AMS Qualifying Exam (June 2016): Probability Questions

Solve any three of the following four problems.

All problems are weighted equally. On this cover page write which three problems you want graded.

problems to be graded:

__________________________________________________________

Name (PRINT CLEARLY), ID number

__________________________________________________________
1. Let $X$ and $Y$ be two independent exponential random variables with respective parameters $\lambda$ and $\mu$. Assume that $\mu < \lambda$. Let $Z$ be equal to $X$ with probability $\mu/\lambda$ and equal to $X + Y$ with probability $1 - \mu/\lambda$. Find the probability distribution function of $Z$.

2. Two points $X$ and $Y$ are independently and uniformly selected inside of a circle $A$ with radius $r$, i.e., each has density $\frac{1}{\pi r^2}$ inside of the circle. Let $d$ be the distance between $X$ and $Y$. Denote by $B$ the circle centered at $X$ with radius $d$. What is the probability that circle $B$ is contained in circle $A$? *Hint: consider the largest circle centered at $X$ and inscribed in $A$.*

3. Let $X$ and $Y$ be jointly continuous with joint probability density function

$$f(x, y) = \frac{1}{x}, \quad 0 \leq y \leq x \leq 1.$$ 

Find the probability density function of $Z = X + Y$.

4. Let $X$ and $Y$ be two independent integer-valued random variables with $P(X = i) = (e - 1)e^{-i}$, $P(Y = j) = \frac{1}{(e-1)j!}$ for $i, j = 1, 2, \ldots$. Let $U = \max\{U_1, U_2, \ldots, U_Y\}$, where $\{U_i\}$ is a sequence of i.i.d. uniform random variables on $(0, 1)$. Assume that the sequence $\{U_i\}$ is also independent of $X$ and $Y$. Find the probability distribution function of $Z = X - U$. 