Tiny Mirror Asymmetry in Electron Scattering Confirms the Inconstancy of the Weak Coupling Constant

The standard model of particle theory predicts that all three fundamental coupling “constants” vary with distance. But demonstrating the variation for the weak interactions required an experimental tour de force.

Spin-polarized beams of high-energy electrons scattering off unpolarized electrons or nuclei are particularly useful for examining the very slight preference for left-handed or right-handed screws in the realm of fundamental particles. But because that preference is measured in parts per 10 million, such experiments require extraordinary care. For longitudinally polarized electron spins, any difference between the scattering of electrons spinning like left- and right-handed screws violates parity conservation—that is, mirror symmetry. Electromagnetic interactions, which dominate electron scattering, strictly respect parity conservation. But the weak interactions do not.

In 2002, the E158 collaboration at SLAC began measuring the tiny fractional difference between the cross sections for the elastic scattering of left- and right-polarized 50-GeV electrons off electrons in an unpolarized liquid-hydrogen target. The polarized beam electrons were accelerated in the laboratory’s 3-km-long linac. The experimental setup, just published, gives a right–left asymmetry measurement precise enough to demonstrate for the first time that the fundamental coupling “constant” of the weak interactions does in fact vary with the energy–momentum scale at which it is measured.1 This so-called running of the weak coupling constant was an important, unconfirmed prediction of the standard model of particle theory.

Running constants

In fact, the standard model predicts such oxymoronic running for all three coupling constants of the theory: strong, electromagnetic, and weak. It’s a characteristic of gauge-invariant field theories like those that constitute the standard model. The Lorentz-invariant magnitude $Q$ of the four-momentum transferred between interacting particles is an inverse measure of their distance of closest approach. The variation of the constants with distance (and therefore with $Q$) is a consequence of the creation and destruction of virtual particle–antiparticle pairs. Those evanescent pairs effectively shield the interacting particles from each other by polarizing the vacuum, much as a polarized dielectric shields capacitor plates.

For electromagnetism and the strong nuclear force, the running of the coupling constants is well established. The fine-structure constant $\alpha = 1/137$, the square of the electron’s charge $e$ in natural units, really describes electromagnetic coupling only in the limit of large distance or, equivalently, negligible $Q$. As charged particles get closer, there’s less vacuum shielding between them. At $10^{-16}$ cm ($Q$ near 100 GeV), the effective $\alpha(Q)$ has increased to about 1/128.

Whereas the shielding of electric charge is due mostly to virtual $e^+e^-$ and other fermion pairs, the running of the strong nuclear coupling constant is dominated by virtual pairs of massless spin-1 gluons. It turns out that, unlike spin-1/2 lepton or quark pairs, pairs of spin-1 particles such as gluons or the heavy $W$ bosons that mediate the weak interaction actually provide anti-shielding between interacting particles. That is, the strong coupling between quarks decreases as they approach each other. At zero separation, it vanishes altogether. That’s called asymptotic freedom (see PHYSICS TODAY, December 2004, page 21).

For the weak couplings, the standard-model prediction is more complicated. The heavy $Z^0$ boson is the neutral partner of the charged $W$s. These three mediators of the weak force couple to each other and to all other known particles. The expected $Q$ dependence of the $Z^0$'s weak coupling constant, $g_w$, is not monotonic. For $Q$ between 1 and 100 GeV, virtual quark–antiquark pairs dominate the vacuum polarization and $g_w$, like $e$, should increase with $Q$. But then, starting near 160 GeV, twice the $W$ mass, virtual $W$ pairs begin to contribute significant anti-shielding and $g_w$ should begin to decrease.

All that running of the weak coupling constant has, until now, been theoretical expectation. Only at $Q \approx 91$ GeV, the mass of the $Z^0$, is there a precise determination of $g_w$. That was accomplished with high-statistics studies of $Z^0$ production in $e^+e^-$ colliding-beam experiments at SLAC and CERN. The principal purpose of the E158 experiment has been to measure the weak coupling at much lower $Q$ with enough precision to confront the standard-model prediction.

Significant departures from the standard model would point to new physics at high energies not yet explored directly. Despite E158's modest beam energy, the experiment would be sensitive, for example, to the existence of a neutral cousin of the $Z^0$ as heavy as 1000 GeV. Such speculative objects appear in theories that seek to unify the strong and electroweak interactions.

A tiny asymmetry

The E158 collaboration, led by Krishna Kumar (University of Massachusetts, Amherst), Emily Hughes (Caltech), Yuri Kolomenski (University of California, Berkeley), and Paul Souder (Syracuse University), set out to measure the right–left asymmetry

$$A = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$$

$\sigma_R$ and $\sigma_L$ are the cross sections for left- and right-handed electron scattering off unpolarized electrons or nuclei. The mixing parameter is inversely proportional to $g_w^2$, the square of the weak coupling constant. The curve showing the standard-model prediction is anchored at $M_Z$, the mass of the $Z^0$ boson, where the mixing parameter has been precisely determined. Lying 6 standard deviations higher, the new SLAC E158 point demonstrates the running of the mixing parameter. The point at still lower $Q$, from a 1999 measurement of parity violation in cesium atoms, is consistent with both the predicted running and no running at all. (Adapted from ref. 1.)
with a combined statistical and systematic uncertainty of about 15%. The cross sections $\sigma_+$ and $\sigma_-$ are for $e^+e^-$ elastic scattering near 90° in the center-of-mass frame, with right- and left-polarized 50 GeV electrons. The acceptance range of center-of-mass scattering angles included in the measured cross sections was about 45°.

The invariant magnitude $Q$ of the four-momentum transferred between the colliding electrons is given, in any reference frame, by $Q^2 = (\Delta p)^2 - (\Delta E)^2$, where $\Delta p$ and $\Delta E$ are the transferred momentum and energy. In the experiment, $Q$, the weighted mean value of $Q$, was 0.16 GeV.

Rather than measuring $g_w(Q)$ directly, experiment E158 determined $\sin^2 \theta_W$, the mixing parameter of the standard model’s unified electroweak theory, formulated in the 1960s by Steven Weinberg, Sheldon Glashow, and Abdus Salam. The theory’s most striking prediction was the existence of the $Z^0$. The empirical weak mixing angle $\theta_W$, often called the Weinberg angle, describes the relative admixture of photonlike (isosinglet) and $W$-like (isotriplet) components in the $Z^0$’s makeup. The mixing parameter is related to the effective (running) electromagnetic and weak coupling constants by

$$\sin^2 \theta_W(Q) = e^2(Q) / G_F^2(Q).$$

The standard-model prediction for the running of $\sin^2 \theta_W$ is shown by the curve in figure 1.

For elastic $e^+e^-$ scattering at center-of-mass scattering angles near 90°, the electroweak mixing parameter for a given $Q$ is related to the right–left asymmetry by

$$A(Q) \approx \sqrt{2[1 - 4 \sin^2 \theta_W(Q)] \times G_F^2} R_0^2 e^{iQ},$$

where $G_F$ is the Fermi constant, which characterizes the strength of the weak interactions in the low-energy limit.

The asymmetry comes from the interference term between the photon-exchange amplitude, which dominates the elastic scattering, and the much smaller $Z^0$-exchange amplitude (see the leading-order Feynman diagrams in figures 2a and 2b). The predicted running of the mixing parameter is calculated from higher-order loop diagrams like figures 2c and 2d. Whereas fermionic quark loops contribute to vacuum screening of the weak coupling, the $W$-boson loops make antiscreening contributions. In effect, the loop diagrams introduce a slight $Q$ dependence into the $Z^0$’s admixture of photonlike and $W$-like components.

**Precision**

For all its expected running, $\sin^2 \theta_W$ shouldn’t stray very far from 1/4. Therefore, given the form of equation 2, the fractional error in the determination $\sin^2 \theta_W$ is about 20 times smaller than the fractional error in measuring the asymmetry. A useful comparison of the mixing parameter at $Q = 0.16$ GeV with the standard-model prediction requires that $\sin^2 \theta_W$ be determined to better than 1%. So Kumar and company only had to measure $A$ to better than 20%. But because the predicted magnitude of the right–left asymmetry was only about $10^{-7}$, that meant that they had to accumulate more than $10^{16}$ elastic collisions. And they had to pay exquisite attention to systematic errors.

The requisite event rate was much too high for event-by-event analysis.

Relative scattering rates with right- and left-polarized beams had to be measured by the light generated in electromagnetic calorimeters made of alternating layers of copper and fused silica.

Spin-polarized electrons were created at the linac’s upstream end by circularly polarized light hitting a specially developed gallium arsenide photocathode. The light began as linearly polarized laser light; it was circularly polarized by an intervening Pockels cell. One could reverse the polarization sense simply by reversing the voltage applied to the cell.

The electron beam was delivered to the hydrogen target in pulses at 120 Hz. Successive pulses were randomly given left- or right-circular polarization. “If you leave it at one polarization for longer intervals,” says Kumar, “inevitable drift in the electronics washes out the tiny asymmetry signal.” One also had to compensate for small but insidious correlations between the sign of the cell voltage and the geometry of the emerging laser and electron beams. It was crucial that the electron beam’s path through the target have no polarization dependence. To that end, an elaborate beam-monitoring system constantly fed small voltage corrections back to the Pockels cell.

For a given cell polarity, one could reverse the electron beam’s circular polarization either by inserting a half-wave plate in the laser beam or reducing the final beam energy by a few GeV—just enough to reduce the precession of the electron spin in a downstream bending magnet by precisely half a cycle. Such steps were taken every few days. The experiment ran for a total of about 3000 hours in 2002 and 2003. “We knew we had beaten the systematics,” says Kumar, “when we got the same asymmetry, within statistics, for all four combinations of beam energies and half-wave plates.”

The final measured asymmetry, $A = (-131 \pm 17) \times 10^{-8}$, yields $\sin^2 \theta_W = 0.2397 \pm 0.0013$ for the effective mixing parameter at $Q = 0.16$ GeV. As shown in figure 1, that’s within one
standard deviation of the standard-model prediction. And it demonstrates the anticipated running by being more than six standard deviations above the precisely measured anchor point at $Q = 91$ GeV, the $Z^0$ mass.

The figure also shows a 1999 measurement at much lower $Q$ by Stephen Bennett and Carl Wieman at The University of Colorado, based on parity-violating atomic-physics effects in cesium. The point is consistent with the standard model prediction. But, being less than three standard deviations from the point at $M_Z$, it’s also barely consistent with no running of $\sin^2\theta_W$. Such measurements are burdened by intricate details of atomic physics. “But recent advances in atomic theory are significantly reducing the uncertainties,” says Wieman.

From the good agreement of the E158 measurement with the standard-model prediction, the collaboration concludes that the mass of any heavier recurrence of the $Z^0$ would have to be at least 1000 GeV.

**End of an era**

In the mid-1970s, experimental evidence for the now-standard electroweak-unification theory was shaky. Searches for the predicted atomic-physics parity violation had yielded conflicting results, and the $Z^0$ had not yet been discovered. But then in 1978, Charles Prescott and Vernon Hughes (Emlyn’s father) mounted E122, the pioneering precursor of E158 at the SLAC linac. Studying electron–deuteron scattering by much the same technique that was upgraded for E158, they got the first definitive evidence for parity violation attributable to the $Z^0$. Although their determination of $\sin^2\theta_W$ had a 10% uncertainty, the result was a historic confirmation of electroweak unification. One year later, Glashow, Salam, and Weinberg shared the Nobel Prize.

The successful completion of E158 marks the passing of an era at SLAC. For almost 40 years, the 3-km linac has been delivering high-energy electrons—and later, positrons—to particle-physics experiments at its downstream end (see figure 3). Now the experimental area is being cleared to make way for the Linac Coherent Light Source, a free-electron-laser facility fed by electrons from the linac, that will provide x-ray beams of unprecedented brightness for experiments in condensed-matter physics, materials science, and biology (see PHYSICS TODAY, May 2005, page 26).

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**References**