < \bar{\alpha} \\ \alpha, & \bar{\alpha} < \alpha \leq 1 \\ \in [1-\bar{\alpha}, \bar{\alpha}] & \alpha = \bar{\alpha} \end{cases}$$

Next notice that there is no equilibrium where $\alpha > \bar{\alpha}$ since $\Pi_E(\alpha) < 0$ for $\alpha > \bar{\alpha}$ and $\Pi_E(\frac{1}{2}) = 0$. Any maximizer of $\Pi_E(\alpha)$ in $[\frac{1}{2}, \bar{\alpha})$ can be supported in a subgame perfect equilibrium by the following equilibrium strategy of M:

$$\bar{\sigma}_M(\alpha) = \begin{cases} 1-\alpha, & \frac{1}{2} \leq \alpha < \bar{\alpha} \\ \alpha, & \bar{\alpha} \leq \alpha \leq 1 \end{cases}$$

Note that when M plays $\bar{\sigma}_M(\alpha)$, E obtains $-\gamma(\bar{\alpha})$ when $\alpha = \bar{\alpha}$, and does not have an incentive to deviate to $\bar{\alpha}$.

Our next goal is to find the maximizer of $\Pi_E(\alpha)$ in $[\frac{1}{2}, \bar{\alpha})$. Define for all

$$\alpha \in [\frac{1}{2}, \bar{\alpha}),$$

$$f(\alpha) = \Pi_E(\alpha) = 2\alpha - 1 - \gamma(\alpha)$$

Since $\gamma(\alpha)$ is strictly convex, $f(\alpha)$ is strictly concave. The FOC for the maximizer of $f(\alpha)$ is

$$f'(\alpha) = 2 - \gamma'(\alpha) \leq 0$$

and equality holds if the maximizer α^* is an interior point.

We can extend the definition of f to $[\frac{1}{2}, \bar{\alpha}]$ and define

$$f(\alpha) = \begin{cases} 2\alpha - 1 - \gamma(\alpha), & \frac{1}{2} \leq \alpha < \bar{\alpha} \\ \frac{1-b-c}{1-b+c} - \gamma(\bar{\alpha}), & \alpha = \bar{\alpha} \end{cases}$$

Then f is continuous in $[\frac{1}{2}, \bar{\alpha}]$ and strictly concave, and hence it has a unique maximizer. Note that E obtains $f(\bar{\alpha})$ when she chooses $\alpha = \bar{\alpha}$ if M is playing

$$\bar{\bar{\sigma}}_M(\alpha) = \begin{cases} 1-\alpha, & \frac{1}{2} \leq \alpha \leq \bar{\alpha} \\ \alpha, & \bar{\alpha} < \alpha \leq 1 \end{cases}$$

Let us consider three subcases regarding the behavior of $f'(\alpha)$.

Subcase 1: $\gamma\left(\frac{1}{2}\right) < 2$ and $\gamma(\bar{\alpha}) > 2$. In this case since γ' is continuous, by the Mean

Value Theorem the maximizer α^* of f is interior, namely $\frac{1}{2} < \alpha^* < \bar{\alpha}$ and

$$\gamma'(\alpha^*) = 2. \text{ Also } \Pi_E(\alpha^*) = f(\alpha^*) > f\left(\frac{1}{2}\right) = 0.$$

Subcase 2: $\gamma\left(\frac{1}{2}\right) \geq 2$. In this case the unique maximizer is $\alpha^* = \frac{1}{2}$ and E obtains

zero.

Subcase 3: $\gamma(\bar{\alpha}) \leq 2$. In this case $\alpha^* = \bar{\alpha}$ maximizes $f(\alpha)$.

Claim. $f(\bar{\alpha}) > 0$.

Proof: Note that there exists $\hat{\alpha}$, $\frac{1}{2} < \hat{\alpha} < \bar{\alpha}$ s.t.

$$\frac{\gamma(\bar{\alpha}) - \gamma\left(\frac{1}{2}\right)}{\bar{\alpha} - \frac{1}{2}} = \gamma'(\hat{\alpha}) < 2$$

Since $\gamma\left(\frac{1}{2}\right) = 0$,

$$\gamma(\bar{\alpha}) < 2\bar{\alpha} - 1$$

and

$$f(\bar{\alpha}) = \frac{1-b-c}{1-b+c} - \gamma(\bar{\alpha}) > \frac{1-b-c}{1-b+c} - \frac{2(1-b)}{1-b+c} + 1 = 0$$

The graph of $\Pi_E(\alpha)$ is depicted below,

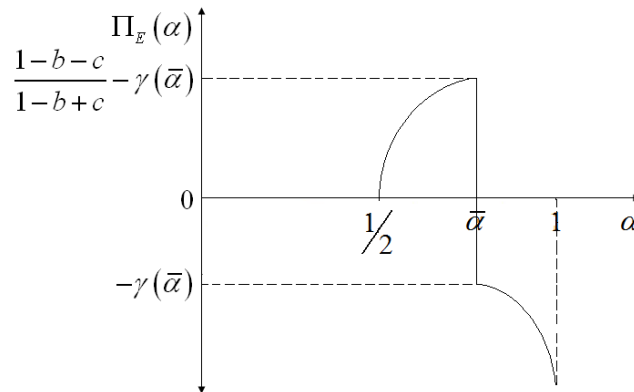


Figure 8

Note that there is no equilibrium where M chooses at $\alpha = \bar{\alpha}$ to invest with probability higher than $1 - \bar{\alpha}$. If M invests with probability higher than $1 - \bar{\alpha}$, E obtains less than $f(\bar{\alpha})$. Since f is continuous at any point in $\left[\frac{1}{2}, \bar{\alpha}\right]$ (including at $\bar{\alpha}$), E has an incentive to slightly lower α below $\bar{\alpha}$.

We conclude that the unique subgame perfect equilibrium outcome in this case is $\alpha = \bar{\alpha}$, and M invests in capacity with probability $1 - \bar{\alpha}$. ■

Case 2: $b + c \geq 1$. In this case the entrant obtains zero payoff even if the cost $\gamma(\alpha)$ is zero, and her payoff is negative if the cost is positive, unless $\alpha = \frac{1}{2}$. Hence the entrant in this case does not build an IS, that is $\alpha^* = \frac{1}{2}$.

We summarize this case by the following proposition.

Proposition 4. Suppose that $b + c \geq 1$. Then any subgame perfect equilibrium of Γ_0 and for every increasing cost function $\gamma(\alpha)$ with $\gamma(\frac{1}{2}) = 0$, $\alpha^* = \frac{1}{2}$.

Example: Consider a specific case of the cost function $\gamma(\alpha)$ where

$$\gamma(\alpha) = k \left(\frac{1}{1-\alpha} - 2 \right)$$

The only interesting case is where $b + c < 1$. In this case $\bar{\alpha} = \frac{1-b}{1-b+c} > \frac{1}{2}$. By (1),

$$\Pi_E(\alpha) = \begin{cases} 2\alpha - 1 - k \left(\frac{1}{1-\alpha} - 2 \right), & \frac{1}{2} \leq \alpha < \bar{\alpha} \\ -k \left(\frac{1}{1-\alpha} - 2 \right), & \bar{\alpha} < \alpha < 1 \\ \in \left[-k \left(\frac{1}{1-\bar{\alpha}} - 2 \right), \frac{1-b-c}{1-b+c} - k \left(\frac{1}{1-\bar{\alpha}} - 2 \right) \right], & \alpha = \bar{\alpha} \end{cases}$$

Claim. (1) Suppose that $\frac{2c^2}{(1-b+c)^2} < k < \frac{1}{2}$. Then $\alpha^* = 1 - \sqrt{\frac{k}{2}}$ and $\frac{1}{2} < \alpha^* < \bar{\alpha}$. E

obtains a net payoff of $(1 - \sqrt{2k})^2 > 0$.

(2) Suppose that $k \geq \frac{1}{2}$. Then $\alpha^* = \frac{1}{2}$ and E does not operate an IS on M and obtains zero expected payoff.

(3) Suppose that $0 < k \leq \frac{2c^2}{(1-b+c)^2}$. Then $\alpha^* = \bar{\alpha}$, M chooses at $\bar{\alpha}$ to invest in capacity with probability $1 - \bar{\alpha}$ and E obtains a net payoff of $(1-b-c) \left(\frac{1}{1-b+c} - \frac{k}{c} \right)$.

Proof: Note that $\gamma(\alpha) = \frac{k}{(1-\alpha)^2}$ and $\alpha^* = (\gamma')^{-1}(2) = 1 - \sqrt{\frac{k}{2}}$. Also,

$$\gamma\left(\frac{1}{2}\right) = 2 \text{ iff } k \geq \frac{1}{2}$$

and

$$\gamma(\bar{\alpha}) = 2 \text{ iff } k = \frac{2c^2}{(1-b+c)^2}$$

Now the proof follows immediately by Proposition 3. ■

Let Γ_1 be defined like Γ_0 except that we assume now that M does not observe the choice α of E before he chooses whether to invest or not.

6. Asymmetric information about the precision of IS.

In this section we assume that the precision α of the IS is the private information of its owner, E. The incumbent, who doesn't know α assigns a continuous density

probability $f(\alpha) > 0$ to every α , $\frac{1}{2} \leq \alpha \leq 1$ and $\int_{\frac{1}{2}}^1 f(\alpha) d\alpha = 1$. In other words, E

knows the game G_α which is actually being played while M doesn't know what game G_α is being played, but knows that α is chosen according to $f(\alpha)$, and this is commonly known.

Let u_M and u_E be the utilities of the two firms from the various outcomes. As in the previous section (see Figure 3), it is assumed that

$$\begin{aligned} u_M(I, E) &= b & u_E(I, E) &= -1 \\ u_M(I, NE) &= c & u_E(I, NE) &= 0 \\ u_M(NI, E) &= 0 & u_E(NI, E) &= 1 \\ u_M(NI, NE) &= 1 & u_E(NI, NE) &= 0 \end{aligned}$$

Suppose that M chooses I with probability p and NI with probability $1 - p$.

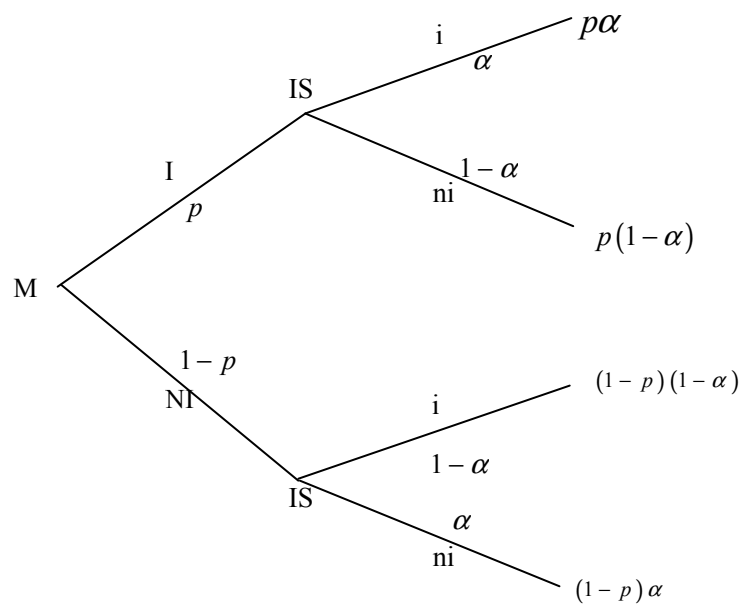


Figure 8

The probability that E assigns to the event that M expands his capacity after observing the signal i is

$$Prob_E(I|\alpha, i) = \frac{\alpha p}{\alpha p + (1-\alpha)(1-p)}$$

Similarly

$$Prob_E(NI|\alpha, i) = \frac{(1-\alpha)(1-p)}{\alpha p + (1-\alpha)(1-p)}$$

$$Prob_E(I|\alpha, ni) = \frac{(1-\alpha)p}{(1-\alpha)p + \alpha(1-p)}$$

$$Prob_E(NI|\alpha, ni) = \frac{\alpha(1-p)}{(1-\alpha)p + \alpha(1-p)}$$

Let $\Pi_E(E|\alpha, i)$ be the expected payoff of E if the signal is i and if she enters the market. Then

$$\begin{aligned} \Pi_E(E|\alpha, i) &= Prob_E(I|\alpha, i)u_E(I, E) + Prob_E(NI|\alpha, i)u_E(NI, E) = \\ &= \frac{1-p-\alpha}{\alpha p + (1-\alpha)(1-p)} \end{aligned} \quad (1)$$

Similarly

$$\Pi_E(E|\alpha, ni) = \frac{\alpha - p}{(1-\alpha)p + \alpha(1-p)} \quad (2)$$

$$\Pi_E(NE|\alpha, i) = 0 \quad (3)$$

$$\Pi_E(NE|\alpha, ni) = 0 \quad (4)$$

Given p , by (1) and (3), if E receives the signal i she prefers E on NE iff

$$\frac{1-p-\alpha}{\alpha p + (1-\alpha)(1-p)} > 0$$

or equivalent iff $\alpha < 1-p$.

That is, if E receives the signal i she will enter if $\alpha < 1 - p$ and she will not enter $\alpha > 1 - p$. If $\alpha = 1 - p$ E is indifferent between entering and not entering the market. Similarly, if E receives the signal ni she enters the market iff

$$\frac{\alpha - p}{(1 - \alpha)p + \alpha(1 - p)} > 0$$

or equivalently iff $\alpha > p$.

We can write now the best reply strategy of E as a function of the signal she receives.

$$s_E(i|\alpha, p) = \begin{cases} NE & p > \frac{1}{2}, \frac{1}{2} < \alpha \leq 1 \\ E & p \leq \frac{1}{2}, \frac{1}{2} < \alpha < 1 - p \\ NE & p \leq \frac{1}{2}, 1 - p < \alpha \leq 1 \\ \text{any strategy} & p \leq \frac{1}{2}, \alpha = 1 - p \end{cases} \quad (5)$$

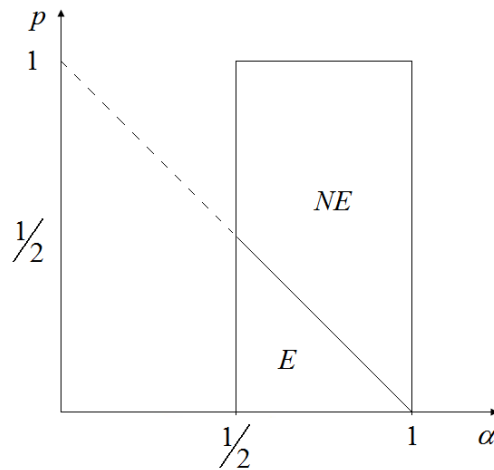


Figure 9: $s_E(i|\alpha, p)$

$$s_E(ni|\alpha, p) = \begin{cases} E & p < \frac{1}{2}, \frac{1}{2} < \alpha \leq 1 \\ NE & p \geq \frac{1}{2}, \frac{1}{2} < \alpha < p \\ E & p \geq \frac{1}{2}, p < \alpha \leq 1 \\ \text{any strategy} & p \geq \frac{1}{2}, \alpha = p \end{cases} \quad (6)$$

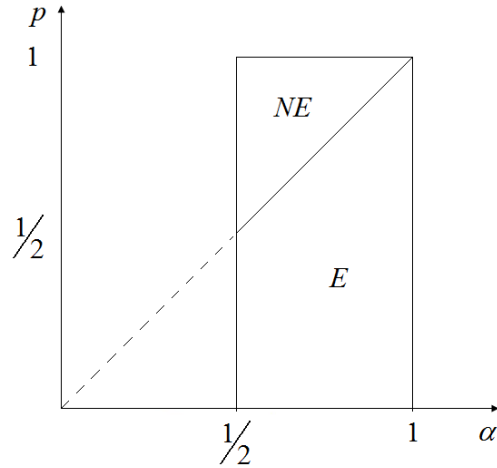


Figure 10: $s_E(ni|\alpha, p)$

Let $E(\alpha) = \int_{\frac{1}{2}}^1 \alpha f(\alpha) d\alpha$ be the expected value of α . Namely $E(\alpha)$ is the expected

quality of the IS from the perspective of the uninformed M.

The next proposition shows that the strategic behavior of the two players in the asymmetric information case is qualitatively similar to the case where the precision of the IS is common knowledge.

Proposition 7. Suppose that $E(\alpha) \neq \bar{\alpha}$. Then the game has a unique perfect Bayesian equilibrium.

- (1) If $E(\alpha) > \bar{\alpha}$, there exists \bar{p}_1 , $\frac{1}{2} < \bar{p}_1 < 1$ such that M expands his capacity with probability \bar{p}_1 . If the signal is ni , E does not enter the market if $\alpha < \bar{p}_1$ and she enters

if $\alpha > \bar{p}_1$. If the signal is i , E does not enter the market irrespective of the precision α of the IS.

(2) If $E(\alpha) < \bar{\alpha}$, there exists \bar{p}_2 , $0 < \bar{p}_2 < \frac{1}{2}$, such M expands his capacity with probability \bar{p}_2 . If the signal is ni , E enters the market irrespective of the precision α of the IS. If the signal is i , E does not enter the market if $\alpha > 1 - \bar{p}_2$ and she enters the market if $\alpha < 1 - \bar{p}_2$.

Proof: Appears in the Appendix.

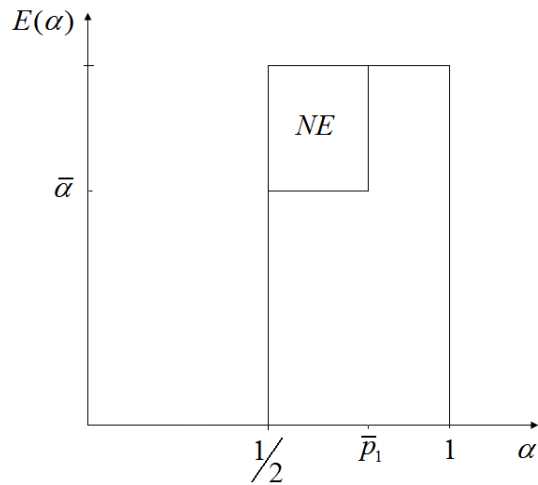


Figure 11: The decision of E when the signal is ni

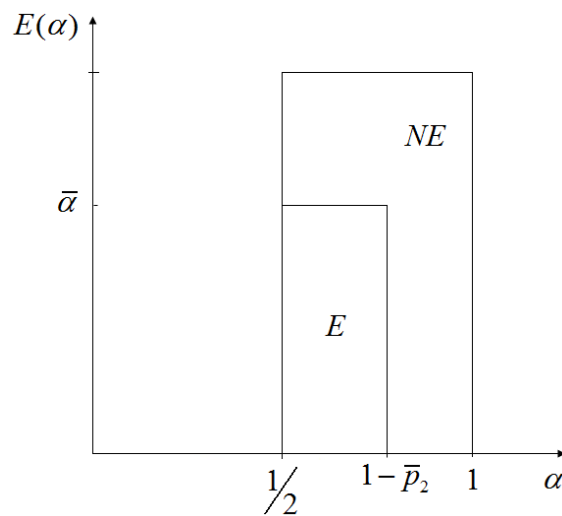


Figure 12: The decision of E when the signal is i

Unlike the case where α is commonly known, the equilibrium strategy of E as a function of α is a pure action (enter or not enter the market with probability 1). The action of E depends on both the expected and the actual precision of the IS. If the expected precision of IS does not exceed $\bar{\alpha}$, E following the signal ni will enter the market irrespective of the actual precision. Furthermore, E will enter the market even if she receives the signal i and if the actual precision α is relatively small ($\alpha < 1 - \bar{p}_2$), otherwise she will not enter the market. If on the other hand, the expected precision of the IS exceeds $\bar{\alpha}$, E following the signal ni will enter the market if the actual precision is relatively high ($\alpha > \bar{p}_1$) and will not enter the market otherwise.

This result is quite consistent with the case where α is commonly known. When α is commonly known, the actual precision and the expected precision are the same. If it does not exceed $\bar{\alpha}$, E, following the signal ni , will enter the market with probability 1. If it exceeds $\bar{\alpha}$, E following ni will randomize between entering and not entering. In the asymmetric information case the mixing is obtained by varying the pure action of E as a function of α . Namely, E enters the market if $\alpha > \bar{p}_1$ and stays out if $\alpha < \bar{p}_1$.

Next we analyze the expected payoff of the entrant. Let $\pi_E(\alpha)$ be the equilibrium expected payoff of E when the precision of the IS is α .

Proposition 8.

(1) Suppose that $E(\alpha) > \bar{\alpha}$. Then for all α in the interval $(\frac{1}{2}, \bar{p}_1)$ E does not enter the market, irrespective of the signal sent by the IS, and $\pi_E(\alpha)$ is zero in this interval.

For all α in $(\bar{p}_1, 1)$, $\pi_E(\alpha)$ is strictly increasing.

(2) Suppose that $E(\alpha) < \bar{\alpha}$. Then for all α in the interval $(\frac{1}{2}, 1 - \bar{p}_2)$ E enters the market, irrespective of the signal sent by the IS, and $\pi_E(\alpha)$ is positive and constant in this interval. On the other hand, for all α in $(1 - \bar{p}_2, 1)$, $\pi_E(\alpha)$ is strictly increasing.

Proof See Appendix.

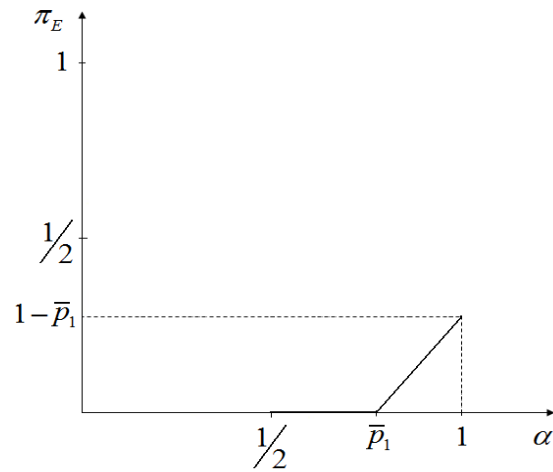


Figure 13: The expected payoff of E when $E(\alpha) > \bar{\alpha}$

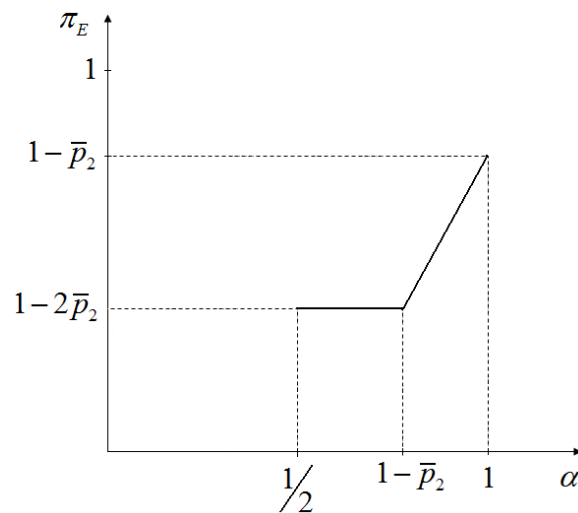


Figure 14: The expected payoff of E when $E(\alpha) < \bar{\alpha}$

In the case where α is common knowledge we showed that if $b > 1 - c$, then E is best off not operating an IS on M, independently how accurate is the IS. If $b < 1 - c$, we showed that the best IS for E is one with a precision $\bar{\alpha}$. In contrast to the common knowledge case, Proposition 4 shows that in the asymmetric case E is always best off with a perfect IS. The payoff of E as function of α is constant up to a certain point (the constant and the point do depend on whether $E(\alpha) > \bar{\alpha}$ or $E(\alpha) < \bar{\alpha}$) and there after it is strictly increasing.

Finally let us analyze the equilibrium payoff of M.

Proposition 9.

- (1) If $E(\alpha) > \bar{\alpha}$, $\pi_M(\alpha)$ is constant for $\frac{1}{2} < \alpha < \bar{p}_1$ and $\pi_M(\alpha)$ is strictly decreasing for $\bar{p}_1 < \alpha < 1$ if $\frac{1}{2} < \bar{p}_1 < \frac{1}{1-b+c}$, and is strictly increasing if $\frac{1}{1-b+c} < \bar{p}_1 < 1$.
- (2) If $E(\alpha) < \bar{\alpha}$, $\pi_M(\alpha)$ is constant for $\frac{1}{2} < \alpha < 1 - \bar{p}_2$ and is strictly decreasing for $1 - \bar{p}_2 < \alpha < 1$.
- (3) Suppose that $E(\alpha) > \bar{\alpha}$, if c is sufficiently close to 1, then $\bar{p}_1 > \frac{1}{1-b+c}$ and $\pi_M(\alpha)$ is strictly increasing for $\bar{p}_1 < \alpha < 1$. If c is sufficiently small, $\bar{p}_1 < \frac{1}{1-b+c}$ and $\pi_M(\alpha)$ is strictly decreasing.

Proof See Appendix.

Example: Consider the uniform distribution case where $f(\alpha) = 2$ for $\frac{1}{2} \leq \alpha \leq 1$ and $f(\alpha) = 0$ otherwise. Note that $E(\alpha) = \frac{3}{4}$ and $E(\alpha) > \bar{\alpha}$ implies that $b + 3c > 1$.

It can be shown that $\bar{p}_1 = \frac{(c-b) + \sqrt{c^2 + b - bc}}{1-b+c}$ and the equilibrium ex-post expected

payoff $\pi_M(\alpha)$ of M is increasing in α for $\bar{p}_1 < \alpha < 1$ iff $\frac{1}{2} < c < 1$ and

$$0 < b < \frac{-(1-c) + \sqrt{c^2 + 6c - 3}}{2}.$$

For every other values of b and c s.t. $0 < b < c < 1$ $\pi_M(\alpha)$ is strictly decreasing.

In the common knowledge case M is always best off when E perfectly detects his action. The asymmetric case yields different results. Up to a certain value of α the ex-post expected payoff of M is constant. If M believes that the expected precision of the IS is relatively small he is best off if E does not spy on him or if E operates an IS with low accuracy. Indeed in this case with high probability M does not expand capacity believing that E is likely not to detect him. As α becomes larger E is more likely to

detect M's action and she is likely to enter. If on the other hand, $E(\alpha)$ is relatively high and the reward, c , from deterring E from entering the market when M expands capacity, is sufficiently close to 1, then (and only then) M benefits from large values of α and M benefits the most from a perfect IS. The reason is that when $E(\alpha) > \bar{\alpha}$ M believes that it is likely that E operates a highly accurate IS and in this case E is likely to detect his action. Hence, M expands his capacity with relatively high probability and for large α expanding capacity is likely to generate the signal i , which induces E to stay out. In this case M obtains c . Hence, the larger is c the larger is the probability that M expands capacity. Also the more accurate the IS is the higher is the probability that E stays out and the higher is the payoff of M.

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Appendix

Proof of Proposition 1

Note that the game has no equilibrium in pure strategies (since $\frac{1}{2} < \alpha < 1$, $0 < c < 1$ and $b > 0$). And since $\alpha > \frac{1}{2}$ there is no equilibrium where the Entrant mixes its three pure strategies.

Consider first the case where the Entrant mixes its strategies (E, E) and (E, NE) . Note that in this case the strategy NI of the Incumbent is strictly dominated by its strategy I. Hence the Incumbent will deviate from $(p, 1-p)$ to I.

When the Entrant mixes (E, E) and (NE, NE) , $p = \frac{1}{2}$ and the Entrant obtains zero.

But the Entrant will deviate to (E, NE) because $\alpha - \frac{1}{2} > 0$ since $\alpha > \frac{1}{2}$.

If the Entrant mixes (E, NE) and (NE, NE) , $p = \alpha$ and the Entrant obtains zero. If the Entrant deviates to (E, E) , it obtains $1 - 2\alpha < 0$, since $\alpha > \frac{1}{2}$. Hence, the Entrant will not deviate.

Note that in this case, if the Incumbent chooses the strategy I he obtains

$[b + (c - b)\alpha]q + c(1 - q)$, and if he chooses its strategy NI he obtains

$(1 - \alpha)q + (1 - q)$. Hence, in equilibrium,

$$[b + (c - b)\alpha]q + c(1 - q) = (1 - \alpha)q + (1 - q)$$

Solving for q we have

$$q = \frac{1 - c}{\alpha(1 - b + c) + b - c}$$

We can conclude that the game has the following equilibrium point: the M's equilibrium strategy is $(p, 1 - p)$ and the E's strategy is $(0, q, 0, 1 - q)$ where $p = \alpha$ is the probability that M invests and

$$q = \frac{1-c}{\alpha(1-b+c)+b-c}$$

The expected payoffs of the firms are

$$\Pi_M = \frac{\alpha(2c-b)+b-c}{\alpha(1-b+c)+b-c}$$

$$\Pi_E = 0$$

■

Proof of Proposition 2

Consider four cases:

Case 1. The Entrant mixes only the two pure strategies (E, E) and (E, NE) and assigns zero probability to the pure strategy (NE, NE) . The following figure shows the resulting game:

		E	(E, E)	(E, NE)
	M			
p	I		$b, -1$	$b + (c - b)\alpha, -1 + \alpha$
1-p	NI		$0, 1$	$1 - \alpha, \alpha$

Figure 15

This case is relevant only if $\alpha < \bar{\alpha} = \frac{1-b}{1-b+c}$. If $\alpha > \bar{\alpha}$, the strategy NI of the Incumbent is strictly dominated by its strategy I, and M will choose I with probability 1, which contradicts the fact that the game has no equilibrium in pure strategies. Let $\alpha < \bar{\alpha}$ and suppose that the Incumbent chooses the mixed strategy $(p, 1-p)$ where $0 < p < 1$. If the Entrant chooses (E, E) , she obtains an expected payoff of $-p + (1-p)$, while her expected payoff is $(-1 + \alpha)p + \alpha(1-p)$ if she chooses (E, NE) .

Since the Entrant mixes two pure strategies it must be that

$$-p + (1-p) = (-1+\alpha)p + \alpha(1-p).$$

Solving for p we obtain $p^* = 1-\alpha$.

The payoff of the Entrant is $2\alpha-1$ and she has no incentive to deviate to (NE, NE) because $2\alpha-1 > 0$.

Suppose next that the Entrant mixes her two pure strategies (E, E) and (E, NE) with probabilities q and $1-q$ respectively, where $0 < q < 1$. If the Incumbent chooses the strategy I, he obtains $bq + [b + (c-b)\alpha](1-q)$; and if he chooses NI, he obtains $(1-\alpha)(1-q)$. Since $0 < p^* < 1$ it must be that

$$bq + [b + (c-b)\alpha](1-q) = (1-\alpha)(1-q)$$

Solving for q we have

$$q^* = \frac{1-b-\alpha(1+c-b)}{1-\alpha(1+c-b)}$$

We summarize this first case in the following lemma:

Lemma 1. Suppose that $\frac{1}{2} < \alpha < \bar{\alpha}$. Then the game has the following equilibrium point: the Incumbent plays $(p^*, 1-p^*)$ and the Entrant plays $(q^*, 1-q^*, 0, 0)$ where

$$p^* = 1-\alpha$$

$$q^* = \frac{1-b-\alpha(1+c-b)}{1-\alpha(1+c-b)}$$

The expected payoffs are

$$\Pi_M^* = \frac{(1-\alpha)b}{1-\alpha(1+c-b)}$$

$$\Pi_E^* = 2\alpha-1$$

also they both increase in α .

Case 2. The Entrant mixes only the two pure strategies (E, E) and (NE, NE) and assigns zero probability to (E, NE) . The resulting game is

	E	
M	(E,E)	(NE,NE)
I	b, -1	c, 0
NI	0, 1	1, 0

Figure 16

Suppose first that the Incumbent mixes his two strategies I and NI and plays $(p, 1-p)$.

If the Entrant chooses (E, E) , she obtains $-p + (1-p)$, and if she chooses (NE, NE) , she obtains zero. Since the Entrant mixes these two strategies, $-p + (1-p) = 0$.

Solving for p we obtain $\hat{p} = \frac{1}{2}$.

The expected payoff of the Entrant is zero. If the Entrant deviates to (E, NE) , she

obtains $(-1 + \alpha)\hat{p} + \alpha(1 - \hat{p}) = \alpha - \frac{1}{2}$, which is positive.

Hence, the Entrant is better off deviating to (E, NE) , and there is no equilibrium where the Entrant mixes only the two pure strategies (E, E) and (NE, NE) .

Case 3. The Entrant mixes her two pure strategies (E, NE) and (NE, NE) .

		E	(E, NE)	(NE, NE)
		M		
p	I		$b + (c - b)\alpha, -1 + \alpha$	$c, 0$
1-p	NI		$1 - \alpha, \alpha$	$1, 0$

Figure 17

This case is relevant only if $\alpha > \bar{\alpha} = \frac{1-b}{1-b+c}$. If $\alpha < \bar{\alpha}$, the strategy I of M is strictly dominated by the strategy NI, and the Incumbent will play NI purely, contradicting the fact that the game has no pure strategy equilibrium. Assume therefore that $\alpha > \bar{\alpha}$. If E chooses (E, NE) , she obtains $(-1 + \alpha)p + \alpha(1 - p)$; and if she chooses (NE, NE) , she obtains zero. Hence in equilibrium, $(-1 + \alpha)p + \alpha(1 - p) = 0$.

Solving for p we obtain $\hat{p} = \alpha$, and E obtains zero. If E deviates to (E, E) , she obtains $-\hat{p} + (1 - \hat{p}) = 1 - 2\alpha$, which is negative. Hence, the Entrant has no incentive to deviate.

Suppose next that E mixes her two pure strategies (E, NE) and (NE, NE) with probabilities q and $1 - q$ respectively. Note that if M chooses the strategy I he obtains $[b + (c - b)\alpha]q + c(1 - q)$, and if he chooses his strategy NI it obtains $(1 - \alpha)q + (1 - q)$. Hence, in equilibrium,

$$[b + (c - b)\alpha]q + c(1 - q) = (1 - \alpha)q + (1 - q)$$

Solving for q we have

$$\hat{q} = \frac{1 - c}{\alpha(1 - b + c) + b - c}$$

We summarize this case in the following lemma:

Lemma 2. Suppose that $\bar{\alpha} < \alpha < 1$. Then, the game has a unique equilibrium point: the Incumbent's strategy is $(\hat{p}, 1 - \hat{p})$ and the Entrant's strategy is $(0, \hat{q}, 0, 1 - \hat{q})$ where

$$\hat{p} = \alpha$$

$$\hat{q} = \frac{1-c}{\alpha(1-b+c)+b-c}$$

The expected payoffs of the firms are

$$\hat{\Pi}_M = \frac{\alpha(2c-b)+b-c}{\alpha(1-b+c)+b-c}$$

$$\hat{\Pi}_E = 0$$

The expected payoff of M increases in α .

Case 4. The Entrant mixes the three strategies (E, E) , (E, NE) and (NE, NE) . By

cases 1 and 3 we must have that $p^* = \hat{p}$. It can be easily shown that $p^* = \hat{p}$ iff $\alpha = 1/2$.

Consequently, there is no equilibrium in this case as we assumed that $\alpha > 1/2$.

Case 5. The Entrant chooses (E, NE) purely. This case is relevant only if

$\alpha = \bar{\alpha} = \frac{1-b}{1-b+c}$. If $\alpha \neq \bar{\alpha}$ by cases 1 and 3 we know that there is no equilibrium in

which E chooses one strategy purely. Assume therefore that $\alpha = \bar{\alpha}$.

		E	(E, E)	(E, NE)	(NE, NE)
		M			
p	I		$b, -1$	$\frac{c}{1-b+c}, -\frac{c}{1-b+c}$	$c, 0$
1-p	NI		$0, 1$	$\frac{c}{1-b+c}, \frac{1-b}{1-b+c}$	$1, 0$

Figure 18

If E plays (E, NE) purely, M is indifferent between choosing I or NI and he plays

$(\tilde{p}, 1 - \tilde{p})$. In this case E obtains

$$-\frac{c}{1-b+c}\tilde{p} + \frac{1-b}{1-b+c}(1-\tilde{p}) = \frac{1-b-\tilde{p}(1-b+c)}{1-b+c}$$

In order for E not to deviate from (E, NE) to (NE, NE) ,

$$\frac{1-b-\tilde{p}(1-b+c)}{1-b+c} \geq 0$$

or equivalently,

$$\tilde{p} \leq \frac{1-b}{1-b+c}$$

In order for E not to deviate from (E, NE) to (E, E) ,

$$\frac{1-b-\tilde{p}(1-b+c)}{1-b+c} \geq 1-2\tilde{p}$$

or equivalently,

$$\tilde{p} \geq \frac{c}{1-b+c}$$

Note that $\frac{1-b}{1-b+c} > \frac{1}{2}$ since we are assuming $1-c > b$. Then, $\frac{1-b}{1-b+c} > \frac{c}{1-b+c}$ and

we can conclude that $\frac{c}{1-b+c} \leq \tilde{p} \leq \frac{1-b}{1-b+c}$.

We summarize this case in the following lemma:

Lemma 3. Suppose that $\alpha = \bar{\alpha}$. Then, the game has multiple equilibrium points: the

Incumbent strategy is $(\tilde{p}, 1-\tilde{p})$ where $\frac{c}{1-b+c} \leq \tilde{p} \leq \frac{1-b}{1-b+c}$ and the Entrant's

strategy is (E, NE) .

The expected payoffs of the firms are

$$\tilde{\Pi}_M = \frac{c}{1-b+c}$$

$$\tilde{\Pi}_E \in \left[0, \frac{1-b-c}{1-b+c} \right]$$

Next let us prove that the unconditional probability that E enters is decreasing in α .

Consider first the case $\frac{1}{2} < \alpha < \bar{\alpha}$. By Lemma 1 the Entrant enters the market with probability 1 if the signal is ni , and with probability q^* if the signal is i . Hence the probability of the Entrant enters the market is

$$Prob(E) = Prob(ni) + q^* Prob(i)$$

To calculate $Prob(ni)$ and $Prob(i)$ consider the following tree:

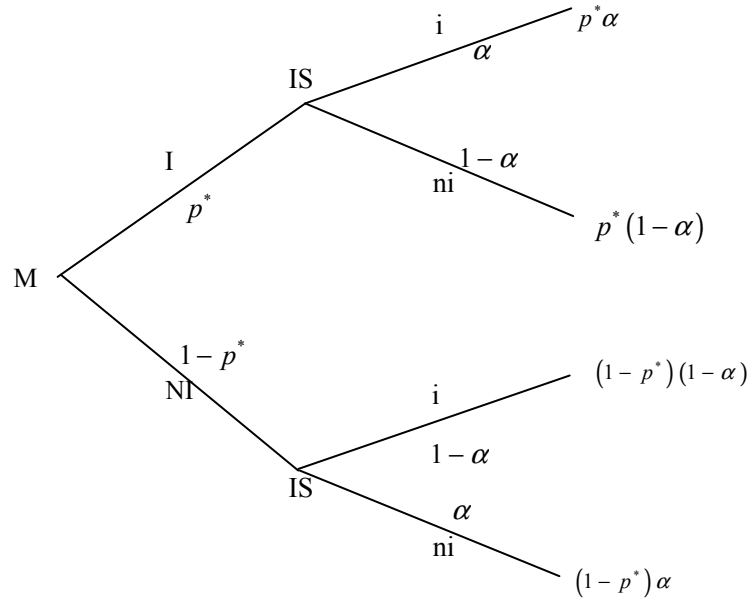


Figure 19

We have

$$Prob(ni) = p^*(1-\alpha) + (1-p^*)\alpha = (1-\alpha)^2 + \alpha^2$$

and

$$Prob(i) = p^*\alpha + (1-p^*)(1-\alpha) = 2(1-\alpha)\alpha$$

Since

$$q^* = \frac{1-b-\alpha(1+c-b)}{1-\alpha(1+c-b)}$$

we have

$$Prob(E) = \frac{1+2b\alpha^2 - \alpha(1+c+b)}{1-\alpha(1+c-b)}$$

And it is decreasing in α ,

$$\frac{\partial \text{Prob}(E)}{\partial \alpha} = \frac{2b \left[\left[2 - (1+c-b)\alpha \right] \alpha - 1 \right]}{\left[1 - \alpha(1+c-b) \right]^2} < 0$$

since $\left[2 - (1+c-b)\alpha \right] \alpha - 1 = -(\alpha-1)^2 + (b-c)\alpha^2 < 0$.

Next assume that $\bar{\alpha} < \alpha < 1$. By Lemma 2, the probability that E enters is

$$\text{Prob}(E) = \hat{q} \text{Prob}(ni) + 0 \text{Prob}(i)$$

Using the tree in Figure 18 (replacing p^* by \hat{p}) we have

$$\text{Prob}(ni) = \hat{p}(1-\alpha) + (1-\hat{p})\alpha = 2(1-\alpha)\alpha$$

Since

$$\hat{q} = \frac{1-c}{\alpha(1-b+c) + b-c}$$

we have

$$\text{Prob}(E) = \frac{2(1-c)(1-\alpha)\alpha}{\alpha(1-b+c) + b-c}$$

And it is decreasing in α since

$$\frac{\partial \text{Prob}(E)}{\partial \alpha} = \frac{-2(1-c) \left[(\alpha-1)^2 (c-b) + \alpha^2 \right]}{\left[\alpha(1-b+c) + b-c \right]^2} < 0$$

■

Proof of Proposition 7.

(1) Suppose that $E(\alpha) > \bar{\alpha}$ where $\bar{\alpha} = \frac{1-b}{1-b+c}$. Consider first the case where M

expands his capacity with probability $\frac{1}{2} < p < 1$. Let $E_\alpha \Pi_M(p)$ be the expected

payoff of M, if he plays the mixed strategy $(p, 1-p)$. In this case, M expands his

capacity with probability p , the signal i is observed with probability α and the signal

ni is observed with probability $1-\alpha$. Hence

$$E_{\alpha} \Pi_M (p) = p \left[\int_{\frac{1}{2}}^1 \alpha u_M (I, NE) f(\alpha) d\alpha + \int_{\frac{1}{2}}^p (1-\alpha) u_M (I, NE) f(\alpha) d\alpha + \int_p^1 (1-\alpha) u_M (I, E) f(\alpha) d\alpha \right] +$$

$$+ (1-p) \left[\int_{\frac{1}{2}}^p \alpha u_M (NI, NE) f(\alpha) d\alpha + \int_p^1 \alpha u_M (NI, E) f(\alpha) d\alpha + \int_{\frac{1}{2}}^1 (1-\alpha) u_M (NI, NE) f(\alpha) d\alpha \right]$$

Since $u_M (I, E) = b$, $u_M (I, NE) = c$, $u_M (NI, E) = 0$ and $u_M (NI, NE) = 1$

$$E_{\alpha} \Pi_M (p) = p \left[c \int_{\frac{1}{2}}^1 \alpha f(\alpha) d\alpha + c \int_{\frac{1}{2}}^p (1-\alpha) f(\alpha) d\alpha + b \int_p^1 (1-\alpha) f(\alpha) d\alpha \right] +$$

$$+ (1-p) \left[\int_{\frac{1}{2}}^p \alpha f(\alpha) d\alpha + \int_{\frac{1}{2}}^1 (1-\alpha) f(\alpha) d\alpha \right]$$

Since $\int_{\frac{1}{2}}^1 f(\alpha) d\alpha = 1$

$$E_{\alpha} \Pi_M (p) = p \left[c + (b-c) \int_p^1 (1-\alpha) f(\alpha) d\alpha \right] + (1-p) \left[1 - \int_p^1 \alpha f(\alpha) d\alpha \right] \quad (1A)$$

Note that E observes neither the mixed strategy played by M nor his actual action. She only observes the signal sent by the IS. Hence, if M unilaterally deviates from his mixed strategy $(p, 1-p)$ to any other strategy, the strategy of E (as a function of her type α and the signal observed) does not change, but the probabilities of the signals do change. In equilibrium, M should be indifferent between playing $(p, 1-p)$ and playing either one of his pure strategies, since $0 < p < 1$. That is

$$E_{\alpha} \Pi_M (0) = E_{\alpha} \Pi_M (1) \quad (2A)$$

By (1A) and (2A)

$$c + (b-c) \int_p^1 (1-\alpha) f(\alpha) d\alpha = 1 - \int_p^1 \alpha f(\alpha) d\alpha \quad (3A)$$

Let

$$g(p) \equiv c - 1 + (b - c) \int_p^1 (1 - \alpha) f(\alpha) d\alpha + \int_p^1 \alpha f(\alpha) d\alpha$$

be defined for all $\frac{1}{2} \leq p \leq 1$. Since $f(\alpha)$ is continuous in α , $g(p)$ is continuously differentiable in p . Also

$$g\left(\frac{1}{2}\right) = -1 + b + (1 - b + c) E(\alpha)$$

By our assumption $E(\alpha) > \frac{1-b}{1-b+c}$, $g\left(\frac{1}{2}\right) > 0$. Since $g(1) = c - 1 < 0$ by the Mean

Value Theorem there is \bar{p}_1 , $\frac{1}{2} < \bar{p}_1 < 1$, such that $g(\bar{p}_1) = 0$.

Next

$$\begin{aligned} g'(p) &= -(b - c) f(p) - (1 - b + c) p f(p) = \\ &= [-(1 - b + c) p - (b - c)] f(p) \end{aligned}$$

Since $f(p) > 0$

$$g'(p) > 0 \text{ iff } p < \frac{c - b}{1 - b + c}$$

Hence $g(p)$ is decreasing for $\frac{1}{2} \leq p < 1$ since $\frac{c - b}{1 - b + c} < \frac{1}{2}$. Since

$g\left(\frac{1}{2}\right) > 0$ and $g(1) < 0$ then g intersects the x-axis only once. Namely, there is a

unique \bar{p}_1 such that $g(\bar{p}_1) = 0$ and $\frac{1}{2} < \bar{p}_1 < 1$. Then \bar{p}_1 is the unique solution of

(3A) and $\bar{p}_1 > \frac{1}{2}$ which is consistent with our assumption.

Next observe that there is no equilibrium strategy $(\bar{p}, 1 - \bar{p})$ such that $\bar{p} \leq \frac{1}{2}$ and

$E(\alpha) > \frac{1-b}{1-b+c}$. Otherwise (3A) should be replaced by

$$c + (b - c) \int_{\frac{1}{2}}^1 (1 - \alpha) f(\alpha) d\alpha = 1 - \int_{\frac{1}{2}}^1 \alpha f(\alpha) d\alpha$$

This implies that

$$E(\alpha) = \frac{1-b}{1-b+c},$$

a contradiction. We conclude that whenever $E(\alpha) > \frac{1-b}{1-b+c}$ there exists a unique

equilibrium. M chooses to expand capacity with probability $\bar{p}_1 > \frac{1}{2}$ and E takes an

action as described in (5) or (6). Namely, if E observes the signal i , she doesn't enter

the market irrespective of her type α . If the signal is ni E does not enter the market iff

$$\frac{1}{2} < \alpha < \bar{p}_1.$$

(2) Suppose next that $E(\alpha) < \frac{1-b}{1-b+c}$. Consider the case where M expands his

capacity with probability p , $0 < p < \frac{1}{2}$. Similarly to the previous case

$$E_\alpha \Pi_M(p) = p \left[\int_{\frac{1}{2}}^{1-p} \alpha u_M(I, E) f(\alpha) d\alpha + \int_{1-p}^1 \alpha u_M(I, NE) f(\alpha) d\alpha + \int_{\frac{1}{2}}^1 (1-\alpha) u_M(I, E) f(\alpha) d\alpha \right] +$$

$$+ (1-p) \left[\int_{\frac{1}{2}}^1 \alpha u_M(NI, E) f(\alpha) d\alpha + \int_{\frac{1}{2}}^{1-p} (1-\alpha) u_M(NI, E) f(\alpha) d\alpha + \int_{1-p}^1 (1-\alpha) u_M(NI, NE) f(\alpha) d\alpha \right]$$

Since $u_M(I, E) = b$, $u_M(I, NE) = c$, $u_M(NI, E) = 0$ and $u_M(NI, NE) = 1$

$$E_\alpha \Pi_M(p) = p \left[b + (c-b) \int_{1-p}^1 \alpha f(\alpha) d\alpha \right] + (1-p) \left[\int_{1-p}^1 (1-\alpha) f(\alpha) d\alpha \right] \quad (4A)$$

In equilibrium where $0 < p < 1$ we have

$$E_\alpha \Pi_M(0) = E_\alpha \Pi_M(1) \quad (5A)$$

By (4A) and (5A) we have

$$b + (c-b) \int_{1-p}^1 \alpha f(\alpha) d\alpha = \int_{1-p}^1 (1-\alpha) f(\alpha) d\alpha \quad (6A)$$

Let

$$m(x) \equiv b + (c-b) \int_x^1 \alpha f(\alpha) d\alpha - \int_{1-p}^1 (1-\alpha) f(\alpha) d\alpha$$

be defined for all $\frac{1}{2} \leq x \leq 1$. Clearly $m(x)$ is continuous and differentiable.

By our assumption

$$m\left(\frac{1}{2}\right) = (1-b+c)E(\alpha) + (b-1) < 0$$

Also

$$m(1) = b > 0$$

In addition

$$m'(x) = [1 - (1-b+c)x]f(x)$$

Since $f(x) > 0$

$$m'(x) > 0 \text{ iff } x < \frac{1}{1-b+c}$$

Hence m increases for $\frac{1}{2} \leq x < \frac{1}{1-b+c}$ and decreases for $\frac{1}{1-b+c} < x \leq 1$. Since

$m\left(\frac{1}{2}\right) < 0$ and $m(1) > 0$ then m intersects the x-axis only once. Namely, there is a

unique \bar{x} such that $m(\bar{x}) = 0$ and $\frac{1}{2} < \bar{x} < 1$. Thus there exists a unique $0 < \bar{p}_2 < 1$

such that \bar{p}_2 is the unique solution of (6A), and $\bar{p}_2 < \frac{1}{2}$ which is consistent with our assumption.

Next it is easy to verify (similarly to the previous case) that there is no equilibrium

where $p \geq \frac{1}{2}$ while $E(\alpha) < \frac{1-b}{1-b+c}$. We conclude that whenever $E(\alpha) < \frac{1-b}{1-b+c}$

there exists a unique equilibrium: M expands his capacity with probability \bar{p}_2 ,

$0 < \bar{p}_2 < \frac{1}{2}$. As for E, if the signal is i , E enters the market iff $\alpha < 1 - \bar{p}_2$. If the signal

is ni , E enters the market, irrespective of α .

It is also easy to verify that there is no equilibrium where M is playing a pure strategy.

Suppose that M expands his capacity with probability 1. The strategy of E is not to enter the market irrespective of the signal or of α . Hence, M obtains c . If he does not

expand his capacity and E does not enter the market, he obtains $1 > c$. Similarly, if in equilibrium, M does not expand his capacity with probability 1, then E's best reply strategy is to enter the market irrespective of α and M obtains 0. If he expands capacity, he obtains $b > 0$. Consequently, M is better off deviating from any one of his pure strategies. This completes the proof of the proposition. ■

Proof of Proposition 8.

By (6) and (7) it is easy to verify that

$$\pi_E(\alpha) = \begin{cases} 1 - 2\bar{p}_2 & , & E(\alpha) < \bar{\alpha}, \frac{1}{2} < \alpha < 1 - \bar{p}_2 \\ \alpha - \bar{p}_2 & , & E(\alpha) < \bar{\alpha}, 1 - \bar{p}_2 < \alpha < 1 \\ 0 & , & E(\alpha) > \bar{\alpha}, \frac{1}{2} < \alpha < \bar{p}_1 \\ \alpha - \bar{p}_1 & , & E(\alpha) > \bar{\alpha}, \bar{p}_1 < \alpha < 1 \end{cases}$$

and the proof follows immediately. ■

Proof of Proposition 9:

By (6), (7) and Proposition 7 it is easy to verify that

$$\pi_M(\alpha) = \begin{cases} \bar{p}_2 b & , & E(\alpha) < \bar{\alpha}, \frac{1}{2} < \alpha < 1 - \bar{p}_2 \\ [\bar{p}_2(1-b+c) - 1] \alpha + 1 - \bar{p}_2(1-b) & , & E(\alpha) < \bar{\alpha}, 1 - \bar{p}_2 < \alpha < 1 \\ \bar{p}_1 c + (1 - \bar{p}_1) & , & E(\alpha) > \bar{\alpha}, \frac{1}{2} < \alpha < \bar{p}_1 \\ [\bar{p}_1(1-b+c) - 1] \alpha + 1 - \bar{p}_1(1-b) & , & E(\alpha) > \bar{\alpha}, \bar{p}_1 < \alpha < 1 \end{cases}$$

Let us prove that $g\left(\frac{1}{1-b+c}\right) > 0$ if c is sufficiently close to 1.

$$g\left(\frac{1}{1-b+c}\right) = c - 1 + (b - c) \int_{\frac{1}{1-b+c}}^1 (1 - \alpha) f(\alpha) d\alpha + \int_{\frac{1}{1-b+c}}^1 \alpha f(\alpha) d\alpha$$

If c is sufficiently close to 1, $g\left(\frac{1}{1-b+c}\right)$ is sufficiently close to

$$\begin{aligned}
& (b-1) \int_{\frac{1}{2-b}}^1 (1-\alpha) f(\alpha) d\alpha + \int_{\frac{1}{2-b}}^1 \alpha f(\alpha) d\alpha \\
&= (b-1) \int_{\frac{1}{2-b}}^1 f(\alpha) d\alpha + (2-b) \int_{\frac{1}{2-b}}^1 \alpha f(\alpha) d\alpha \\
&> (b-1) \int_{\frac{1}{2-b}}^1 f(\alpha) d\alpha + \int_{\frac{1}{2-b}}^1 f(\alpha) d\alpha = b \int_{\frac{1}{2-b}}^1 f(\alpha) d\alpha > 0
\end{aligned}$$

Since $g'(p) < 0$ for all $p, \frac{1}{2} < p < 1$, and $g\left(\frac{1}{1-b+c}\right) > 0$ for c sufficiently large, we

have that $\bar{p}_1 > \frac{1}{1-b+c}$.

■