Sovereign Default Resolution Through Maturity Extension

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Abstract

Sovereign default episodes are resolved by restructuring the debt through renegotiations and implemented by bond swaps and resumption of debt service payments. This process provides debt relief for the sovereign and partially compensates lenders for their losses. In the data, the bulk of such debt relief is implemented by extending the maturity of the debt rather than changing its face value. We augment a standard maturity choice model with a post-default renegotiation phase and study whether it can replicate this observed maturity extension. The model is successful in generating this and other key features of renegotiations and maturity choice, but critically only when we assume that sovereigns continue to be excluded temporarily from financial markets after renegotiation, consistent with the observation that countries do not immediately resume borrowing. In contrast, a version of the model where market access is restored promptly features a counterfactual reduction in maturity, a puzzle for the standard model.

JEL: F34, G11, H63

Keywords: sovereign debt, maturity choice, maturity extension, sovereign default, renegotiation

1. Introduction

Emerging markets are vulnerable to debt crises which often result in outright default. Recently, similar dynamics proved relevant for the European Union "periphery" including Euro Area members such as Spain and Greece. One option in the policy-makers' toolkit is the active management of the maturity structure of the debt, over the business cycle and in the run-up of crises. Broner et al. (2013) document that emerging markets issue shorter-term debt when financial conditions worsen. A relatively recent quantitative literature on maturity choice (e.g. Arellano and Ramanarayanan (2012), Hatchondo et al. (2016), Sánchez et al. (2018)) studies the trade-offs faced under endogenous default risk, yet much less is known about the maturity structure determined as part of the bond swaps used to resolve default episodes. These swaps implement new

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terms for the lenders by replacing the old bonds, on which the sovereign stopped making payments, with new obligations with a new face value and maturity structure. This restructuring process is a necessary but not sufficient first step towards regaining access to international financial markets. In parallel, a body of largely empirical work has focused on the haircuts suffered by lenders. It relates bond-level and aggregate haircuts to debt relief, the length of market exclusion, and yields following the eventual return to markets. The comprehensive account of Cruces and Trebesch (2013) concludes that *maturity extension* is the main mechanism for debt relief: bond swaps reduce overall indebtedness, yet a greater share of the debt takes the form of long-term bonds compared to the composition of debt at the time of default. This suggests that maturity choice during restructuring is of vital importance for understanding the resolution of default episodes, the reduction in debt burden, and lenders' eventual compensation.

We provide a quantitative analysis of the resolution of default crises via bond swaps and the incentives that lead countries to negotiate maturity extensions. This requires a joint characterization of maturity choice both at issuance, during normal market access, and as part of the restructuring process. The resulting model brings together separate but inherently related strands of the literature on sovereign debt: it embeds Yue (2010)'s Nash bargaining setup, for the resolution of default episodes, in the standard maturity choice model, e.g. Arellano and Ramanarayanan (2012). While in good credit standing the sovereign can issue both short- and long-term debt and the eventual renegotiations following default alter both the maturity structure of the debt as well as its overall level. We assume lenders are risk-neutral and competitive, resulting in bond prices that are actuarially fair. As standard in this class of models, we focus on Markov equilibria, so that the sovereign lacks commitment over future default decisions and debt issuance behavior. Default is potentially desirable because it supports higher levels of consumption, due to the suspension of debt service payments, but costly because it leads to temporary financial autarky and an exogenous loss of income, which captures in reduced-form any real costs or disruptions caused by default. The model's success in generating the maturity extension in the data hinges critically on the inclusion of a post-renegotiation market exclusion spell. The absence of new issuance immediately following renegotiations is a ubiquitous feature of the data, which we interpret through the lens of the model as evidence of continued market exclusion. Using the dataset compiled by Cruces and Trebesch (2013) we find that, on average, countries are excluded for roughly 3 years prior to restructuring and an additional 4 afterwards. These estimates are not sensitive to the definition of market re-access and outliers.

The post-renegotiation exclusion is essential for inducing maturity extension because it calls for the sovereign to resume debt service payments before it is able to tap international markets again: with short-term debt, the sovereign risks having to make relatively large principal payments without the option to roll them over, which is costly from a consumption-smoothing point of view. Instead, with long-term debt the sovereign makes a smooth sequence of relatively small payments. The intuition here mirrors the interaction of maturity with roll-over risk emphasized by Bocola and Dovis (2020), but rather than concerning the risk of markets "shutting down" in a self-fulfilling manner, here the sovereign risks delays in reentry.

In the version of the model with immediate reentry there is no such risk of continued exclusion and consideration related to debt dilution dominate instead: after renegotiations debt and default risk are low and the relatively impatient government starts accumulating debt fast. Expectations of eventual high indebtedness depress current long-term bond prices and make compensating defaulted lenders using such instruments costly. This is a manifestation of the inefficiency inherent in dilutable long-term debt, as discussed by e.g. Hatchondo et al. (2016). In this setting defaults are settled with mostly short-term debt, which the country can roll over, but which does not suffer from dilution.

The takeaway from the analysis of these two environments is that maturity choice in renegotiation is shaped by the same forces in play during normal market access, namely a tension between consumption smoothing, the hedging benefit of long-term debt, and debt dilution. In the empirically-relevant case of post-renegotiation exclusion, extending maturity supports consumption smoothing for the country while its downside, dilution, is kept in check temporarily by the government's inability to resume borrowing immediately.

Our project is related to all work on emerging market long-term debt and maturity choice but most closely to two recent papers. In terms of theory, Aguiar et al. (2019) study a standard maturity choice model and prove that inaction in secondary markets as well as no further issuance of long-term debt are optimal in their environment, where the sovereign is paying down an exogenous initial stock of debt. Their stark result reflects the choice of shocks facing the sovereign, in particular abstracting from the income dynamics common in quantitative work. Their findings rationalize why buybacks in secondary markets are uncommon and why they are unlikely to improve the country's overall position, cf. Bulow and Rogoff (1988). Quantitatively, Sánchez et al. (2018) also study renegotiation, but do so within the maturity choice framework of Sánchez et al. (2018). They consider a rich quantitative model augmented with several extensions, including portfolio adjustment costs and exogenous sudden-stop shocks. They share our focus on maturity extension and list four ingredients that contribute to their model's ability to generate it: lenders' aversion to face-value haircuts (possibly due to balance sheet and regulatory concerns), post-renegotiation exclusion, debt dilution, and income dynamics around default events. In their model, renegotiations are modeled as an alternating-offers bargaining game. In contrast, here we identify a puzzle in the standard, workhorse maturity choice model in the literature and offer a minimal resolution by arguing that post-renegotiation exclusion is sufficient to replicate the extension in the data.

In generating the maturity extension behavior found in the data, we have abstracted from several recent developments in the sovereign default literature, chiefly production and an endogenous cost of default as in Mendoza and Yue (2012), the possibility of self-fulfilling crises following Cole and Kehoe (2000), lender risk-aversion or richer pricing kernels, e.g. Lizarazo (2013) or Aguiar et al. (2016), capital and investment, e.g. Gordon and Guerron-Quintana (2018), or foreign reserves, e.g. Bianchi et al. (2018), to name but a few. These extensions likely have first-order consequences for the choice of maturity at issuance and during renegotiation. We cannot rule out that one or some combination of them could induce the maturity extension behavior observed in the data. We model negotiations by following Yue (2010), using a cooperative game theory solution concept, in contrast to some of the recent work using non-cooperative bargaining, as in Benjamin and Wright (2011) and more recently Asonuma and Joo

(2020) and Sánchez et al. (2018). One final noteworthy omission from the model is abstracting from involvement by international financial institutions (IFIs) and conditionality, as discussed by Boz (2011) and more recently by Fink and Scholl (2016). Emerging markets borrow from IFIs intermittently but these could potentially play an important role in renegotiations and during debt crises. In the case of the Greece 2012 event, the European Commission was actively involved in the restructuring process, as discussed in Zettelmeyer et al. (2013).

One key challenge in the study of maturity choice and long-term debt more broadly is the poor convergence properties of this class of models, across a wide range of numerical methods and strategies. Here we follow Sánchez et al. (2018) and Gordon (2019) and use discrete choice methods. Conceptually they are similar to Chatterjee and Eyigungor (2012)'s iid income shocks with continuous support, as they induce a comparable "randomization" across issuance choices in a way that loosens the tight interconnection between bond prices and issuance policies, but do so in a more tractable way and with more modest computational requirements. Appendix A reviews the method and details its use here.

We calibrate the model to Greece and its 2012 restructuring event, as well as key findings in a broader sample of events, detailed in Section 2. The behavior we aim to replicate is robustly observed across most if not all default episodes, at least since the early 1990s. The Greece 2012 event has the advantage of readily available high quality data on cash flows prior to and following the bond swap as well as haircuts at the instrument level, provided by Zettelmeyer et al. (2013). The parameter values resulting from the calibration are comparable with those reported in reference papers on maturity choice, suggesting that the results here carry over to a broader set of countries and circumstances.

The calibrated model mirrors the behavior of debt maturity in the data, both in terms of choice at issuance and regarding the lengthening of maturity during restructuring. In the data, maturity increases from 6.4 to 8.2 years (measured as Macaulay (1938) duration) while the benchmark model has maturity increasing on average from 6.4 to 10 years. The key assumption supporting this positive result is the inclusion of a post-renegotiation exclusion spell. Absent this feature, we find that a version of the model with immediate market reentry would instead induce a reduction in the maturity of debt, from 7.6 to 5.4 years. In addition to replicating the maturity extension, the model is consistent with the data more broadly: the haircuts applied to lenders following renegotiation are close to the values observed in the Greece 2012 event, an overall 65% haircut with substantial heterogeneity across maturities, debt-to-GDP in the 50% range, a countercyclical trade balance, and consumption more volatile than income. Moreover, model spreads are somewhat low in level and volatility compared to standard emerging market values, but comparable with the Greece data. In the model this is due in part to the mix of recovery and risk-neutral lenders. Hatchondo et al. (2016) assume a richer pricing kernel and can generate greater spread levels and volatility, in a similar model, albeit with exogenous recovery and no change in maturity upon restructuring.

There is one dimension in which the model has some difficulty in matching the data, the average share of short-term debt in total (i.e. the average maturity structure). To address this, in the benchmark specification we introduce a small adjustment cost that penalizes deviations of maturity from a target, average value. This cost breaks indifference over maturity choice in states of the world in which borrowing is essentially risk-free. We document that the maturity extension and the quantitative features of the model do not rely on the presence of this adjustment cost, by reporting calibrations of the benchmark model and the model with immediate reentry where this cost has been eliminated.

The rest of the paper proceeds as follows: Section 2 will summarize key stylized facts about restructuring of sovereign debt and maturity choice, Section 3 lays out the structure of the model and the definition of equilibrium, Section 4 contains the quantitative analysis of the model, both the benchmark version with post-renegotiation exclusion and a version with immediate market reentry, while Section 5 concludes. Two appendices detail the numerical method and report additional robustness checks, respectively.

2. Maturity and Debt Restructuring in the Data

We summarize key features of recent debt restructuring events, drawing on the relevant literature, and identify 4 main stylized facts against which to evaluation the quantitative and qualitative properties of the model.

I. Maturity Extension. We start with our main motivating observation, that virtually all cases involve a robust increase in the maturity of debt. Table 1 summarizes data from Fang et al. (2016) on all cases during the most recent 20 years. We find that debt is relatively short-term prior to restructuring and that, after the exchange, average maturity increases by roughly 6 years. Sturzenegger and Zettelmeyer (2018) confirm the importance of changes in maturity, as part of the rescheduling plan, for events in the '90s and early '00s, in their comprehensive account of defaults during this time frame. The fact that the maturity structure of the debt is altered during bond swaps is important for the haircuts suffered by creditors. Haircuts are implemented with a mix of outright reduction in face value and rescheduling of payments. Cruces and Trebesch (2013) report that many episodes feature no reduction in face value but rather haircuts are applied exclusively through rescheduling.

		Duration (Years)			Haircut	Face Value	Δ Average
	#*n*	Before	After	Δ	(%)	Cut (%)	Maturity
All Episodes	23	4.9	10.4	5.9	36.1	16.5	11.6
Excl. Preemptive	16	5.7	11.4	5.8	42.4	20.0	12.2
Greece 2012	1	6.7	17.7	11.0	64.6	53.5	12.0

Table 1: **Maturity Extension.** Average duration in years, before and after restructuring, haircuts, changes in the face value of debt, and the change in average maturity, across 23 cases (1994–2015). Source: Reinhart et al. (2016) and Fang et al. (2016). Cases marked as "preemptive" are bond swaps without prior default. "Average Maturity" is weighed using principal face value (note, *not* Macaulay duration).

Figure 1 compares the maturity structure of debt before and after the bond swap in the Greece 2012 event. Macaulay duration increases from 6.4 to 8.2 years while the average decay rate of payments decreased from about 18% to 4%. This latter measure can be understood as a data counterpart to the decay rate of payments used to model long-term debt in quantitative model of sovereign default, including in Section 3.

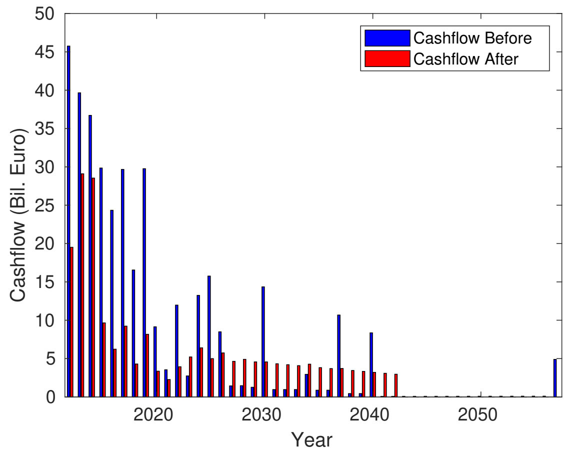

Figure 1: **Greece 2012 Cashflows.** Cashflows scheduled by outstanding, defaulted debt ("before") and resulting from the bond swap ("after"). The bond swap reschedules a substantial subset of payments after 2020 while reducing payments due within the first 5-10 years. Source: Zettelmeyer et al. (2013)

II. Market Exclusion, Pre- and Post-Restructuring. A second feature of particular interest to our quantitative exercise is the nature and length of market exclusion following default and after restructuring. Table 2 reports on this dimension of the data. We note that sovereigns who defaulted are excluded both prior and following their restructuring events. During this latter exclusion, the country is making payments on the new debt but is unable to tap international private markets. We find that countries are excluded on average 2 to 2.5 years prior to their restructurings and roughly 4 to 5 years in their aftermath. These magnitudes are robust to the exclusion of preemptive restructurings (which are coded with a "before" exclusion of 0) and of highly indebted poor countries (HIPC). Cruces and Trebesch (2013) compare the features of their more recent dataset to the results of Gelos et al. (2011) who consider a sample ending in 2000, in particular on whether harsher haircuts are followed by lengthier exclusions and higher spreads upon reentry.

III. Haircut Heterogeneity. Asonuma et al. (2017) study the recent 1999–2015 sample and conclude that short-term debt suffers higher haircuts compared to long-term instruments. They report that a 10-year instrument will receive a haircut that is 3-11% smaller than a 1-year instrument, depending on the empirical specification used. Haircut heterogeneity is particularly striking in the Greece 2012 case, as summarized by Table 3. The overall haircut applied was roughly 65%, with debt due within one year suffered haircuts in excess of 78-80%, while instruments due in 10+ years received haircuts of 50% or less. In the case of Greece, this robust, monotonic relation between

Length of Exclusion	#*n*	Years
Before Restructuring		
All Episodes	17	1.9
Excluding Preemptive	10	3.2
Excluding S&M 2004	16	2.6
After Restructuring		
All Episodes	67	5.0
Excluding HIPC	47	3.7

Table 2: **Exclusion Duration.** The average number of years between default and renegotiation ("before") and between renegotiation and market access ("after"). Based on the dataset of Cruces and Trebesch (2013), Table 2 and Table A2 in their Online Appendix. "S&M 2004" is the Serbia and Montenegro restructuring of 2004, listed only as having defaulted "since the 1990s." HIPC = "highly indebted poor country."

residual maturity and haircuts is induced by the "one-size-fits-all" nature of the swap, a format in which lenders receive shares of the new portfolio proportional to the face value of the bonds they hold, irrespective of residual maturity. Section 3.3 in the paper by Zettelmeyer and coauthors provides examples of other cases where such a format was used.

SZ Haircut (%)	
Residual duration	-2.90
	(0.10)
Constant	78.12
	(0.40)
#*n*	198
R^2	0.93

Table 3: **Haircut Heterogeneity, Greece 2012.** Regression of haircut on residual duration, at the level of individual bonds, in the Greece 2012 restructuring case. Source: Zettelmeyer et al. (2013). Heteroskedasticity-robust standard errors in parentheses.

IV. Countercyclical Maturity. Rodrik and Velasco (1999) and Broner et al. (2013) are among the first to document the systematic response of maturity to deteriorating conditions. They find that issuance of new debt shifts into short-term debt in response to increases in the cost of borrowing, during crises and weak fundamentals. Arellano and Ramanarayanan (2012) confirm this pattern by estimating yield curves and documenting the response of maturity and the term premium to worsening financial conditions.

Evidence on default events and their resolution, beyond the 4 stylized facts emphasized here, is provided by Sturzenegger and Zettelmeyer (2018, 2007) and Tomz and Wright (2013).

3. Model

We develop a quantitative sovereign default model with a choice of maturity both at issuance and as part of the post-default debt swap process. The model features two distinct types of market exclusion: first, after default but prior to renegotiating, and second, following the renegotiation, while making arrears payments on the restructured debt.

There are two types of agents: a risk-averse sovereign and a continuum of competitive, risk-neutral international lenders. In the tradition of Eaton and Gersovitz (1981), markets are incomplete in that the sovereign can borrow by issuing state-uncontingent instruments only, under lack of commitment. The sovereign can choose not to service its outstanding debt, in which case it enters a default state to be described below. Finally, market access is eventually regained following negotiations of a debt swap. We compare the mix of short- and long-term debt prior to default with the one resulting from the swap.

3.1. Debt Maturity and Payment Schedule

The sovereign can issue long-term bonds as in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), with a choice of two maturities, Short and Long, as in Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016). A unit of a bond of maturity $i \in \{S, L\}$ issued this period schedules an infinite stream of debt service payments, starting next period, given by

$$\kappa_i,\ (1-\delta_i)\kappa_i,\ (1-\delta_i)^2\,\kappa_i,\ (1-\delta_i)^3\,\kappa_i,\ \ldots$$

where κ_S and κ_L are scaling parameters, and by assumption short-term debt has a greater decay rate than the long-term bond, $\delta_S > \delta_L$. Let r be the risk-free interest rate at which lenders can borrow or lend internationally, assumed constant throughout. Then, the risk-free price for bond of maturity i is

$$q_i^{\text{rf}} = \kappa_i \sum_{\tau=0}^{\infty} (1+r)^{-(1+\tau)}\,(1-\delta_i)^{\tau} = \frac{\kappa_i}{\delta_i + r} \tag{1}$$

and the risk-free Macaulay duration is

$$D_i = \frac{1}{q_i^{\text{rf}}} \sum_{\tau=0}^{\infty} (1+\tau)\,(1+r)^{-(1+\tau)}\,(1-\delta_i)^{\tau}\,\kappa_i^{\tau} = \frac{1+r}{\delta_i + r}. \tag{2}$$

We use risk-free duration throughout, for both data and model, in order to focus on the changes in quantities, size and timing of payments, rather than movements in bond prices. We normalize the risk-free bond prices for both maturities to unity, $q_i^{\text{rf}} = 1$, by setting $\kappa_i = \delta_i + r$. This normalization leaves each bond's duration unchanged but facilitates comparisons of debt levels across maturities and simplifies expressions characterizing the overall debt stock.

For a portfolio consisting of b_S units of the short bond and b_L units of the long bond, the Macaulay duration can be computed as

$$\begin{aligned} D\left(b_S, b_L\right) &= \frac{1}{q_S^{\mathrm{rf}} b_S + q_L^{\mathrm{rf}} b_L} \sum_{\tau=0}^{\infty} (1+\tau)(1+r)^{-(1+\tau)} \left[(1-\delta_S)^{\tau} \kappa_S^{\tau} b_S + (1-\delta_L)^{\tau} \kappa_L^{\tau} b_L\right] \\ &= \frac{b_S}{b_S + b_L} D_S + \frac{b_L}{b_S + b_L} D_L = D_S + (D_L - D_S) \frac{b_L}{b_S + b_L}. \end{aligned} \tag{3}$$

For the computation of the duration of new issuance, i.e. excluding buy-backs, we follow Arellano and Ramanarayanan (2012) and define

$$D^{\text{Issue}}\left(\ell_S^+, \ell_L^+\right) = D_S + (D_L - D_S) \frac{\ell_L^+}{\ell_S^+ + \ell_L^+} \tag{4}$$

where $\ell_i^+ = \max\{0, \ell_i\}$ so that buy-backs (negative issuance) are not considered. Finally, for a bond trading a market price q_i, the yield-to-maturity spread s_i is implicitly defined by

$$q_i = \frac{\kappa_i}{\delta_i + (r + s_i)} \tag{5}$$

where the term in parentheses in the denominator is the yield-to-maturity of the bond.

3.2. The Sovereign

The sovereign starts each period with b_S and b_L outstanding units of the short and long bond respectively and an endowment realization y, drawn from a Markov process with support $\mathbb{Y}$. We use the convention that positive b is debt. The state is given by the tuple $\langle y, b_S, b_L\rangle \in \mathbb{Y} \times \mathbb{R} \times \mathbb{R}$. The government may decide to exert its default option and receive state-contingent value V^d. Alternatively, it can continue making debt service payments and achieve value V^p, in which case we say that the country is in good credit standing.

$$V(y, b_S, b_L) = \max_{d\in\{0,1\}} dV^d(y, b_S, b_L) + (1-d)V^p(y, b_S, b_L) \tag{6}$$

In each period with good credit standing, the sovereign will auction ℓ_S units of the short bond and ℓ_L units of the long bond. Whenever $\ell_i < 0$ the country is retiring, buying back some of its outstanding bonds. The stock of debt at the start of the period $\langle b_S, b_L\rangle$ and the new issuance $\langle \ell_S, \ell_L\rangle$ determine the stock of debt at the start of the next period $\left\langle b_S', b_L'\right\rangle$ via the stock-flow identity $b_i' = (1-\delta_i) b_i + \ell_i$, so that we can think of the government as either choosing issuance or the new debt stock.

The sovereign makes debt service payments and chooses consumption together with new issuance, subject to the budget constraint

$$c + \kappa_S b_S + \kappa_L b_L = y + q_S(y, b_S', b_L') \ell_S + q_L(y, b_S', b_L') \ell_L - \phi(b_S', b_L'), \tag{7}$$

taking the bond price schedules q_S and q_L as given. ϕ is an adjustment cost function that penalizes deviations from the targeted *average* maturity of debt, as in Bocola and Dovis

(2020). The role of this function is to anchor average maturity to the value in the data by breaking the near-indifference over maturity structure in the risk-free region. We report results with and without this cost function. The value achieved under continued debt service satisfies

$$\begin{aligned} V^p(y, b_S, b_L) = \max_{c, b'_S, b'_L} & \, u(c) + \beta\, \mathbb{E}_{y'|y}\, V(y', b'_S, b'_L) \\ \text{s.t. } & b'_i = (1-\delta_i)\, b_i + \ell_i, \;\; \text{for all } i \in \{S, L\} \\ & \text{and (7)} \end{aligned} \tag{8}$$

The choice of default triggers an endowment penalty and temporary exclusion from world markets. For an endowment realization of y, the country's actual income is $h(y) \leq y$ whenever the sovereign is in default. Afterwards, the country will bargain with its creditors with probability η each period. Following this restructuring process, the country resumes debt service payments but only reenters financial markets with probability η^a. This implies that on average the country will be excluded from financial markets for $1/\eta$ periods before restructuring and for an additional $1/\eta^a$ periods following the resumption of debt service payments. Thus, the value achieved while in default, prior to restructuring satisfies

$$V^d(y, b_S, b_L) = u[h(y)] + \beta\, \mathbb{E}_{y'|y}\{(1-\eta)\, V^d(y', b_S, b_L) + \eta V^a(y', \gamma_S, \gamma_L)\}. \tag{9}$$

where $\langle \gamma_S, \gamma_L \rangle$ is the new debt owed to the creditors, following the bond swap, to be characterized shortly.

After restructuring, the country resumes payments, while still unable to tap international market. We allow the country to pay down its debt faster than required by the decay rate of each maturity, but not to borrow ($\ell_i \leq 0$). Then, the value achieved following the renegotiation is given by

$$\begin{aligned} V^a(y, b_S, b_L) = \max_{c, b'_S, b'_L} & \, u(c) + \beta\, \mathbb{E}_{y'|y}\{\eta^a V(y', b'_S, b'_L) + (1-\eta^a) V^a(y', b'_S, b'_L)\} \\ \text{s.t. } & \ell_i = b'_i - (1-\delta_i)\, b_i \leq 0, \;\; \text{for all } i \in \{S, L\} \\ & c + \kappa_S b_S + \kappa_L b_L = y + q^a_S(y, b'_S, b'_L)\, \ell_S + q^a_L(y, b'_S, b'_L)\, \ell_L \end{aligned} \tag{10}$$

The bond price schedules q^a_i reflect both the country's temporary exclusion from borrowing but also its eventual return to market and any future defaults. We will characterize all bond price schedules once we lay out the structure of renegotiation.

For reference, we denote $\mathcal{B}_i(y, b_S, b_L)$ the borrowing policies while in good credit standing and $\mathcal{B}^a_i(y, b_S, b_L)$ the policies while excluded, following the renegotiation.

3.3. Debt Renegotiation and Haircuts

The government and its bond holders agree to swap the defaulted debt $\langle b_S, b_L \rangle$ for a new portfolio $\langle \gamma_S, \gamma_L \rangle$. We model the debt swap negotiation using the Generalized Nash Bargaining solution. We assume that all creditors are represented by a single committee which aims to maximize the overall value of the new bonds. We discuss the allocation of the new bonds to the old bond holders shortly.[1] We set the threat point for the government to the value it can achieve in permanent autarky while subject to the output loss penalty. The corresponding threat point for the creditors is 0, i.e. no recovery. This is standard but possibly not without loss of generality.

$$V^{\text{aut}}(y) = u\left[h(y)\right] + \beta_{y'|y}\,\mathbb{E}\,V^{\text{aut}}(y') \tag{11}$$

The sovereign's surplus from successfully renegotiating, with resulting bonds $\langle\gamma_S, \gamma_L\rangle$, is given by the difference between the value associated with resuming payments and the one for autarky,

$$\Delta_{\text{Sov}}(y, \gamma_S, \gamma_L) = V^a(y, \gamma_S, \gamma_L) - V^{\text{aut}}(y) \tag{12}$$

The surplus of international creditors is given by the the market value of the new debt,

$$\Delta_{\text{Cre}}(y, \gamma_S, \gamma_L) = \left[\kappa_S + (1-\delta_S)q_S^a(y, \gamma_S', \gamma_L')\right]\gamma_S + \left[\kappa_L + (1-\delta_L)q_L^a(y, \gamma_S', \gamma_L')\right]\gamma_L \tag{13}$$

This expression reflects the fact that payments resume immediately in the renegotiation period, and that the sovereign will eventually resume borrowing and potentially default again, so that the value of the tail of payments is priced at q_i^a. The tail is evaluated at $\gamma_i' = \mathcal{B}_i^a(y, \gamma_S, \gamma_L)$ since the sovereign has the option to start paying down the new debt immediately.

The Nash program for a sovereign that defaulted with outstanding debt $\langle b_S, b_L\rangle$ and current endowment realization of y is

$$\begin{aligned} &\underset{\gamma_S,\gamma_L}{\arg\max}\ \left[\Delta_{\text{Sov}}(y, \gamma_S, \gamma_L)\right]^{\alpha}\left[\Delta_{\text{Cre}}(y, \gamma_S, \gamma_L)\right]^{1-\alpha} \\ &\quad \text{s.t. } \Delta_{\text{Sov}} \geq 0 \text{ and } \Delta_{\text{Cre}} \geq 0 \end{aligned} \tag{14}$$

where α is the sovereign's bargaining power parameter. Note that the defaulted debt portfolio (b_S, b_L) does not enter the Nash program and, as a consequence, γ_S and γ_L are functions of the endowment realization y alone. This further implies that the value of default is independent of the defaulted portfolio. This is Yue (2010)'s *"bygones are bygones"* result. Nash bargaining maximizes and splits the total surplus, an inherently forward-looking object. The value created by resolving the default is independent of pre-default debt levels, as the lenders have the ability to "hold up" the sovereign regardless of whether it defaulted on more or less debt.[2]

A final remark on the bargaining setup here is that the joint surplus can always be positive, in all states of the world, so that conditional on negotiating, the program (14) always has a solution and the threat point is never visited in equilibrium. This is related to the observation that because lenders have an outside option of zero, any strictly positive but arbitrarily small recovery will induce positive surpluses for both parties.

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[1] It can be shown easily that the same bargaining outcome, in terms of maturity composition and haircuts, can be achieved if we were to allow short and long debt holders to form separate committees, with distinct bargaining powers, as long as the sovereign's bargaining power is fixed. This result reflects the homogeneity of lenders in terms of attitude towards risk and outside options, as assumed here.

[2] Wang (2019) finds that replacing Nash bargaining with the Kalai-Smorodinsky axioms breaks the "bygones" result and introduces a role for legacy debt in renegotiations.

The policies resulting from the solution to (14) are denoted $\Gamma_i(y)$. The value of default can be reduced to

$$V^d(y) = u[h(y)] + \beta\, \mathbb{E}_{y'|y}\{(1-\eta)\, V^d(y') + \eta V^a[y', \Gamma_S(y'), \Gamma_L(y')]\}. \tag{15}$$

The new bonds are allocated to the old bond holders using the following rule, motivated by the evidence on heterogeneous haircuts in Asonuma et al. (2017) and the one-size-fits-all offer in the Greece 2012 event: the old short bonds receive a share $\mathcal{S}_S = \mu_S b_S / (\mu_S b_S + \mu_L b_L)$ of the new portfolio, made up of units of both maturities, while the rest are assigned to the old long bonds, a share $\mathcal{S}_L = 1 - \mathcal{S}_S$. On a per-unit basis, each old unit of the short bond is swapped for $\mathcal{S}_S \gamma_S / b_S$ new units of short and $\mathcal{S}_S \gamma_L / b_S$ units of long. We normalize $\mu_L = 1$ so that μ_S is a parameter controlling the "priority" of a short bond units relative to long-term units. We define the aggregate haircut as the percent change in risk-free, present face value

$$H_{\text{Agg}} = 1 - \frac{\Delta^{\text{rf}}_{\text{Cre}}}{q^{\text{rf}}_S b_S + q^{\text{rf}}_L b_L} = 1 - \frac{(1+r)(\gamma_S + \gamma_L)}{b_S + b_L}. \tag{16}$$

or, alternatively, using market prices

$$H^{\text{Mrkt}}_{\text{Agg}} = 1 - \frac{\Delta_{\text{Cre}}}{q^a_S(y, \gamma_S, \gamma_L)\, b_S + q^a_L(y, \gamma_S, \gamma_L)\, b_L} \tag{17}$$

Similarly, we define the haircut applied to maturity $i \in \{S, L\}$ as

$$H_i = 1 - \mathcal{S}_i \frac{\Delta^{\text{rf}}_{\text{Cre}}}{q^{\text{rf}}_i b_i} = 1 - \mathcal{S}_i \frac{(1+r)(\gamma_S + \gamma_L)}{b_i} \tag{18}$$

and a corresponding version based on market prices,

$$H^{\text{Mrkt}}_i = 1 - \mathcal{S}_i \frac{\Delta_{\text{Cre}}}{q^a_i(y, \gamma_S, \gamma_L)\, b_i}. \tag{19}$$

The $1 + r$ term in the numerator of the risk-free expressions reflects the timing of the renegotiation: payments are resumed immediately, rather than next period.

3.4. Lenders and Bond Prices

We assume that international investors are competitive, risk-neutral, and that they can borrow or lend freely at a constant risk-free rate. Then, in order for them to break even, the bond price for maturity i must satisfy

$$\begin{aligned} q_i(y, b'_S, b'_L) = \frac{1}{1+r}\, \mathbb{E}_{y'|y}\{ & d(y', b'_S, b'_L)\, \chi_i(y', b'_S, b'_L) \\ & + [1 - d(y', b'_S, b'_L)]\, [\kappa_i + (1-\delta_i)\, q_i(y', b''_S, b''_L)]\} \end{aligned} \tag{20}$$

here $b''_i = \mathcal{B}_i(y', b'_S, b'_L)$ is the stock of maturity i that the sovereign will choose next period, conditional on not defaulting, d is the sovereign's default policy function and χ is the expected recovery rate, implied by the debt swap procedure described in the previous section, which satisfies

$$\chi_i\left(y, b'_S, b'_L\right) = \frac{1}{1+r}\,\mathbb{E}_{y'|y}\left\{(1-\eta)\,\chi_i\left(y', b'_S, b'_L\right) + \eta\mathcal{S}_i\,\frac{\Delta_{\mathrm{Cre}}\left(y', \gamma_S, \gamma_L\right)}{b_i}\right\} \tag{21}$$

where $\gamma_i = \Gamma_i(y')$ is the the new debt of maturity i bargained in renegotiation. Finally, the debt serviced while still excluded, prior to the eventual return to market, is priced using bond price schedules that satisfy

$$q_i^a\left(y, b'_S, b'_L\right) = \eta^a q_i\left(y, b'_S, b'_L\right) + (1-\eta^a)\frac{1}{1+r}\{\kappa_i + (1-\delta_i)\,\mathbb{E}_{y'|y}\,q_i^a\left(y', b''_S, b''_L\right)\} \tag{22}$$

where $b''_i = \mathcal{B}_i^a(y', b'_S, b'_L)$, using the relevant borrowing policies. Note that with probability η^a the sovereign accesses markets, can default again, and the relevant value of the bond is the one under good credit standing (q_i).

The Model without Post-Restructuring Exclusion. In the analysis of our results, we will stress the role of the post-restructuring exclusion for maturity extension. To emphasize this point, we will compare our benchmark model with an alternative version in which default is followed by renegotiations and market access immediately, so that

$$V^d\left(y, b_S, b_L\right) = u\left[h\left(y\right)\right] + \beta\,\mathbb{E}_{y'|y}\{(1-\eta)\,V^d\left(y', b_S, b_L\right) + \eta V^p\left(y', \gamma_S, \gamma_L\right)\}. \tag{23}$$

and the relevant surpluses during bargaining are

$$\Delta_{\mathrm{Sov}}\left(y, \gamma_S, \gamma_L\right) = V^p\left(y, \gamma_S, \gamma_L\right) - V^{\mathrm{aut}}\left(y\right) \tag{24}$$

and

$$\Delta_{\mathrm{Cre}}\left(y, \gamma_S, \gamma_L\right) = \left[\kappa_S + (1-\delta_S)q_S\left(y, \gamma'_S, \gamma'_L\right)\right]\gamma_S + \left[\kappa_L + (1-\delta_L)q_L\left(y, \gamma'_S, \gamma'_L\right)\right]\gamma_L \tag{25}$$

for the sovereign and lenders, respectively. Since the post-restructuring exclusion regime is never visited, its associated value function (V^a), bond prices (q_i^q), and policies ($\mathcal{B}_i^a$) are now redundant. The structure of the model remains otherwise unaltered.

3.5. *Equilibrium*

Let $\mathbb{S} = \mathbb{Y} \times \mathbb{R} \times \mathbb{R}$ denote the state. A Recursive Markov Equilibrium consists of

(a) Value functions $V, V^a, V^p : \mathbb{S} \to \mathbb{R}$, $V^d : \mathbb{Y} \to \mathbb{R}$
(b) Default $d : \mathbb{S} \to \{0, 1\}$ and borrowing policies $\ell_S, \ell_L, \mathcal{B}_S, \mathcal{B}_L, \mathcal{B}_S^a, \mathcal{B}_L^a : \mathbb{S} \to \mathbb{R}$,
(c) Nash solution policies $\Gamma_S, \Gamma_L : \mathbb{Y} \to \mathbb{R}$,
(d) Bond price schedules and recovery rates $q_S, q_L, q_S^a, q_L^a, \chi_S, \chi_L : \mathbb{S} \to \mathbb{R}$

such that

1. The value functions satisfy equations (6), (8), (9), and (10),
2. The sovereign's policies solve programs (6), (8), and (10),
3. The Nash solution solves the program (14),
4. International investors break even and bond prices satisfy (20), (22), and (21).

4. Quantitative Analysis

4.1. Calibration

We calibrate the model to a yearly frequency. The constant risk-free rate is set to 3.2%, as in Arellano and Ramanarayanan (2012). We set $\delta_S = 1$ and $\delta_L = 0.071$ so that the two maturities have risk-free Macaulay durations of 1 and 10 years respectively. The short-term bond is thus one-period debt. We will compare model spreads with yields for Greece relative to comparable German instruments. Over the sample period German yields are trending downwards and near zero, therefore we use a reference value for the model risk-free rate in order to avoid having to augment the model with risk-free rate dynamics consistent to the ones in the data. Hatchondo et al. (2016) study a related version of the model with risk-averse lenders and a time-varying risk-free rate, but restrict attention to an exogenous recovery rate, without a change in maturity.

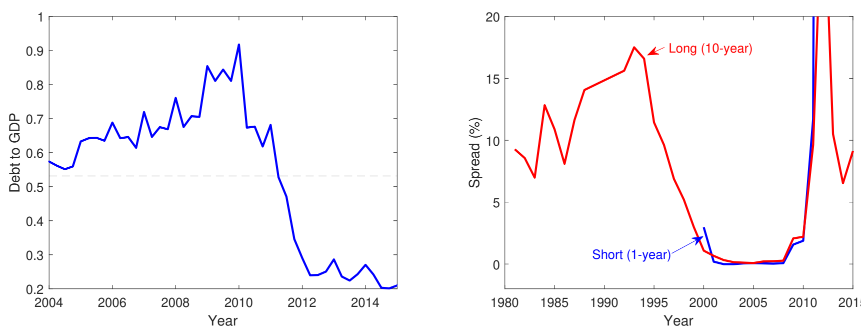


Figure 2: **Greece Data.** *Left Panel:* External debt to GDP, bonds and notes only, (solid blue) and sample average (dashed black). *Right Panel:* 1-year and 10-year spreads (over German bonds). Source: Global Financial Data (2015).

The endowment process is assumed AR(1), its parameters (autocorrelation ρ and innovation standard deviation σ_ε) are estimated using OECD National Accounts data and the process is discretized using the standard Tauchen (1986) method.

$$\log y_t = \rho \log y_{t-1} + \sigma_\varepsilon \varepsilon, \quad \varepsilon \sim \text{ iid } \mathcal{N}(0,1) \tag{26}$$

All annual data is detrended using the Hodrick-Prescott filter, with a parameter value of 100. The data counterpart for the endowment process is GDP minus Gross Capital Formation, given that we abstract from investment and production. The utility function has a constant coefficient of relative risk aversion σ, set to 2 in line with the literature,

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} = 1 - c^{-1} \tag{27}$$

The default output cost follows Chatterjee and Eyigungor (2012),

$$h(y) = y - \max\left\{0,\ \lambda_0 y + \lambda_1 y^2\right\}. \tag{28}$$

with $\lambda_0 \leq 0 \leq \lambda_1$ and no output cost for endowments draws lower than $-\lambda_0/\lambda_1$. The adjustment cost for deviations from the average maturity structure is given by

$$\phi(b'_S, b'_L) = \iota\left(\frac{b'_S}{b'_S + b'_L} - \overline{S}\right)^2 \tag{29}$$

where the parameter ι controls the average magnitude of this cost.

The remaining parameters, related to discounting (β), the output cost of default (λ_0, λ_1), length of exclusions (η, η^a), bargaining parameters (α, μ_S) and mean maturity ($\iota, \overline{S}$) are picked to match key moments: total debt to GDP, trade balance countercyclicality, the relative volatility of consumption to endowment, the volatility of short-term spreads, the lengths of exclusion pre- and post-swap, overall haircut, the haircut applied to short-term debt, and the average share of short-term debt in total. Tables 4 and 5 include the calibrated parameter values while Table 6 reports the model's fit over the targeted and untargeted moments. Table B.8 in Appendix B documents the robustness of the main results to key alterations to the calibration.

	Value	Description
σ	2	CRRA
r	3.2%	Lenders' risk-free rate
δ_S	1.0	1-year short duration
δ_L	0.071	10-year long duration
ρ	0.9	Endowment autocorrelation
σ_ε	0.02	Endowment variance

Table 4: **Calibration I.** Parameters calibrated outside the model.

We follow the same strategy in the calibration of the model with immediate market access. The main difference is due to the calibration of the pre-renegotiation exclusion, which is targeted to the overall exclusion in the data, an expected exclusion of 7 years. Table B.9 in the Appendix reports the robustness checks for this model.

We remark that the data moments for Greece are not dissimilar from reference values used in the literature on emerging markets, in particular consumption is more volatile than income, the trade balance to GDP ratio is countercyclical, and recovery rates are in line with the full sample but somewhat lower for Greece 2012. The main difference relates to bond yield spreads, which are lower for Greece than compared to the prototypical emerging market. The data values are sensitive to the inclusion of observations in the period of zero or near-zero spreads prior to the crisis, a time during which Greece borrowed at similar rates to Germany, as seen in the right panel of Figure 2. In the model, we can replicate low spreads due to the presence of substantial recovery. Appendix B shows that spreads would be higher if haircuts were smaller, closer to the full sample average of default events.

Ruling Out Certain Default. We further restrict problems (8) and (14), for good credit standing issuance and the Nash bargaining respectively (the latter only for the case of immediate return to markets) with a minimum bond price for long-term debt. This prevents, as discussed in Hatchondo et al. (2016), booms in consumption prior to default

	Benchmark Model	Immediate Reentry	Targeted Moment
β	0.94	0.935	Total debt to GDP
λ_0	–0.85	–0.7	Corr NX/GDP, GDP
λ_1	1.0	0.78	Std C / Std GDP
ι	0.02	0.01	Std spread Short
$\overline{S}$	0.33	0.33	Share of Short in total debt
η	0.33	0.14	Pre-restructuring exclusion
η^a	0.25	—	Post-restructuring exclusion
α	0.945	0.9	Haircut, overall
μ_S	0.5	0.45	Haircut, 1-year

Table 5: **Calibration II.** Parameters set as to match selected moments.

in states of the world with low endowment draws, in which the country would want to receive from lenders the discounted present value of recovery today, and default next period with probability 1. This happens whenever the maximum amount of resources lenders would be willing to transfer to the country ($q_S b'_S + q_L b'_L$) is weakly lower than the expected recovery value ($\chi_S b'_S + \chi_L b'_L$), but equal to it for arbitrarily high borrowing. In these low endowment states, the output cost of default $y - h(y)$ is low or null, due to the concavity of h, so that default would be deterred by exclusion alone. Quantitatively, for values of y that are $2.5 - 3$ standard deviations below mean, the benefits from promising certain default and receiving the recovery value now outweighs the cost of financial market exclusion, hence the necessity of implementing such a limit. We set the floor on q_L to 0.7, following Hatchondo et al. (2016), corresponding to a maximum long-term spread of about 17%. In equilibrium this constraint is not binding, as default now dominates in the relevant states.

4.2. Results

We compute the model using the methods detailed in Appendix A, following Sánchez et al. (2018) and Gordon (2019). Table 6 reports key moments, for two cases, with and without post-renegotiation exclusion, both with and without the adjustment cost for deviations from average maturity. We approximate the model's ergodic distribution by simulating the model for a large number of periods (150,000) and drop an initial segment (5,000). We then report moments computed over all periods such that the country is in good credit standing, in that period and 3 periods prior.[3] Table 6 documents that the model can generate targeted moments in line with the data while

[3]This additional requirement is imposed in order to eliminate the influence of behavior following renewed market access: reentry is associated with a small number of periods with unusually high borrowing, as the country quickly returns to levels of debt closer to average. Without excluding these outlier periods, the computed correlation of net exports to GDP and GDP is slightly higher (closer to zero or small and positive). Outside of these particular circumstances, the trade balance is counter-cyclical, as induced by the standard bond pricing mechanism. The other moments are largely unaffected by this selection criterion.

also inducing a maturity extension consistent with the broad evidence reported in Section 2. At the same time, the model features spreads (level and volatility, short- and long-term debt) comparable with Greece data.[4] The table reports 4 calibrations: the benchmark model, with post-renegotiation exclusion and the adjustment cost ι, a version of the model with immediate reentry and the adjustment cost, and versions of the two models in which the adjustment cost has been set to zero and all other parameters unaltered. In the Greece 2012 data, maturity was extended from 6.4 to 8.2 years while the benchmark model generates an increase from 6.4 to 10.0 years. This value implies that the Nash solution calls for lenders to be compensated exclusively with long-term debt. In contrast, with immediate market reentry, maturity is counterfactually reduced, from 7.6 to 5.4 years. In the cases without adjustment cost, even though the model does not match average maturity, the main findings concerning maturity choice during the swap are unaffected: the benchmark model with post-restructuring exclusion features maturity extension while the one with immediate reentry behaves counterfactually on this dimension.

Maturity Choice. Figure 3 summarizes the model's maturity choice behavior. It plots average duration chosen at various level of endowment, both during good credit standing, with normal market access, and during the bond swap. The behavior is consistent with the data in two key dimension: first, the model generates a reduction of maturity in bad times, low endowment realizations induce portfolio choices with a greater share of short-term debt and therefore lower duration, and second, the bond swap compensates lenders using long-term debt only, so that maturity is extended in renegotiation. Table 7 reports regressions of the chosen duration (for either the stock of debt or issuance alone) on short-term spreads or the state variables, using model-generated data. Consistent with our forth stylized fact in Section 2, maturity is shortened during times of high spreads and when the endowment realization is low.

Bond Pricing and Dilution. Figure 4 replicates a key finding in the maturity choice literature. It plots the two bond price schedules, for Short- and Long-term debt against the corresponding own quantity, keeping the debt of the other maturity constant at zero. The Short bond price schedule offers better prices (lower yields) than the corresponding Long schedule, but in a way that is more elastic (steeper) to the amount borrowed. Aguiar et al. (2019) discuss the implications of these elasticity differences for incentives to borrow and the maturity structure.

Analysis. The use of long-term debt in restructuring reflects the incentives for consumption-smoothing while in post-renegotiation exclusion. Since short-term debt is one-period, its use requires an immediate payment to the lenders, at the time of the swap, without the country having access to additional borrowing through financial markets. Given the targeted level of haircuts, using short-term debt would call for a substantial reduction in consumption at the time of the swap. In contrast, the use of long-term debt requires

[4]The statistics of spreads reported in the Greece 2012 column are computed relative to analogous instruments for Germany, over 2000–2012 due to data limitations. The right panel of Figure 2 plots 1-year and 10-year spreads for Greece from Global Financial Data (2015).

	Data	Benchmark Model	Immediate Reentry	No Adj. $\iota = 0$	Immediate $\iota = 0$
Targeted					
Std C / Std GDP	1.2	1.2	1.2	1.2	1.2
Corr NX/GDP, GDP	–0.15	–0.26	–0.28	–0.27	–0.26
Share Short in Total Debt	0.33	0.34	0.32	0.70	0.66
Total Debt to GDP	0.53	0.49	0.40	0.47	0.42
Std Short (1-year)	1.0	1.0	1.1	0.2	0.3
Pre-restruct. Exclusion	3.0†	3.0	7.0	3.0	7.0
Post-restruct. Exclusion	4.0†	4.0	—	4.0	—
Haircut, overall	0.65	0.65	0.62	0.64	0.65
Haircut, 1-year	0.75	0.78	0.80	0.70	0.76
Bond Swap					
Duration Pre-restruct.	6.4††	6.4	7.6	2.8	5.4
Duration Post-restruct.	8.2††	10.0	5.4	10.0	5.3
Haircut, 10-year	0.50	0.56	0.55	0.38	0.51
Spreads (%)					
Mean Short (1-year)	0.6	0.3	0.5	0.1	0.2
Mean Long (10-year)	0.7	0.2	0.4	0.1	0.1
Std Long (10-year)	0.8	0.2	0.2	0.1	0.1

Table 6: **Moments.** Comparison of key moments in the data with model ergodic distribution statistics. Data column is Greece, except values marked with † which are computed with the Cruces and Trebesch (2013) dataset in Section 2. Business cycle moments are based on yearly OECD data over 1980–2010, spreads are computed over 2000-2012, Debt to GDP is External Debt from Global Financial Data (2015), averaged over 2004–2015. See Figure 2 for time paths of these series. †† The maturity extension is from 6.4 to 8.2 years for Greece 2012 and from 4.9 to 10.4 in the full sample of recent events (under a modestly different definition). Model numbers are sample statistics from a 150,000 period simulation, after dropping the initial 5,000 observations/years. Observations are included in the calculation if the country is in good credit standing in the current period and in 3 periods prior.

	D_{t+1}	D_{t+1}	D^{Issue}_{t+1}	D^{Issue}_{t+1}
$\text{Sp}_{S,t+1}$	–29.57		–10.51	
	(0.18)		(0.34)	
y_t		1.46		9.47
		(0.08)		(0.12)
$b_{S,t}$		–2.90		4.04
		(0.08)		(0.12)
$b_{L,t}$		0.64		–13.58
		(0.07)		(0.11)
$\#n$	140,533	140,533	140,533	140,533
R^2	0.16	0.03	0.01	0.24

Table 7: **Maturity Choice.** Regressions of the chosen duration on either the short-term spread or on state variables (fundamentals), with simulated data from the Benchmark model. Stock measures reflect duration of the stock of debt (b'_S, b'_L) while the Issue(ance) measures capture issuance excluding buy-backs (ℓ^+_S, ℓ^+_L).

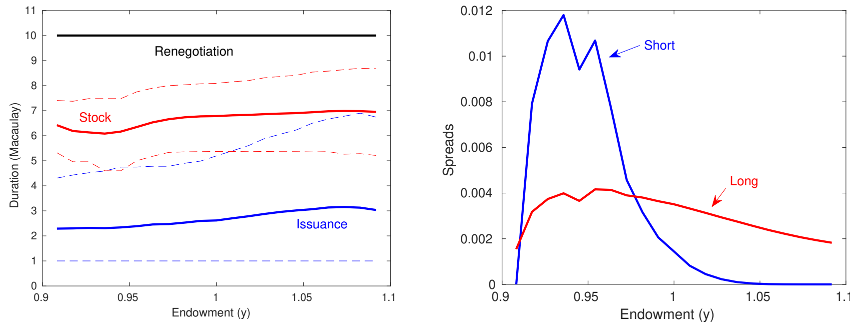


Figure 3: **Maturity Choice and Spreads.** *Left Panel:* Average Macaulay duration chosen during good credit standing, stock (blue) and new issuance (red), and as part of the bond swap when debt is renegotiated (black), across the level of the endowment, averaged over the simulation and 0.1 and 0.9 percentiles (dashed lines). For the same endowment level (y), different maturity mixes are chosen at different times due to variation in the maturity structure of outstanding debt and due to the discrete choice shocks used in computation. See Appendix A for a discussion of the method. *Right Panel:* Average Short- (blue) and Long-term (red))debt spread, versus endowment, conditional on good credit standing.

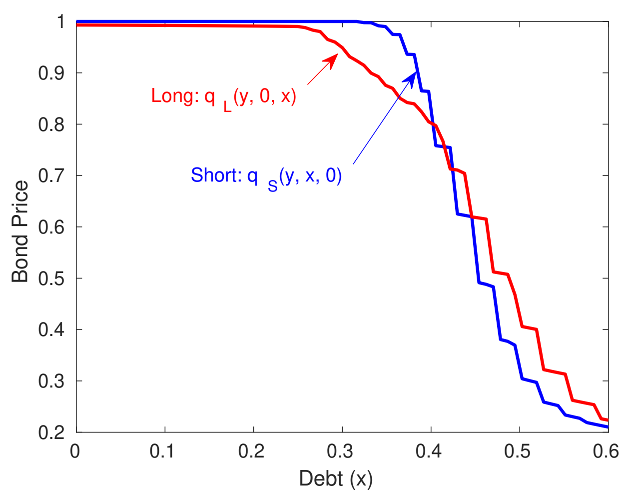


Figure 4: **Bond Price Schedules.** The price of Short- and Long-term debt, against own quantity, keeping other maturity at zero. Short-term borrowing offers better prices yet it is more sensitive to the issued quantity, the q_S schedule is steeper.

small losses of consumption, not only at the time of the renegotiation but in all subsequent periods. Moreover, since the country continues to be excluded for markets, these payments are safe from additional dilution, at least until the country's eventual access to new issuance. In practice, these forces are reflected in the sovereign's surplus in renegotiation $\Delta_{\text{Sov}}(y,\gamma_S,\gamma_L)$, in particular its steeper slope in γ_S than in γ_L, as displayed in Figure 5. The net result is that the use of exclusively long-term debt is an effective way to deliver value to lenders while at the same time avoiding large, costly jumps in consumption around the swap. Behavior in the model with immediate market re-access is markedly different: in this setting the sovereign would dilute the lenders' long-term claim, the same period, by borrowing in excess of the newly renegotiated debt. This additional source of inefficiency for long-term debt, as discussed by Hatchondo et al. (2016), leads the Nash bargaining to select short-term debt as the preferred instrument with which to compensate lenders. This will not require a sizable consumption drop on the part of the country as it can and does use financial markets to borrow, unlike in the full model. This finding highlights the role of dilution not only at the time of issuance, during good credit history, but also in terms of evaluating post-restructuring behavior and lenders' recovery.

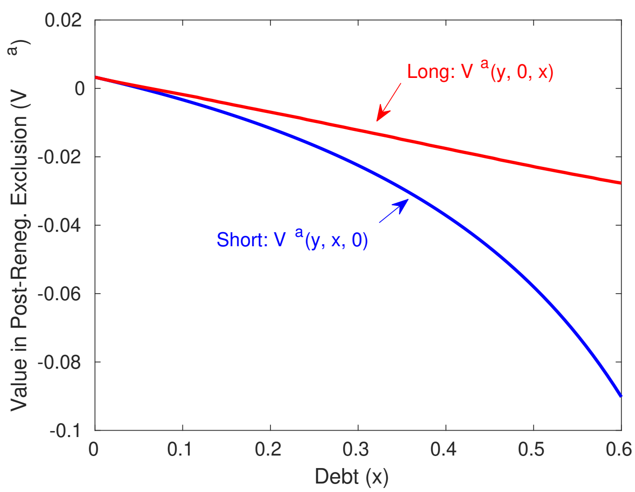


Figure 5: **Sovereign's Value in Renegotiation.** The value achieved by the sovereign in post-renegotiation exclusion V^a is substantially steeper in short-term debt compared to long-term obligations, reflecting the inability to tap international markets for roll-over and therefore the need for a larger consumption reduction.

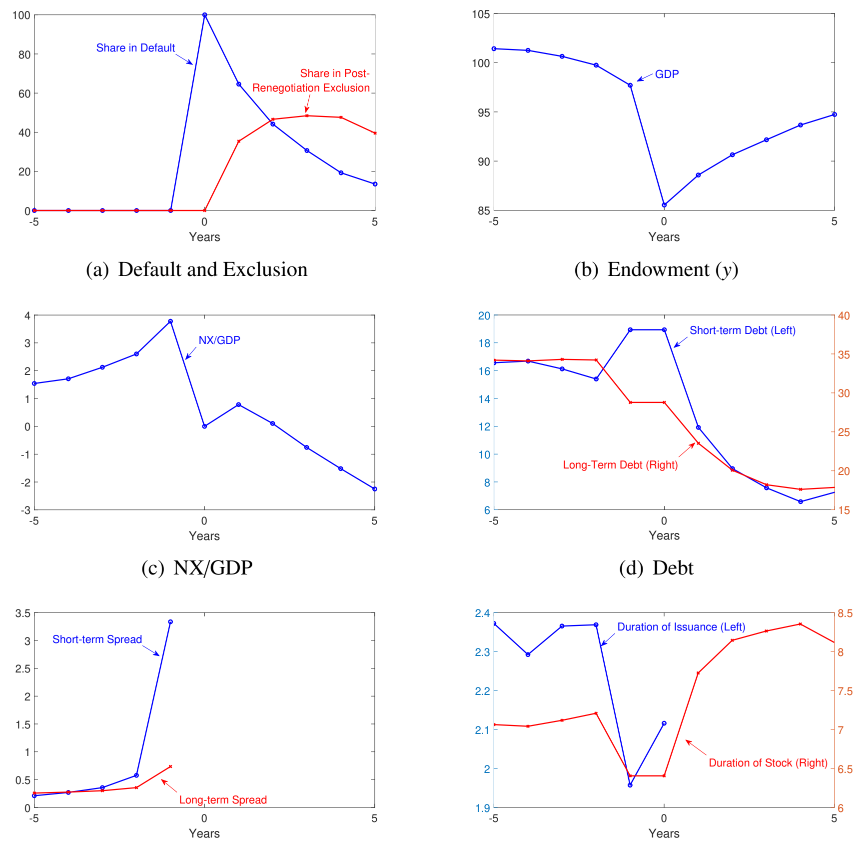
Event Study. To better showcase dynamics around defaults, Figure 6 plots a synthetic event by averaging over the default episodes in a long simulation of the benchmark model. We restrict attention to defaults that follow at least 5 periods of market access. Default happens in year 0 and we display 5 years before and after the default. In panel (a) we see the share of "economies" in default in blue, which spikes to 100% in year 0, as well as the share of "economies" in post-renegotiation exclusion. The remainder have normal market access. By 5 years after the default, about 10% of "economies" are still in post-default exclusion while under 40% are in post-renegotiation exclusion, with over 50% accessing international markets again. Panel (b) plot the time path of the endowment (or GDP) over this event. Defaults are preceded by low endowment realizations and cause a substantial drop in endowment due to the penalty function $h(y)$. The benefits of defaulting are apparent in panel (c), which plots the ratio of Net Exports to GDP. The country is a net exporter prior to its default, servicing the debt in the face of worsening income and a tight bond price schedules. Upon default, the current account balances as debt service is suspended. Panel (d) plots the level of debt by maturity (short and long), while panel (f) shows the associated Macaulay duration for both debt issuance and the stock of debt. Prior to default, the sovereign reduces maturity (notably in year -1), as duration is procyclical. As the country renegotiates, maturity is extended to levels greater than in the years prior to default (e.g. a comparison of years -4 through -1 to 3 and later). Finally, panel (e) plots short- and long-term spreads in the run-up to default. On average spreads are low but increase as default becomes more likely, with a greater response by short-term spreads (going from 0.2 to 3.5% over roughly 3 years), reflecting the higher volatility of short-term yields and the lower recovery rate for this instrument. Long-term spreads also increase, from about 0.2% to over 0.6%, just prior to default.

5. Conclusion

We have argued that a standard model of maturity choice for sovereign debt, once augmented with post-default renegotiations, replicates major features of bond swaps in the data, including the extension of maturity and haircut heterogeneity. We propose that the continued exclusion of the country from financial markets after restructuring plays a critical role in rationalizing the use of long-term debt. In contrast, if a sovereign were to immediately resume borrowing, the resulting dilution of the long-term debt would render it an unattractive instrument during renegotiation, and the country would instead repay lenders largely with a one-time, lump-sum payment, corresponding to the short-term bond. Lengthening the maturity of debt strikes a balance between the country's consumption-smoothing concerns, in light of continued lack of access to markets, and the need to compensate lenders, as to eventually regain good credit standing.

Limitations and Future Work. In establishing these results we have assumed that market exclusion, prior to and following renegotiation, is exogenous and of random length. As a side-effect, the model cannot address the question of whether sovereigns are unable or unwilling to borrow in international markets following debt swaps. In the model, if possible the sovereign would choose to resume borrowing immediately (due to relative impatience) more so since bond prices would be high following the reduction in debt burden in renegotiation. The Markov structure of the equilibrium and the absence of private information create no scope for reputation or other mechanisms that could rationalize the observed absence of new borrowing. Finally, the model does not incorporate a role for IFIs such as the IMF and for conditionality[5] both at the time of

[5]As studied in Boz (2011) and Fink and Scholl (2016) and documented by Reinhart and Trebesch (2016).



(a) Default and Exclusion

(b) Endowment (y)

(c) NX/GDP

(d) Debt

(e) Spreads

(f) Duration

Figure 6: **Default Event Study.** Averages of key variables over a 11 year window around default events. Default period is normalized to 0. We collect all default events in the main, long simulation of the model conditional on no default or exclusion in the 5 years prior.

issuance under normal market access, or throughout the restructuring process. These limitations suggest productive avenues for future work on maturity choice and resolution of default episodes.

Appendix A. Numerical Solution

Models with long-term debt or maturity choice raise substantial computational challenges, in part due to the role of dilution in bond pricing. With long-term debt, the bond price schedule reflects the market value of the tail of payments, evaluated at equilibrium issuance policies. In parallel, issuance policies are determined in response to the bond price schedules. This interdependence induces divergent behavior during computation. Several approaches in the recent literature have addressed this issue by creating some residual uncertainty in issuance policies by means of small iid shocks. Chatterjee and Eyigungor (2012) use such a shock for income with continuous support, weakening the tight link between issuance and prices. Unfortunately, their method requires additional computational burden due to the need to find income shock values that leave the sovereign indifferent between discrete borrowing options, potentially many such levels. For our application, we will instead follow Sánchez et al. (2018) and Gordon (2019) and use standard discrete choice methods. We require that the borrowing policies take values in a discrete set and subject each potential choice to an iid draw from a Gumbel distribution, as in the multinominal logit model. In each state $\langle y, b_S, b_L \rangle$ we associate to each choice $\left\langle b'_S, b'_L \right\rangle$ a shock $\varepsilon_{b'_S, b'_L}$ so that now the ex-ante value achieved under repayment is

$$V^p\left(y, b_S, b_L\right) = \mathbb{E}_{\varepsilon_{b'_S, b'_L}} \max\left\{W(y, b_S, b_L, b'_S, b'_L) + \rho\, \varepsilon_{b'_S, b'_L}\right\} \tag{A.1}$$

where ρ is a parameter controlling the precision of the taste shocks. Under standard assumptions concerning the distribution of the shocks, choice probabilities conditional on repayment are given by

$$G\left(b'_S, b'_L \middle| y, b_S, b_L\right) = \frac{\exp\left[W(y, b_S, b_L, b'_S, b'_L)/\rho\right]}{\sum_{x'_S, x'_L} \exp\left[W(y, b_S, b_L, x'_S, x'_L)/\rho\right]} \tag{A.2}$$

In the model we treat the default option symmetrically with the borrowing choices but nest it out in order to address the "blue bus red bus" problem associated with changing the number of grid points for bonds.

$$V\left(y, b_S, b_L\right) = \mathbb{E}_{\varepsilon_d, \varepsilon_p}\left\{V^p\left(y, b_S, b_L\right) + \rho_D\, \varepsilon_p, V^d\left(y, \right) + \rho_D\, \varepsilon_d\right\} \tag{A.3}$$

so that each state is associated with a default probability given by

$$d\left(y, b_S, b_L\right) = \frac{\exp\left[V^d(y)/\rho_D\right]}{\exp\left[V^d(y)/\rho_D\right] + \exp\left[V^p(y, b_S, b_L)/\rho_D\right]} \tag{A.4}$$

Finally, we augment the Nash problem (14) with analogous taste shocks, one for each $\langle \gamma_S, \gamma_L \rangle$ option. As a result, the level of the endowment at the time of the swap does not fully determine the debt level and maturity mix of the exit portfolio, as some uncertainty remains, associated with the taste shocks. The precision parameter associated with the Nash taste shocks is ρ_N.

In the numerical exercise, convergence is achieved under 1,000 iterations, with full updating of the q schedules each iteration, for precision parameters in the 1.0e-5 – 3.0e-5 range. We compute the model under the tightest taste shocks consistent with convergence within 1,000 iterations to at most a 1.0e-5 change in the bond price schedules and 1.0e-6 change in value functions. The reported results are for ρ equal to 1.0e-5 for all precision parameters, for both models (with and without post-swap exclusion). An alternative approach would be to use these precision parameters to target additional quantitative features of the data. We use precision parameter values as small as possible, given convergence criteria, and confirm that the main takeaway (the role of post-swap exclusion in generating maturity extension) is not sensitive to these parameters.

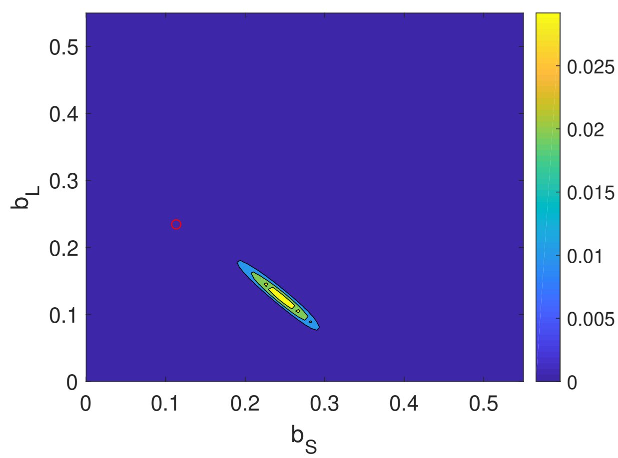


Figure A.7: **Discrete Choice Method.** This figure illustrates the choice probabilities in the model's equilibrium, in a representative state: we pick average endowment and an initial portfolio with an average short-to-long mix (red circle). The heat map represents choice probabilities over $\left\langle b'_S, b'_L \right\rangle$.

Figure A.7 plots representative choice probabilities from the computed model, for a given state. The bulk of the probability mass is tightly centered around the option with the highest associated payoff. The taste shocks induce some degree of additional uncertainty over the maturity composition of the new portfolio and less over the overall level of debt. In this particular example state the sovereign will, on average, increase indebtedness and shift the portfolio into a shorter-term structure. The taste shocks generate some uncertainty about the magnitude of these changes but not their sign.

For both models, the endowment support is discretized over 21 points spanning ±3 standard deviations of the unconditional distribution, while the two bonds are restricted to grids of 120 points each, equally spaced between 0 and 60% of average endowment (normalized to $\overline{y} = 1$).

Appendix B. Robustness

Table B.8 reports quantitative results for robustness exercises. Column [1] is the benchmark result in Table 6, replicated here for convenience. Column [2] labeled "Shorter Exclusion" reports the case where the post-renegotiation exclusion is substantially reduced, by setting η_a to 0.75 (compared to the benchmark value of 0.25). The pattern of maturity extension is unaltered, suggesting that the inability to roll over the debt upon restructuring is key, rather than the expected length of the exclusion. Column [3] eliminates haircut heterogeneity by fixing $\mu_S = 1$. In recovery, short- and long-term bond holders receive equal treatment. The main consequence of this alteration is a decrease in short-term spreads and an increase in long-term ones, as well a a reduction in spread volatility. The main result concerning maturity extension is preserved. Column [4] considers a reduction in aggregate haircuts (across all maturities), implemented by lowering the bargaining power of the sovereign to $\alpha = 0.9$ from the benchmark value of 0.945 This induces an aggregate haircut of 50% (versus tot 65% in the benchmark and the data). As a result, spreads are higher and more volatile, while the maturity extension is somewhat more dramatic, from under 5 years to 10, compared to the change from 6.7 to 10 in the benchmark calibration. Column [5] reports results under an alternative assumption about the direct cost of default, in particular if we assume that the country's endowment is subject to the $h(y)$ penalty function also during the post-renegotiation exclusion spell, not only prior to renegotiation, as in the main analysis. This set of results continues to exhibit maturity extension, from 7.2 to 10 years on average. The overall higher cost of default enables the country to sustain more debt at lower spreads.

Together, these robustness exercises suggest that the key mechanisms responsible for the maturity extension is the incentive to smooth debt service and thus consumption following the resumption of payments, a form of roll-over risk, rather than the calibration of the renegotiation and recovery process. Table B.9 documents the results of robustness checks on the version of the model with immediate return to market. The failure of this version of the model to replicate the maturity extension stylized fact is a robust finding across these alternative calibrations.

	[1] Benchmark Model	[2] Shorter Exclusion	[3] Uniform Haircuts	[4] Lower Agg. Haircut	[5] Continued Penalty
Parameter Change	—	$\eta_a = 0.75$	$\mu_S = 1.0$	$\alpha = 0.9$	—
Targeted					
Std C / Std GDP	1.2	1.2	1.2	1.2	1.3
Corr NX/GDP, GDP	–0.26	–0.27	–26	-0.25	-0.23
Share Short in Total Debt	0.34	0.34	0.34	0.33	0.30
Total Debt to GDP	0.49	0.46	0.49	0.62	0.92
Std Short (1-year)	1.0	1.2	0.8	1.6	0.3
Pre-restruct. Exclusion	3.0	3.0	3.0	3.0	3.0
Post-restruct. Exclusion	4.0	1.3	4.0	4.0	4.0
Haircut, overall	0.65	0.64	0.66	0.50	0.81
Haircut, 1-year	0.78	0.76	0.66	0.63	0.89
Bond Swap					
Duration Pre-restruct.	6.4	5.9	6.7	4.8	7.2
Duration Post-restruct.	10.0	10.0	10.0	10.0	10.0
Haircut, 10-year	0.56	0.51	0.66	0.23	0.78
Spreads (%)					
Mean Short (1-year)	0.3	0.3	0.2	0.4	0.1
Mean Long (10-year)	0.2	0.3	0.3	0.5	0.1
Std Long (10-year)	0.2	0.2	0.1	0.4	0.1

Table B.8: **Robustness Calibrations, Benchmark Model.**

	[1] Immediate Reentry	[2] Shorter Exclusion	[3] Uniform Haircuts	[4] Lower Agg. Haircut
Parameter Change	—	$\eta = 0.33$	$\mu_S = 1.0$	$\alpha = 0.8$
Targeted				
Std C / Std GDP	1.2	1.2	1.2	1.2
Corr NX/GDP, GDP	–0.28	–0.28	–0.27	–0.30
Share Short in Total Debt	0.32	0.29	0.34	0.32
Total Debt to GDP	0.40	0.32	0.40	0.50
Std Short (1-year)	1.1	1.9	1.0	1.1
Pre-restruct. Exclusion	7.0	4.3	7.0	7.0
Haircut, overall	0.62	0.23	0.63	0.40
Haircut, 1-year	0.80	0.64	0.63	0.70
Bond Swap				
Duration Pre-restruct.	7.6	7.8	7.3	8.0
Duration Post-restruct.	5.4	4.9	5.4	6.3
Haircut, 10-year	0.55	0.18	0.63	0.31
Spreads (%)				
Mean Short (1-year)	0.5	0.7	0.5	0.5
Mean Long (10-year)	0.4	0.5	0.5	0.3
Std Long (10-year)	0.2	0.4	0.2	0.2

Table B.9: **Robustness Calibrations, Immediate Reentry Model.**

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