Patents and Licenses

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August 30, 2012

Abstract

This article considers the problem of patent licensing in a Cournot oligopoly under a class of general demand functions. We consider two cases, the case where the innovator is an outsider and the one where it is one of the incumbent firms. The licensing policies considered are upfront fees, royalties and combinations of the two. It is shown that (i) for generic values of magnitudes of the innovation, a royalty policy is better than fee or auction provided the industry size is relatively large, (ii) under combinations of fees and royalties, provided the innovation is relatively significant (or the industry size is relatively large), (a) there is always an optimal policy where the innovation is licensed to practically all firms of the industry and (b) any optimal combination includes a positive royalty.

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1 Introduction

A patent grants an innovator monopoly rights over the use of an innovation for a given period of time. It seeks to provide incentive to innovate as well as to disseminate the innovation. Licensing is a standard way through which an innovation can be diffused. Licensing policies in practice take various forms, but they can be classified into three broad categories: licensing by means of upfront fees, royalties, and policies that combine both fees and royalties.

The theoretical literature of patent licensing can be traced back to Arrow (1962) who argued that a perfectly competitive industry provides a higher incentive to innovate than a monopoly. Licensing under oligopoly was first studied by Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986).[1] Considering innovators that are outsiders to the industry, the early literature concluded that licensing by means of upfront fees dominates royalty licensing for an outside innovator (Kamien and Tauman, 1984, 1986; Kamien, Oren and Tauman, 1992). It has been subsequently argued that royalties can can be explained by factors such as informational asymmetry (Gallini and Wright, 1990; Rockett, 1990; Macho-Stadler and Pérez-Castrillo, 1991; Beggs, 1992, Choi, 2001; Sen, 2005a), product differentiation (Muto, 1993; Wang and Yang, 1999; Poddar and Sinha, 2004; Erkal, 2005b), or integer constraint of number of licenses (Sen, 2005b).

The literature has also considered innovators who are one of the incumbent firms in the industry. An incumbent innovator has an additional incentive to use royalties, as they provide the innovator with a competitive edge by raising the effective cost of its rival. This reasoning was put forward by Shapiro (1985), later formalized by Wang (1998) in a Cournot duopoly and extended to a Cournot oligopoly model by Kamien and Tauman (2002).[2]

Considering general licensing policies in combinations of upfront fees and royalties for both outside and incumbent innovators in a Cournot oligopoly, Sen and Tauman (2007) have shown that consumers and the innovator are better off, firms are worse off and the social welfare is improved as a result of licensing. They have also shown that licensing of relatively significant non-drastic innovations[3] involves positive royalty. Combinations of fees and royalties have been also studied in specific duopoly models such as a duopoly with differentiated products (Faulí-Oller and Sandonís, 2002) and a Stackelberg duopoly (Filippini, 2005).

Regarding the relation between industry structure and incentives to innovate, Arrow's initial analysis has been further qualified. It has been shown that, under royalty licensing, the perfectly competitive industry provides the highest incentives to innovate (Kamien and Tauman, 1986). However, when the innovator uses combinations of fees and royalties, the highest incentives are provided by industries where the number of firms is not too large or too small (Sen and Tauman, 2007) although Arrow's basic intuition is still robust in that a perfectly competitive industry always provides a higher incentive than a monopoly.

Other issues addressed in the literature include leadership structure (Kabiraj, 2004; 2005), strategic delegation (Mukherjee, 2001; Saracho, 2002), negative royalties (Liao and

[1] For the early literature, see also Gallini (1984), Gallini and Winter, (1985), Katz and Shapiro (1987) and Muto (1987). More recent papers include Erkal (2005a), Liao and Sen (2005), Giebe and Wolfstetter (2008) and Martin and Saracho (2010).

[2] See also Marjit (1990) and Wang (2002) for other issues pertaining to incumbent innovators.

[3] A cost-reducing innovation is *drastic* (Arrow, 1962) if it significant enough to create a monopoly with the reduced cost; otherwise it is *non-drastic*. See Section 2.5 for further classification of innovations.

Sen, 2005), licensing outcomes under a cooperative-game-theoretic approach (Tauman and Watanabe, 2007; Watanabe and Muto, 2008; Jelnov and Tauman, 2009), scale economies (Sen and Stamatopoulos, 2009b) and selling patent rights (Tauman and Weng, 2012).

The early literature of patent licensing has been reviewed by Kamien (1992). As we have briefly discussed, the literature has since branched out in various directions. Instead of providing a broad overview covering various models, in this article we provide an in-depth analysis of the licensing problem in a Cournot oligopoly framework and present certain general results that hold for a large class of demand functions. This article builds on the following papers, whose basic differences are presented below.

Table 1

	type of innovator	policies	demand
Kamien and Tauman (1986)	outsider	fees, royalties	linear
Kamien, Oren and Tauman (1992)	outsider	fees, royalties	general
Sen (2005b)	outsider	fees, royalties	linear
Sen and Tauman (2007)	outsider, incumbent	combinations	linear

The demand function we consider is the general form considered in Kamien, Oren and Tauman (1992) [KOT]. In this framework, we present a unified approach by considering both outside and incumbent innovator, as well as policies that are combinations of upfront fees and royalties, as in Sen and Tauman (2007). Most of the conclusions obtained with linear demand continue to hold qualitatively under general demand. Specifically, it is shown that (i) for generic values of magnitudes of the innovation, a royalty policy is better than fee or auction provided the industry size is relatively large, (ii) under combinations of fees and royalties, provided the innovation is relatively significant, (a) there is always an optimal policy where the innovation is licensed to practically all firms of the industry and (b) any optimal policy includes a positive royalty.

2 The model

Consider a homogeneous good Cournot oligopoly where $\overline{N}$ is the set of competing firms. Initially any firm $i \in \overline{N}$ produces under the same constant marginal cost $c > 0$. An innovator I has a patent for a new technological innovation that reduces the cost from c to $c - \varepsilon$ ($0 < \varepsilon < c$), so ε is the magnitude of the innovation. For $i \in \overline{N}$, let q_i be the quantity produced by firm i and let $Q = \sum_{i \in \overline{N}} q_i$. As in KOT, the following assumptions are maintained throughout.

A1. The price function or the inverse demand function $p(Q)$: $\mathrm{R}_+ \to \mathrm{R}_+$ is decreasing.

A2. $\exists\, Q^0 > 0$ such that $p(Q) = 0$ for all $Q \geq Q^0$.

A3. For $Q \in [0, Q^0]$, $p(Q)$ is strictly decreasing and twice continuously differentiable.

A4. $\exists\, 0 < Q^c < Q^{c-\varepsilon} < Q^0$ such that $p(0) > p(Q^c) = c > p(Q^{c-\varepsilon}) = c - \varepsilon > 0$.

A5. For $Q \in [0, Q^0)$, the revenue function $Qp(Q)$ is strictly concave: $2p'(Q) + Qp''(Q) < 0$.

A6. For $p \in [0, p(0))$, the price elasticity $\eta(p) := -pQ'(p)/Q(p)$ is increasing.

2.1 Two cases: outside and incumbent innovators

Note that $\overline{N}$ is the set of all competing firms. Denote by $N \equiv \{1, \ldots, n\}$ the set of all firms other than the innovator I, where $n \geq 2$.[4] We consider two cases:

(i) Outside innovator: The innovator I is an outsider to the industry, i.e., it is not one of the firms in $\overline{N}$. For this case $\overline{N} = N$.

(ii) Incumbent innovator: The innovator I is one of the incumbent firms in $\overline{N}$. For this case $\overline{N} = \{I\} \cup N$.

It will be useful to define the indicator variable

$$\lambda = \begin{cases} 0 \text{ if } I \text{ is an outside innovator,} \\ 1 \text{ if } I \text{ is an incumbent innovator} \end{cases} \tag{1}$$

By (1), $|\overline{N}| = n + \lambda$.

2.2 Licensing policies

I can license its technology to some or all firms in N. We consider three types of licensing policies.

Royalty policy: Under this policy I offers to license its innovation at a uniform unit royalty $r \geq 0$. A firm that accepts to be a licensee uses the innovation and pays r to I for every unit it produces.

Upfront fee or auction policy: Under this policy I offers to license the innovation to k firms $(1 \leq k \leq n)$ by using an upfront fee. I determines the upfront fee to extract the maximum possible surplus from the licensees. The best way to do this is through an auction policy[5] where I auctions off k licenses (possibly with a minimum bid) and the upfront fee that a licensee pays is its winning bid. The minimum bid is required for $k = n$, as without that no firm will place a positive bid since each one is guaranteed to have a license. So a typical upfront fee policy is k for $1 \leq k \leq n-1$ and $(n, \underline{b})$ for $k = n$, where $\underline{b} \geq 0$ is the minimum bid.

Auction plus royalty (AR) policy: This policy is a combination of upfront fee and royalty. Under this policy I announces a rate of royalty $r \geq 0$ and then auctions off k licenses. Any licensee pays its winning bid as upfront fee. In addition, it pays I the royalty r for each unit it produces. As before, I needs to specify a minimum bid for $k = n$. So a typical AR policy is (k, r) for $1 \leq k \leq n-1$ and $(n, r, \underline{b})$ for $k = n$, where k is the number of licenses auctioned off, $r \geq 0$ is the unit royalty and $\underline{b} \geq 0$ is the minimum bid.

When an innovation of magnitude ε is licensed with rate of royalty r, the effective unit cost of a licensee is $c - (\varepsilon - r)$. As a firm has unit cost c without a license, no firm will accept a policy with $r > \varepsilon$. So we can restrict $r \in [0, \varepsilon]$.

[4] The case of $n = 1$ is straightforward and does not capture all strategic aspects of the licensing. For this reason we consider $n \geq 2$. For certain results, we shall further restrict $n \geq 3$.

[5] As shown in Katz and Shapiro (1985) and Kamien and Tauman (1986), compared to a flat upfront fee, an auction generates more competition that increases the willingness to pay for the license.

Define $\delta := \varepsilon - r$. The variable $\delta \in [0, \varepsilon]$ is the *effective magnitude* of the innovation when the rate of royalty is r. Henceforth the policies will be expressed in terms of δ. We shall denote royalty policies by δ and AR policies by (k, δ) and $(n, \delta, \underline{b})$. Auction policies are special AR policies with $r = 0$ or $\delta = \varepsilon$, so they correspond to (k, ε) and $(n, \varepsilon, \underline{b})$.

2.3 The licensing game

For $\lambda \in \{0, 1\}$, the strategic interaction between I and the firms in N is modeled as the *licensing game* G_λ that has the following stages.

Stage 1: In Stage 1, I announces a royalty policy δ, or an AR policy: (k, δ) or $(n, \delta, \underline{b})$.

Stage 2: In case a royalty policy is announced in Stage 1, firms in N simultaneously decide whether to become a licensee or not. In case of an AR policy, firms in N bid simultaneously for the license and k highest bidders win the license (ties are broken randomly).

Stage 3: Firms in $\overline{N}$ compete in quantities. Under AR policy, if a firm wins the license with bid b and produces q, it pays $b + rq = b + (\varepsilon - \delta)q$ to I. Under royalty policy, a licensee who produces q pays $rq = (\varepsilon - \delta)q$ to I.

We confine to Subgame Perfect Nash Equilibrium (SPNE) outcomes of G_λ.

2.4 The Cournot oligopoly game $\mathcal{C}_\lambda^n(k, \delta)$

For a licensing policy that has royalty $r = \varepsilon - \delta$, the effective unit cost of a licensee is $c - \varepsilon + r = c - \delta$. The unit cost of a non-license is c. If I is an incumbent innovator (i.e., $\lambda = 1$), then it produces using the new technology, so its unit cost is $c - \varepsilon$. Therefore if there are k licensees, in Stage 3 of G_λ a Cournot oligopoly game is played with $n + \lambda$ firms where λ firms (firm I, if it is an incumbent firm) have cost $c - \varepsilon$, k firms (licensees) have cost $c - \delta$ and $n - k$ firms (non-licensees) have cost c. Denote this game by $\mathcal{C}_\lambda^n(k, \delta)$.

To determine SPNE of G_λ, we need to characterize Nash Equilibrium (NE) of $\mathcal{C}_\lambda^n(k, \delta)$ for all k and δ. For $1 \leq k \leq n$, define $H^k : [0, p(0)] \to R$ as

$$H^k(p) := p[1 - 1/k\eta(p)] \text{ for } p \in [0, p(0)) \text{ and } H^k(p(0)) = p(0) \tag{2}$$

From A1-A6, it follows that $H^k(p)$ is continuous and strictly increasing for $p \in [0, p(0)]$. This function will be useful for our analysis.

The monopoly problem. To begin with, consider the case of a monopolist who has the new technology so its unit cost is $c - \varepsilon$. The problem of the monopolist is to choose $Q \geq 0$ to maximize $\phi_M^\varepsilon(Q) = (p(Q) - c + \varepsilon)Q$. Since $\phi_M^\varepsilon(0) = 0$ and $\phi_M^\varepsilon(Q) = -(c - \varepsilon)Q < 0$ for $Q \geq Q^0$ (by Assumption A2) it is sufficient to consider $Q \in [Q, Q^0]$. Then $p(Q)$ is strictly decreasing (by A3) and the problem is equivalent to choosing $p \in [0, p(0)]$ to maximize the monopoly profit at price p, given by

$$F(p) := (p - c + \varepsilon)Q(p) \tag{3}$$

By A4-A6, the monopoly problem has a unique solution. Let $p_M(\varepsilon)$ be the monopoly price under cost $c - \varepsilon$. Then $0 < p_M(\varepsilon) < p(0)$ and $p_M(\varepsilon)$ is the unique solution of the equation

$$H^1(p) = c - \varepsilon \tag{4}$$

Denote the monopoly profit by $\pi_M(\varepsilon)$, i.e., $\pi_M(\varepsilon) \equiv F(p_M(\varepsilon))$.

The case where the incumbent innovator becomes a monopolist. Having solved the monopoly problem, let us now dispose off the case where ε, δ are such that the incumbent innovator becomes a monopolist with its technology. The proof follows from KOT by using assumptions A1-A6.

Lemma 1 *Consider the case of an incumbent innovator (i.e. $\lambda = 1$). Let $0 \leq \delta \leq \varepsilon$ be such that $\varepsilon \geq \delta + (c-\delta)/\eta(c-\delta)$. Then for any $1 \leq k \leq n$, the game $\mathcal{C}_1^n(k,\delta)$ has a unique NE where a natural monopoly is created with firm I and all other firms drop out of the market. The NE price $p_1^n(k,\delta)$ equals the monopoly price $p_M(\varepsilon)$ and $c - \varepsilon < p_M(\varepsilon) \leq c$ [equality iff $\varepsilon = \delta + (c-\delta)/\eta(c-\delta)$].*

The case where all licensees obtain positive profit. Now we consider the case $\varepsilon < \delta + (c-\delta)/\eta(c-\delta)$ where for $\lambda = 1$ (incumbent innovator), all licensees obtain positive profit. For $\lambda = 0$ (outside innovator) licensees always obtain positive profit. To present the result for both cases together, the result in Lemma 2 is presented under the inequality $\lambda\varepsilon < \delta + (c-\delta)/\eta(c-\delta)$ which always holds for $\lambda = 0$. It will be useful to define for $\lambda \in \{0,1\}$,

$$\delta_\lambda(0) := c/\eta(c) \text{ and } \delta_\lambda(k) := (c/\eta(c) - \lambda\varepsilon)/k \text{ for } k \geq 1 \tag{5}$$

The proof follows from the equilibrium conditions of the resulting Cournot oligopoly.

Lemma 2 *Let $\lambda \in \{0,1\}$, $1 \leq k \leq n$ and $\delta \in [0,\varepsilon]$ and suppose $\lambda\varepsilon < \delta + (c-\delta)/\eta(c-\delta)$. The game $\mathcal{C}_\lambda^n(k,\delta)$ has a unique NE. The NE has the following properties.*

(a) *The NE price $p_\lambda^n(k,\delta)$ is continuous and strictly decreasing in δ.*

(b) *Suppose $\delta < \delta_\lambda(k)$. Then $c < p_\lambda^n(k,\delta) < p(0)$ and $p_\lambda^n(k,\delta)$ is the unique solution of $H^{n+\lambda}(p) = c - (k\delta + \lambda\varepsilon)/(n+\lambda)$ over $p \in [0, p(0)]$. All firms obtain positive profit.*

(c) *Suppose $\delta \geq \delta_\lambda(k)$. Then $c - \delta < p_\lambda^n(k,\delta) \leq c$ [equality iff $\delta = [c/\eta(c) - \lambda\varepsilon]/k$] and $p_\lambda^n(k,\delta)$ is the unique solution of $H^{k+\lambda}(p) = c - (k\delta + \lambda\varepsilon)/(k+\lambda)$ over $p \in [0, p(0)]$. A $(k+\lambda)$-firm natural oligopoly is created where k licensees and firm I (if $\lambda = 1$) obtain positive profit and the $n-k$ non-licensees drop out of the market.*

(d) *For $\delta \leq \delta_\lambda(k)$, the NE price $p_\lambda^n(k,\delta)$ and NE outputs of firms depend only on the product $k\delta$.*

The case of $\lambda = 0$ is direct from KOT. The case of $\lambda = 1$ follows by extending KOT.[6]

2.5 Classification of innovations

Drastic innovation A cost-reducing innovation is *drastic* (Arrow, 1962) if the monopoly price under the new technology does not exceed the old cost c; otherwise it is non-drastic. Thus, a sole user of a drastic innovation can become a monopolist with the reduced cost.

[6] KOT consider the case of an outside innovator, so there are two sets of cost-asymmetric firms: licensees and non-licensees. In the case of an incumbent innovator there are three sets: the innovator, licensees and non-licensees. Due to this reason, extending the KOT framework is required for $\lambda = 1$.

If only firm has the innovation, its cost is $c-\varepsilon$ and the cost of any other firm is c. For $\lambda = 0$ (outside innovator), taking $k = 1$ and $\delta = \varepsilon$, it follows that an innovation of magnitude ε is drastic if $\mathcal{C}_0^n(1, \varepsilon)$ is a natural monopoly. So by Lemma 2(c) and (5), an innovation of magnitude ε is drastic if $\varepsilon \geq \delta_0(1) \equiv c/\eta(c)$ and it is non-drastic if $\varepsilon < c/\eta(c)$.[7]

Remark 1 By the monotonicity of the function $H^k(p)$ given in (2), we have $H^1(c) \geq H^1(c-\delta)$ for all $\delta \in [0, \varepsilon]$, implying that $c/\eta(c) \leq \delta + (c-\delta)/\eta(c-\delta)$. Therefore if an innovation of magnitude ε is non-drastic (i.e., $\varepsilon < c/\eta(c)$), then $\varepsilon < \delta + (c-\delta)/\eta(c-\delta)$. Then by Lemma 2, it follows that for any non-drastic innovation, all licensees obtain positive profit under any licensing policy.

The notion of drastic innovation can be extended as follows.

***k*-drastic innovation** Let $\lambda \in \{0, 1\}$. For $k \geq 1$, a cost-reducing innovation is *k-drastic for the game G_λ* if k is the minimum number such that if k firms have the innovation, all other firms drop out of the market and a k-firm natural oligopoly is created.

Note that for a k-drastic innovatio: (a) if there are k or more licensees, then all non-licensees drop out of the market and (b) if there are $k-1$ or less licensees, then all firms obtain positive profit.

Observe that a drastic innovation is 1-drastic and any non-drastic innovation is k-drastic for some integer $k \geq 2$. Since $\delta \leq \varepsilon$, for a non-drastic innovation of magnitude ε, the effective magnitude of the innovation is also non-drastic. From Lemma 2, it follows that for $k \geq 2-\lambda$, a cost-reducing innovation of effective magnitude δ is $(k+\lambda)$-drastic for G_λ if $\delta_\lambda(k) \leq \delta < \delta_\lambda(k-1)$. If there are k licensees for such an innovation, a $(k+\lambda)$-firm natural oligopoly is created where the NE price $p_\lambda^n(k, \delta)$ does not exceed c and it equals c if and only if $\delta = \delta_\lambda(k)$.

Exact and non-exact *k*-drastic innovation Let $\lambda \in \{0, 1\}$. For $k \geq 2-\lambda$, a cost-reducing innovation is *exact $(k+\lambda)$-drastic* if $\delta = \delta_\lambda(k)$ (so that $p_\lambda^n(k, \delta) = c$) and it is *non-exact $(k+\lambda)$-drastic* if $\delta_\lambda(k) < \delta < \delta_\lambda(k-1)$ (so that $p_\lambda^n(k, \delta) < c$).

Note that an innovation is non-exact for all but countably many magnitudes. We shall derive the results for generic values of ε and restrict to non-exact innovations.

Remark 2 Taking $\delta = \varepsilon$ in (5), it follows that a cost-reducing innovation of magnitude ε is $(k+\lambda)$-drastic for G_λ if $c/(k+\lambda)\eta(c) \leq \varepsilon < c/(k+\lambda-1)\eta(c)$. It is exact if $\varepsilon = c/(k+\lambda)\eta(c)$ and non-exact if $c/(k+\lambda)\eta(c) < \varepsilon < c/(k+\lambda-1)\eta(c)$.

2.6 Willingness to pay for a license

For $\lambda \in \{0, 1\}$, let $\overline{q}_\lambda^n(k, \delta)$ and $\underline{q}_\lambda^n(k, \delta)$ be the respective NE outputs[8] of a licensee and a non-licensee in $\mathcal{C}_\lambda^n(k, \delta)$. For $\lambda = 1$, let $\widehat{q}_\lambda^n(k, \delta)$ be the NE outputs of firm I. Let $\overline{\phi}_\lambda^n(k, \delta)$, $\underline{\phi}_\lambda^n(k, \delta)$ and $\widehat{\phi}_\lambda^n(k, \delta)$ be the corresponding NE profits. We know (see, e.g., Katz and Shapiro,

[7]Note that for $\lambda = 1$, firm I always has cost $c-\varepsilon$, so it is the only firm that can potentially become a monopolist. When no other firm has the new technology, it is equivalent to the situation where all n firms in N have cost $c-\delta = c$, so that $\delta = 0$. Taking $k = n$ and $\delta = 0$ in Lemma 1, an innovation of magnitude ε is drastic if $\mathcal{C}_1^n(k, 0)$ is a natural monopoly, which occurs if and only if $\varepsilon \geq c/\eta(c)$. Thus, we obtain the same condition as in the case $\lambda = 0$.

[8]See the Appendix for their expressions.

1985) that for $1 \leq k \leq n-1$, the willingness to pay for a license under the AR policy (k, δ) is

$$b_\lambda^n(k,\delta) = \overline{\phi}_\lambda^n(k,\delta) - \underline{\phi}_\lambda^n(k,\delta) = [p_\lambda^n(k,\delta) - c + \delta]\overline{q}_\lambda^n(k,\delta) - \underline{\phi}_\lambda^n(k,\delta) \tag{6}$$

For $k = n$, it is

$$b_\lambda^n(n,\delta) = b_\lambda^n(n,\delta) = \overline{\phi}_\lambda^n(n,\delta) - \underline{\phi}_\lambda^n(n-1,\delta) = [p_\lambda^n(n,\delta) - c + \delta]\overline{q}_\lambda^n(n,\delta) - \underline{\phi}_\lambda^n(n-1,\delta) \tag{7}$$

For the AR policy $(n, \delta, \underline{b})$, it is optimal for the innovator I to set the minimum bid $\underline{b} = b^n(n, \delta)$. Henceforth we denote a policy by simply (k, δ) where it will be implicit that for $k = n$, there is a minimum bid $b^n(n, \delta)$.

2.7 Licensing revenue and payoff of I

For $\lambda = 0$ (i.e. when I is an outsider), I's payoff is simply its licensing revenue. For $\lambda = 1$, I's payoff is the sum of its licensing revenue and its NE profit in the resulting oligopoly game. Note that I's NE profit in $\mathcal{C}_\lambda^n(k, \delta)$ is

$$\widehat{\phi}_\lambda^n(k,\delta) = [p_\lambda^n(k,\delta) - c + \varepsilon]\widehat{q}_\lambda^n(k,\delta) \tag{8}$$

The function $F : [0, p(0)] \to R$ given by $F(p) = (p - c + \varepsilon)Q(p)$ in (3), which presents the monopoly profit at price p under the cost $c - \varepsilon$, will be useful in this section.

• **Royalty policy** For any $\delta \in [0, \varepsilon]$, if I announces a royalty policy δ (i.e. royalty rate $r = \varepsilon - \delta$), then in equilibrium all n firms accept the offer (i.e., $k = n$) and the resulting oligopoly is $\mathcal{C}_\lambda^n(n, \delta)$ where I obtains the royalty payment $nr\overline{q}_\lambda^n(n, \delta) = n(\varepsilon - \delta)\overline{q}_\lambda^n(n, \delta)$. If $\lambda = 1$, I also obtains its NE profit $\widehat{\phi}_\lambda^n(n, \delta)$. By (8) and using the function F, the payoff of I under a royalty policy δ is

$$\Pi_\lambda^n(\delta) = \lambda\widehat{\phi}_\lambda^n(n,\delta) + n(\varepsilon - \delta)\overline{q}_\lambda^n(n,\delta) = F(p^n(n,\delta)) - n\overline{\phi}_\lambda^n(n,\delta) \tag{9}$$

• **AR policy** Under the AR policy (k, δ), in equilibrium: (i) the fee that I obtains is $kb_\lambda^n(k, \delta)$ and (ii) the royalty payment is $kr\overline{q}_\lambda^n(k, \delta) = k(\varepsilon - \delta)\overline{q}_\lambda^n(k, \delta)$. The licensing revenue of I is the sum of these two components. If $\lambda = 1$, I also obtains its NE profit $\widehat{\phi}_\lambda^n(k, \delta)$. From (6), (8) and using the function F, the payoff of I under an AR policy (k, δ) for $1 \leq k \leq n-1$ is

$$\Pi_\lambda^n(k,\delta) = F(p_\lambda^n(k,\delta)) - n\underline{\phi}_\lambda^n(k,\delta) - \varepsilon(n-k)\underline{q}_\lambda^n(k,\delta) \tag{10}$$

From (7), (8) and again using F, the payoff of I under AR policy (n, δ) is

$$\Pi_\lambda^n(n,\delta) = F(p^n(n,\delta)) - n\underline{\phi}_\lambda^n(n-1,\delta) \tag{11}$$

• **Auction policy** Recall that a policy of auctioning off k licenses is equivalent to the AR policy (k, ε), so the payoffs under auction are obtained by taking $\delta = \varepsilon$ in (10) and (11).

Under royalty policies, the problem of I is to choose $\delta \in [0, \varepsilon]$ to maximize $\Pi_\lambda^n(\delta)$ given in (9). Under AR policies, the problem of I is to choose $1 \leq k \leq n$ and $\delta \in [0, \varepsilon]$ to maximize $\Pi_\lambda^n(k, \delta)$ given in (10) and (11). Since these functions are continuous and bounded, these maximization problems have a solution, i.e., there exists an optimal royalty as well as an optimal AR policy.

3 Bound for the payoffs of I

The following result provides an upper bound for the payoffs $\Pi_\lambda^n(\delta)$, $\Pi_\lambda^n(k,\delta)$ and $\Pi_\lambda^n(n,\delta)$ in terms of the function F. This result will be useful to derive more specific properties of optimal licensing policies.

Proposition 1 *Let $n \geq 2$, $1 \leq k \leq n$ and $\delta \in [0,\varepsilon]$.*

(i) $\Pi_\lambda^n(\delta) \leq F(p^n(n,\delta))$ *and* $\Pi_\lambda^n(k,\delta) \leq F(p_\lambda^n(k,\delta))$ *for all* $1 \leq k \leq n$. *Consequently, the maximum that I can obtain is the monopoly profit* $\Pi_M(\varepsilon) \equiv F(p_M(\varepsilon))$.

(ii) *Suppose* $1 \leq k \leq n-1$. *Then* (a) $\Pi_\lambda^n(k,\delta) < F(p_\lambda^n(k,\delta))$ *if* $\delta < \delta_\lambda(k)$ *and* (b) $\Pi_\lambda^n(k,\delta) = F(p_\lambda^n(k,\delta))$ *if* $\delta \geq \delta_\lambda(k)$.

(iii) *Suppose* $k = n$. *Then* (a) $\Pi_\lambda^n(n,\delta) < F(p^n(n,\delta))$ *if* $\delta < c/\delta_\lambda(n-1)$ *and* (b) $\Pi_\lambda^n(n,\delta) = F(p^n(n,\delta))$ *if* $\delta \geq \delta_\lambda(n-1)$.

(iv) *For any non-drastic innovation of magnitude ε (i.e., $\varepsilon < c/\eta(c)$), under any licensing policy I obtains strictly lower than $\pi_M(\varepsilon)$.*

(v) *For any drastic innovation of magnitude ε (i.e., $\varepsilon \geq c/\eta(c)$), an incumbent innovator obtains $\pi_M(\varepsilon)$ by using the innovation exclusively and not licensing. An outside innovator obtains $\pi_M(\varepsilon)$ through the licensing policy $(1,\varepsilon)$, i.e., auctioning off only one license with no royalty.*

Proof. (i): The first part is immediate from (9), (10) and (11). The second part follows by noting that the unique maximum of $F(p)$ is attained at the monopoly price $p_M(\varepsilon)$.

(ii)-(iii): Follow from (10) and (11) by noting that $\underline{q}_\lambda^n(k,\delta)$ and $\underline{\phi}_\lambda^n(k,\delta)$ (NE output and profit of a non-licensee) is positive if $\delta < c/\delta_\lambda(n-1)$ and zero otherwise (Lemma 2).

(iv): We know that when the innovation is non-drastic, then all licensees obtain positive profit for any $\delta \in [0,\varepsilon]$ (see Remark 1), so that $\overline{\phi}_\lambda^n(n,\delta) > 0$. Using this in (9), we have $\Pi_\lambda^n(\delta) < F(p_\lambda^n(n,\delta)) \leq \pi_M(\varepsilon)$, which proves the result for royalty policies.

Next consider an AR policy (k,δ). If either (a) $1 \leq k \leq n-1$ and $\delta < \delta_\lambda(k)$, or (b) $k = n$ and $\delta < \delta_\lambda(n-1)$, then by parts (ii) and (iii), $\Pi_\lambda^n(k,\delta) < F(p_\lambda^n(k,\delta)) \leq \pi_M(\varepsilon)$. So let (a) $1 \leq k \leq n-1$ and $\delta \geq \delta_\lambda(k)$, or (b) $k = n$ and $\delta \geq \delta_\lambda(n-1)$ [implying that $\delta > \delta_\lambda(n)$, since $\delta_\lambda(n-1) > \delta_\lambda(n)$]. Then by Lemma 2(c), it follows that $p_\lambda^n(k,\delta) \leq c$. As the innovation is non-drastic, we have $p_M(\varepsilon) > c$, so that $p_\lambda^n(k,\delta) < p_M(\varepsilon)$. Then from parts (ii) and (iii), we have $\Pi_\lambda^n(k,\delta) = F(p_\lambda^n(k,\delta)) < F(p_M(\varepsilon)) = \pi_M(\varepsilon)$.

(v): Consider a drastic innovation of magnitude ε, i.e., $\varepsilon \geq c/\eta(c)$. First consider $\lambda = 1$ (incumbent innovator). When no other firm is licensed, all firms have unit cost $c - \delta = c$, i.e., $\delta = 0$. For $\delta = 0$, the inequality $\varepsilon \geq \delta + (c-\delta)/\eta(c-\delta)$ of Lemma 1 reduces to $\varepsilon \geq c/\eta(c)$. Then by Lemma 1 it follows that firm I obtains $\pi_M(\varepsilon)$ by using the innovation exclusively.

Next consider $\lambda = 0$ (outside innovator). First note that $\delta_0(1) \equiv c/\eta(c)$ (take $\lambda = 0$ and $k = 1$ in (5)). As the innovation is drastic, we have $\varepsilon > c/\eta(c) \equiv \delta_0(1)$. Taking $k = 1$ and $\delta = \varepsilon$ in (ii), it follows that $\Pi_0^n(1,\varepsilon) = F(p_0^n(1,\varepsilon))$. As $p_0^n(1,\varepsilon)$ equals the monopoly price $p_M(\varepsilon)$, we have $\Pi_0^n(1,\varepsilon) = \pi_M(\varepsilon)$. This completes the proof of the result. ■

Remark 3 Proposition 2 shows that an outside innovator can obtain the monopoly profit $\pi_M(\varepsilon)$ for a drastic innovation by auctioning off one license. However, this is not the unique policy that gives the monopoly profit. Sen and Stamatopoulos (2009a) have shown that for a drastic innovation, there are either $n-1$ or n optimal licensing policies for an outside innovator. The innovator can obtain the monopoly profit by selling k licenses ($1 \leq k \leq n-1$, as well as $k = n$ in some cases) using an AR policy. Each of these policies yield the monopoly profit for the innovator and for all but one of these policies (the one presented in Proposition 1), the rate of royalty is positive.

4 Royalty versus auction policies

For the rest of this article, we focus on non-drastic innovations. We look at generic values of ε and so consider only non-exact innovations. In this section we compare royalty and auction policies and show that for relatively large sizes of industry, royalties are superior for the innovator than auction. The function $F(p) = (p - c + \varepsilon)Q(p)$ (the monopoly profit at price p under cost $c - \varepsilon$) will be useful for our analysis in this section as well. In what follows, we show that $F(c) = \varepsilon Q(c)$ form an upper bound for the payoffs from both royalty and auction. The payoff from royalty can be made arbitrarily close to $F(c)$ by increasing the industry size. However, for any non-exact innovation, the payoff from auction stays bounded away from $F(c)$ as industry size increases. Sen (2005) showed this result for an outside innovator with linear demand. This section extends Sen (2005) to the general demand as well as to the case of incumbent innovators.

4.1 Royalty policy

Suppose the innovation is non-drastic, i.e., $\varepsilon < c/\eta(c)$. Consider the royalty policy $\delta = 0$ (i.e., the rate of royalty is $r = \varepsilon - \delta = \varepsilon$). If this policy is announced, in equilibrium, all firms accept the offer to be licensees and the resulting oligopoly is $\mathcal{C}^n_\lambda(n, 0)$ that has NE price $p^n_\lambda(n, 0)$. We note that $\lim_{n\to\infty} p^n_\lambda(n, 0) = c$ (see Result 1 of the Appendix). The payoff of I under the royalty policy δ is the sum of I's oligopoly profit (if $\lambda = 1$) and royalty payments. This payoff is

$$\Pi^n_\lambda(0) = \lambda[p^n_\lambda(n, 0) - c + \varepsilon]\widehat{q}^n_\lambda(n, 0) + \varepsilon n\overline{q}^n_\lambda(n, 0) = \lambda[p^n_\lambda(n, 0) - c]\widehat{q}^n_\lambda(n, 0) + \varepsilon Q(p^n_\lambda(n, 0)) \quad (12)$$

Since $\lim_{n\to\infty} \widehat{q}^n_\lambda(n, 0) = \lambda\varepsilon Q(c)\eta(c)/c < \infty$ (see Result 2 of the Appendix), by (12),

$$\lim_{n\to\infty} \Pi^n_\lambda(0) = \lim_{n\to\infty} \lambda[p^n_\lambda(n, 0) - c] \lim_{n\to\infty} \widehat{q}^n_\lambda(n, 0) + \lim_{n\to\infty} \varepsilon Q(p^n_\lambda(n, 0)) = \varepsilon Q(c) = F(c) \quad (13)$$

4.2 Auction policy

Let $\lambda \in \{0, 1\}$. Note from (5) that $\delta_0(1) = \delta_1(0) = c/\eta(c)$. Since the innovation is non-drastic, we have $\varepsilon < c/\eta(c) = \delta_0(1) = \delta_1(0)$. Any such innovation is $(m+\lambda)$-drastic for some $m \geq 2 - \lambda$, i.e., $\delta_\lambda(m) \leq \varepsilon < \delta_\lambda(m-1)$. As we consider generic values of ε by assuming that the innovation is non-exact, we have $\delta_\lambda(m) < \varepsilon < \delta_\lambda(m-1)$. Also assume that the industry size is relatively large such that $n \geq 3$ and $n - 1 > m$.

Lemma 3 *For an $(m+\lambda)$-drastic innovation, it is not optimal for I to auction off more than m licenses.*

Proof Note that the policy of auctioning off k licenses is a special AR policy (k, ε). Let $k \geq m$. Since $\delta_\lambda(k)$ is decreasing in k (see (5)) and $\varepsilon > \delta_\lambda(m)$, it follows that $\varepsilon > \delta_\lambda(k)$ for all $k \geq m$. Taking $\delta = \varepsilon$ in Lemma 2(c), it follows that for $k \geq m$, the NE price $p^n_\lambda(k, \varepsilon)$ is less than c and it is the unique solution of $H^{k+\lambda}(p) = c - \varepsilon$ where H is given in (2). Since $H^{k+\lambda}(p)$ is strictly increasing in k, it follows that

$$c > p^n_\lambda(m, \delta) > p^n_\lambda(m+1, \delta) > \ldots > p^n_\lambda(n, \delta) \tag{14}$$

By Proposition 1(ii) and (iii), for $k \geq m$, the payoff of I under the policy (k, ε) equals $\Pi^n_\lambda(k, \varepsilon) = F(p^n_\lambda(k, \varepsilon))$. As the function F is strictly concave and its unique maximum is attained at $p_M(\varepsilon)$, $F(p)$ is increasing for $p < p_M$. Since the innovation is non-drastic, we have $p_M > c$. Then from (14) it follows that $\Pi^n_\lambda(m, \varepsilon) = F(p^n_\lambda(m, \varepsilon)) > \Pi^n_\lambda(k, \varepsilon) = F(p^n_\lambda(k, \varepsilon))$ for all $k \geq m+1$. This proves the result. ■

In view of Lemma 3, consider $k \leq m$. For the policy (m, ε) (i.e., auctioning off m licenses), I obtains $\Pi^n_\lambda(m, \varepsilon) = F(p^n_\lambda(m, \varepsilon)) < F(c)$ (since the innovation is non-exact, we have $\varepsilon > \delta_\lambda(m)$, implying that $p^n_\lambda(m, \varepsilon) < c$).[9] Observe that that for any oligopoly of size $n > m+1$, the payoff $\Pi^n_\lambda(m, \varepsilon)$ is independent of n. Denote

$$\tau_1(\varepsilon) \equiv F(c) - \Pi^n_\lambda(m, \varepsilon) > 0 \tag{15}$$

Note that $\tau_1(\varepsilon)$ does not vary with n.

Now consider auction policies with $k \leq m-1$. Since $\delta_\lambda(k)$ is decreasing in k and $\varepsilon < \delta_\lambda(m-1)$, it follows that $\varepsilon < \delta_\lambda(k)$ for all $k \leq m-1$. Taking $\delta = \varepsilon$ in Lemma 2(b), it follows that for $k \leq m-1$, the NE price $p^n_\lambda(k, \varepsilon)$ is more than c and $\lim_{n\to\infty} p^n_\lambda(k, \varepsilon) = c$ (see Result 3 of the Appendix). Taking $\delta = \varepsilon$ in (10), the payoff of I under the policy (k, δ) is

$$\begin{aligned}\Pi^n_\lambda(k, \varepsilon) &= F(p^n_\lambda(k, \varepsilon)) - n\underline{\phi}^n_\lambda(k, \varepsilon) - \varepsilon(n-k)\underline{q}^n_\lambda(k, \varepsilon) \\ &= F(p^n_\lambda(k, \varepsilon)) - [p^n_\lambda(k, \varepsilon) - c]n\underline{q}^n_\lambda(k, \varepsilon) - \varepsilon(1 - k/n)n\underline{q}^n_\lambda(k, \varepsilon)\end{aligned} \tag{16}$$

Noting that $\lim_{n\to\infty} n\underline{q}^n_\lambda(k, \varepsilon) = Q(c) - Q(c)\eta(c)(k+\lambda)\varepsilon/c$ (see Result 4 of the Appendix) and $F(c) = \varepsilon Q(c)$, by (16), it follows that

$$\begin{aligned}\lim_{n\to\infty} \Pi^n_\lambda(k, \varepsilon) &= F(c) - \lim_{n\to\infty}[p^n_\lambda(k, \varepsilon) - c] \lim_{n\to\infty} n\underline{q}^n_\lambda(k, \varepsilon) - \lim_{n\to\infty} \varepsilon(1 - k/n) \lim_{n\to\infty} n\underline{q}^n_\lambda(k, \varepsilon) \\ &= F(c) - \varepsilon[Q(c) - Q(c)\eta(c)(k+\lambda)\varepsilon/c] = F(c)\eta(c)(k+\lambda)\varepsilon/c\end{aligned} \tag{17}$$

Since $k \leq m-1$, from (17) we conclude that

$$\lim_{n\to\infty} \Pi^n_\lambda(k, \varepsilon) = F(c)\eta(c)(k+\lambda)\varepsilon/c \leq F(c)\eta(c)(m-1+\lambda)\varepsilon/c \tag{18}$$

Recall from (5) that $\delta_\lambda(m-1) = (c/\eta(c) - \lambda\varepsilon)/(m-1)$, so that the inequality $\varepsilon < \delta_\lambda(m-1)$ is equivalent to $\varepsilon < c/(m-1+\lambda)\eta(c)$. Hence $\eta(c)(m-1+\lambda)\varepsilon/c < 1$. Let us denote

[9] For an $(m+\lambda)$-drastic innovation, I obtains $F(c)$ from auctioning off m licenses only if the innovation is exact, i.e., $\varepsilon = \delta_\lambda(m)$, so that $p^n_\lambda(m, \varepsilon) = c$.

$F(c)\eta(c)(m-1+\lambda)\varepsilon/c \equiv F(c)-\tau_2(\varepsilon)$ where $\tau_2(\varepsilon)$ is positive constant. Then from (18), it follows that for all $k \leq m-1$, $\lim_{n\to\infty} \Pi_\lambda^n(k,\varepsilon) \leq F(c)-\tau_2(\varepsilon)$. Now choose a sufficiently small positive constant $\tau_3(\varepsilon) < \tau_2(\varepsilon)$ and let $\tau_4(\varepsilon) = \tau_2(\varepsilon)-\tau_3(\varepsilon) > 0$. Then $\exists\ N_A(\varepsilon) > m+1$ such that for all $k \leq m-1$,

$$\Pi_\lambda^n(k,\varepsilon) < [F(c)-\tau_2(\varepsilon)] + \tau_3(\varepsilon) = F(c)-\tau_4(\varepsilon) \text{ for all } n \geq N_A(\varepsilon) \tag{19}$$

Consider the two constants $\tau_1(\varepsilon)$ in (15) and $\tau_4(\varepsilon)$ in (19) and let $\tau(\varepsilon) = \min\{\tau_1(\varepsilon), \tau_4(\varepsilon)\}$. Then by (15) and (19) it follows that for any k,

$$\Pi_\lambda^n(k,\varepsilon) \leq F(c)-\tau(\varepsilon) \text{ for all } n \geq N_A(\varepsilon) \tag{20}$$

Since $\lim_{n\to\infty} \Pi_\lambda^n(0) = F(c)$, $\exists\ N_R(\varepsilon) > m+1$ such that

$$\Pi_\lambda^n(0) > F(c)-\tau(\varepsilon) \text{ for all } n \geq N_R(\varepsilon) \tag{21}$$

Taking $N(\varepsilon) = \max\{N_A(\varepsilon), N_R(\varepsilon)\}$, it follows from (20) and (21) that for any k, $\Pi_\lambda^n(0) > \Pi_\lambda^n(k,\varepsilon)$ for all $n \geq N(\varepsilon)$. The conclusion is summarized in the following proposition.

Proposition 2 *Let $\lambda \in \{0,1\}$. Consider a non-exact $(m+\lambda)$-drastic innovation, i.e., $\delta_\lambda(m) < \varepsilon < \delta_\lambda(m-1)$ where $m \geq 2-\lambda$. Then $\exists\ N(\varepsilon) > m+1$ such for all $n \geq N(\varepsilon)$, the payoff of I from the royalty policy $\delta = 0$ (i.e., $r = \varepsilon$) is higher than any auction policy.*

5 AR policies: some general results

In this section we consider the more general auction plus royalty (AR) policies. This section is an extension of Sen and Tauman (2007) who completely characterize optimal AR policies under linear demand. Let $n \geq 3$ and consider a non-drastic innovation of magnitude ε, i.e., $\varepsilon < c/\eta(c)$. Certain general properties of optimal AR policies can be derived when the innovation is relatively significant. Specifically, assume that for $\lambda \in \{0,1\}$, $\varepsilon > \delta_\lambda(n-1)$ where $\delta_\lambda(k)$ is given in (5). This implies that the innovation is $(m+\lambda)$-drastic for some $2-\lambda \leq m \leq n-1$.[10] Note from (5) that the inequality $\varepsilon > \delta_\lambda(n-1)$ is equivalent to

$$\varepsilon > c/(n-1)\eta(c) \text{ for } \lambda = 0 \text{ and } \varepsilon > c/n\eta(c) \text{ for } \lambda = 1 \tag{22}$$

These inequalities hold if either ε is relatively significant, or n is relatively large. As before we restrict to generic values of ε by assuming that the innovation is non-exact, i.e., $\exists\ 2 \leq m \leq n-1$ such that $\delta_\lambda(m) < \varepsilon < \delta_\lambda(m-1)$. The function $F(p) = (p-c+\varepsilon)Q(p)$ (the monopoly profit at price p under cost $c-\varepsilon$) will be again useful for our analysis.

Proposition 3 *Let $n \geq 3$. Consider a non-drastic innovation of magnitude ε (i.e., $\varepsilon < c/\eta(c)$) that is non-exact and suppose $\varepsilon > \delta_\lambda(n-1)$ for $\lambda \in \{0,1\}$. Then the following hold.*

(i) *Under any optimal AR policy, firm I obtains at least $F(c)$.*

[10] If m or more firms in N have an $(m+\lambda)$-drastic innovation without any royalty, then a $(m+\lambda)$-firm natural oligopoly is created and the remaining $n-m$ firms drop out of the market. Since $m \leq n-1$, it follows that if $k = n-1$ or $k = n$ firms have the innovation without any royalty, then a $(k+\lambda)$-firm natural oligopoly is created and the remaining $n-m$ firms drop out of the market.

(ii) *Under any optimal AR policy, the NE price of the resulting oligopoly game is at least* c.

(iii) *Let* $1 \leq k \leq n-1$ *and* $\delta \in [0, \varepsilon]$. *For any such* (k, δ), *there exists* $\widetilde{\delta} \in [0, \varepsilon]$ *such that* $\Pi^n_\lambda(n-1, \widetilde{\delta}) \geq \Pi^n_\lambda(k, \delta)$. *Consequently there always exists an optimal AR policy where* $k = n-1$ *or* n.

(iv) *Any optimal AR policy must include positive royalty.*

Proof (i) We prove (i) by showing that there exists an AR policy that yields the payoff $F(c)$ for firm I. Consider the AR policy $(n-1, \delta_\lambda(n-1))$ (since $\delta_\lambda(n-1) < \varepsilon$, this policy has positive royalty $r = \varepsilon - \delta_\lambda(n-1)$). From Proposition 1(ii), it follows that the payoff of I from this policy is $\Pi^n_\lambda(n-1, \delta_\lambda(n-1)) = F(p^n_\lambda(n-1, \delta_\lambda(n-1))$. Since $p^n_\lambda(n-1, \delta_\lambda(n-1) = c$ (by Lemma 2(c)), we have $\Pi^n_\lambda(n-1, \delta_\lambda(n-1)) = F(c)$.

(ii) Since the function $F(p)$ is strictly concave and its unique maximizer is $p_M(\varepsilon)$, it follows that $F(p)$ is strictly increasing for $p < p_M(\varepsilon)$. Since the innovation is non-drastic, we have $c < p_M(\varepsilon)$. Therefore $F(p) < F(c)$ for $p < c$. Consider a policy (k, δ) and denote the resulting NE price $p^n_\lambda(k, \delta)$ by p. By Proposition 1, the payoff from this policy does not exceed $F(p)$. If $p < c$, then $F(p) < F(c)$, so the payoff is strictly lower than $F(c)$. By part (i), such a policy cannot be optimal, proving that the resulting price under any optimal AR policy must be at least c.

(iii) Consider a policy (k, δ) such that $1 \leq k \leq n-1$ and $\delta \in [0, \varepsilon]$. If $\delta > \delta_\lambda(k)$, then $p^n_\lambda(k, \delta) < c$ and using the same reasoning as in part (ii), the payoff from such a policy is strictly lower than $F(c)$ and by part (i), the policy is inferior to $(n-1, \delta_\lambda(n-1))$.

Next consider a policy (k, δ) such that $\delta \leq \delta_\lambda(k)$, in which case $p^n_\lambda(k, \delta) \geq c$. Note from Lemma 2(d) the NE price and NE outputs of firms in $\mathcal{C}^n_\lambda(k, \delta)$ depend only on the product $k\delta$. Consider the AR policy $(n-1, \widetilde{\delta})$ such that $(n-1)\widetilde{\delta} = k\delta$. Then by (5), $\widetilde{\delta} \leq \delta_\lambda(n-1)$ and $p^n_\lambda(n-1, \widetilde{\delta}) \geq c$. From (10), it follows that the first two terms are the same for the policies (k, δ) and $(n-1, \widetilde{\delta})$, while the last term is (weakly) higher for the latter policy. This proves that $\Pi^n_\lambda(n-1, \widetilde{\delta}) \geq \Pi^n_\lambda(k, \delta)$.

(iii) Since the innovation is non-exact, $\exists\ 2 - \lambda \leq m \leq n-1$ such that $\delta_\lambda(m) < \varepsilon < \delta_\lambda(m-1)$. A policy with zero royalty corresponds to $\delta = \varepsilon$ (hence $r = \varepsilon - \delta = 0$). Consider the policy (k, ε) for $k \geq m$. Taking $\delta = \varepsilon$ in Lemma 2, we have $\varepsilon > \delta_\lambda(m) \geq \delta_\lambda(k)$. Therefore, for such a policy, $p^n_\lambda(k, \delta) < c$ and by part (ii), such a policy cannot be optimal.

Next consider a policy (k, ε) where $k \leq m-1$. Taking $\delta = \varepsilon$ in Lemma 2, we have $\varepsilon < \delta_\lambda(m-1) \leq \delta_\lambda(k)$. Therefore, for such a policy $p^n_\lambda(k, \delta) > c$ and $\underline{q}^n(k, \delta)$ is positive. As in (ii), note from Lemma 2(d) in this case the NE price and NE outputs of firms in $\mathcal{C}^n_\lambda(k, \delta)$ depend only on the product $k\delta$. Consider the AR policy $(n-1, \widetilde{\delta})$ such that $(n-1)\widetilde{\delta} = k\varepsilon$, so that $\widetilde{\delta} = k\varepsilon/(n-1) < \varepsilon$ (since $k \leq m-1 < n-1$), implying that the policy $(n-1, \widetilde{\delta})$ has positive royalty. From (10) it follows that the first two terms are the same for the policies (k, ε) and $(n-1, \widetilde{\delta})$, while the last term is strictly higher for the latter policy (this is because $\underline{q}^n(k, \delta) > 0$, since $\varepsilon < \delta_\lambda(k)$). Therefore $\Pi^n_\lambda(n-1, \widetilde{\delta}) > \Pi^n_\lambda(k, \delta)$, proving that a policy with zero royalty is never optimal. Hence any optimal AR policy must include a positive royalty.
■

It is shown in Sen and Tauman (2007) that results of Proposition 3 hold for *any* non-drastic innovation under linear demand. Thus, the conclusions of Sen and Tauman (2007) are robust to the case of general demand provided the industry size is relatively large.

Appendix

NE outputs for the Cournot oligopoly game $\mathcal{C}_\lambda^n(k,\delta)$ Let $\lambda \in \{0,1\}$, $1 \le k \le n$ and $\delta \in [0,\varepsilon]$ and suppose $\lambda\varepsilon < \delta + (c-\delta)/\eta(c-\delta)$. Let $\overline{q}_\lambda^n(k,\delta)$ and $\underline{q}_\lambda^n(k,\delta)$ be the respective NE outputs of a licensee and a non-licensee and for $\lambda = 1$, let $\widehat{q}_\lambda^n(k,\delta)$ be the NE outputs of firm I for the game $\mathcal{C}_\lambda^n(k,\delta)$.

(i) $\delta < \delta_\lambda(k)$: The NE outputs are given below where $p = p_\lambda^n(k,\delta)$:

$$\widehat{q}_\lambda^n(k,\delta) = Q(p)\frac{\lambda(c-\varepsilon)+\lambda\eta(p)(n\varepsilon - k\delta)}{(n+\lambda)c - k\delta - \lambda\varepsilon},\ \ \overline{q}_\lambda^n(k,\delta) = Q(p)\frac{c-\delta+\eta(p)[(n+\lambda-k)\delta - \lambda\varepsilon]}{(n+\lambda)c - k\delta - \lambda\varepsilon}$$

$$\underline{q}_\lambda^n(k,\delta) = Q(p)\frac{c-\eta(p)(k\delta+\lambda\varepsilon)}{(n+\lambda)c-k\delta-\lambda\varepsilon} \tag{23}$$

(ii) Suppose $\delta \ge \delta_\lambda(k)$: The NE outputs are given below where $p = p_\lambda^n(k,\delta)$:

$$\widehat{q}_\lambda^n(k,\delta) = Q(p)\frac{\lambda(c-\varepsilon)+\lambda k\eta(p)(\varepsilon-\delta)}{(k+\lambda)c-k\delta-\lambda\varepsilon},\ \overline{q}_\lambda^n(k,\delta) = Q(p)\frac{c-\delta-\lambda\eta(p)(\varepsilon-\delta)}{(k+\lambda)c-k\delta-\lambda\varepsilon},\ \underline{q}_\lambda^n(k,\delta) = 0 \tag{24}$$

Some limiting properties:

Consider the Cournot oligopoly game $\mathcal{C}_\lambda^n(n,0)$.

Result 1 $\lim_{n\to\infty} p_\lambda^n(n,0) = c$.

Proof Taking $k = n$ and $\delta = 0$ in Lemma 2(a), it follows that $p_\lambda^n(n,0)$ is the unique solution of $H^{n+\lambda}(p) = c - \lambda\varepsilon/(n+\lambda)$. Using the expression of H from (2), this equation can be written as

$$p = [c - \lambda\varepsilon/(n+\lambda)]/[1 - 1/(n+\lambda)\eta(p)] \tag{25}$$

Since $\eta(p)$ is bounded for $p \in (0, p(0))$, the right side of (25) converges to c as $n \to \infty$. This proves the result. ■

Result 2 $\lim_{n\to\infty} \widehat{q}_\lambda^n(n,0) = \lambda\varepsilon Q(c)\eta(c)/c$.

Proof Using Result 1, from (23) it follows that

$$\lim_{n\to\infty} \widehat{q}_\lambda^n(n,0) = \lim_{n\to\infty} Q(p_\lambda^n(n,0))\frac{\lambda(c-\varepsilon)/n + \lambda\eta(p_\lambda^n(n,0))\varepsilon}{(1+\lambda/n)c - \lambda\varepsilon/n} = \lambda\varepsilon Q(c)\eta(c)/c \tag{26}$$

This completes the proof. ■

Let $\delta_\lambda(m) < \varepsilon < \delta_\lambda(m-1)$ for some $m \ge 2-\lambda$ where $\lambda \in \{0,1\}$. Consider an auction policy with $k \le m-1$. Note that such a policy is a special AR policy (k,ε) which results in the Cournot oligopoly game $\mathcal{C}_\lambda^n(k,\varepsilon)$.

Result 3 If $\varepsilon < \delta_\lambda(m-1)$, then $\lim_{n\to\infty} p_\lambda^n(k,\varepsilon) = c$ for any $k \le m-1$.

Proof Taking $\delta = \varepsilon$ in Lemma 2(a), it follows that for $k \leq m-1$, the NE price $p^n_\lambda(k,\varepsilon)$ is more than c and it is the unique solution of $H^{k+\lambda}(p) = c - \varepsilon$ where H is given in (2). Using the expression of H from (2), this equation can be written as

$$p = [c - (k+\lambda)\varepsilon/(n+\lambda)]/[1 - 1/(n+\lambda)\eta(p)] \tag{27}$$

Since $\eta(p)$ is bounded for $p \in (0, p(0))$, the right side of (25) converges to c as $n \to \infty$. This proves the result.

Result 4 If $\varepsilon < \delta_\lambda(m-1)$, then $\lim_{n\to\infty} n\underline{q}^n_\lambda(k,\varepsilon) = Q(c) - Q(c)\eta(c)(k+\lambda)\varepsilon)/c$ for any $k \leq m-1$.

Proof Note from (23) that

$$n\underline{q}^n_\lambda(k,\varepsilon) = Q(p^n_\lambda(k,\varepsilon))\frac{c - \eta(p^n_\lambda(k,\varepsilon))(k+\lambda)\varepsilon}{(1+\lambda/n)c - (k+\lambda)\varepsilon/n} \tag{28}$$

Using Result 3, the result follows from (28). ■

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